Ministry of Higher Education and Scientific Research
Setif 1 University Ferhat Abbas - Algeria
Faculty of Economics, Commerce and Management Sciences
Department of Basic Education

Microeconomics 1 (Lectures & Solved Problems)

Course Pack for First-Year Bachelor's Students

(According to the syllabus prescribed by the Ministry of Higher Education and Scientific Research)

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Content

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Introduction

Introduction:

Microeconomics is one of the two branches of economic theory, which deals with the behavior of individual economic units, such as consumers, workers, and business firms, by explaining how these units make their economic decisions about allocating limited resources, and how they interact in different types of markets. Macroeconomics, the other branch of economic theory, by contrast, deals with aggregate economic quantities, such as interest rates, unemployment, and inflation. Microeconomics has traditionally been one of the most important courses in all economics and business curricula and is a requirement in practically all colleges and universities over the world.

Based on my long experience in teaching “Microeconomics” module for first year bachelor students at the faculty of economics, commerce, and management sciences, Setif 1 University, I have found that the majority of students face significant difficulties in comprehending the content of this module, due to its reliance on mathematical techniques and the economic analysis of concepts and the relationships between different economic variables. For that, this course handout has been designed for those students to provide a clear and simplified explanation of all elements covered in the eight chapters taught during the 1st semester of every academic year. In addition, our goal is to help students apply these concepts in real-world situations.

1. Course Description:

This course is intended to provide students with a thorough understanding of the core concepts and issues of microeconomics. Students will develop understanding of economic models and methods of systematic economic reasoning. The first semester includes seven main chapters, in addition to a preliminary chapter that serves as an introduction to microeconomics. The other remaining chapters focus on the study of consumer and producer behaviors, as illustrated below, in more details, in the course structure section.

2. Learning Objectives:

The aim of this course is to achieve the main following learning objectives:

- Help students to understand and use microeconomic terminology and methods to solve problems, and apply theoretical knowledge in real-world contexts;
- Enable students to acquire a strong foundation that allows them to develop their analytical and interpretive abilities regarding various microeconomic phenomena and behaviors;
- Enable students to understand and use the methods and tools that allow them to analyze the behavior of consumers and producers in the market.

3. Learning Outcomes:

After completing this course, students will be able to:

- Understand the basic concepts, models and tools of microeconomic theory;
- Learn the fundamental topics related to demand and supply sides, measurement of elasticities, government intervention in markets, understanding consumer behavior through depending on both cardinal and ordinal utility theories, as well as comprehending producer behavior through the study of production and cost theories;
- Apply theoretical microeconomic concepts and theories to understand and solve real-life economic problems.

4. Prerequisites:

To effectively study Microeconomics course, students should possess the following knowledge and skills:

- *Familiarity with some essential economic concepts*; such as: scarcity, opportunity cost, demand, supply, market, rationality, utility, marginal analysis, and so on.
- *The ability to think logically and critically*; as the course requires understanding cause and effect relationships, identifying assumptions, drawing conclusions, and using models to predict individual and firm behaviors.

- *Having some basic mathematical skills*; including: algebra (to solve equations and work with formulas), graphs and functions (to understand curves, slopes, and intercepts), and basic calculus, particularly derivatives and optimization (to analyze marginal changes and solve consumer or producer choice problems).
- *Proficiency in English;* as the course is taught entirely in English, therefore, students are expected to have a good level in English to understand the content of lectures and to engage effectively in tutorial sessions.

5. Course Evaluation and Grading:

Throughout the semester, students will be evaluated using several methods that reflect both their continuous engagement and overall understanding of the material. Students will receive two separate grades: one for tutorials, and one for the final exam. The tutorial grade is determined based on three main criteria: attendance, participation, and conducting an in-semester test. The final exam grade is determined based solely on a cumulative written exam conducted at the end of the semester.

- **Tutorial grade**:
  - Attendance: 4 points (20%)
  - Class participation: 6 points (30%)
  - In-semester test: 10 points (50%)
  - **Total**: **20 points (100%)**
- **Final exam grade:**
  - Cumulative written final exam: **20 points (100%)**

The final grade for the course is calculated by giving a weight of **60%** to lectures and **40%** to tutorials.

6. Course Structure:

The first semester includes eight chapters, as outlined below, which are taught through approximately **14** sessions (weeks), starting in mid-September and ending in the third week of December, which marks the beginning of the winter break. The weekly schedule consists of **three (3) hours** allocated for delivering the lectures and **one and a half hours (1.5)** for tutorials.

Chapter 1: Introduction to microeconomics

Chapter 2: Consumer behavior analysis (cardinal utility approach)

Chapter 3: Consumer behavior analysis (ordinal utility approach)

Chapter 4: Demand, supply, and market equilibrium

Chapter 5: The measurement of elasticities

Chapter 6: Market equilibrium applications

Chapter 7: Producer behavior analysis (production theory)

Chapter 8: Producer behavior analysis (costs of production)

Chapter 1: Introduction to Microeconomics

Learning Objectives

By the end of this chapter, students will be able to:

- Understand the meaning of economics and economic problem.
- Understand the meaning of microeconomics and distinguish it from macroeconomics.
- Identify the main themes of microeconomics.
- Explain the key concepts in microeconomics, including opportunity cost, rationality, decisions made at the margin, and efficiency.

1. Meaning of Economics and Economic Problem

1.1. What is Economics?

Economics is *a social science*. However, it is different from other social sciences such as psychology, sociology, and political science, because economists ask different questions, and they answer them using tools that other social scientists find rather exotic (Hall & Lieberman, 2010, p. 1). The origine of this science can be traced to *Adam Smith's* book "An *Inquiry into the Nature and Causes of Wealth of Nations*" published in 1776. The word "Economics" was derived from two Greek words "*oikou*", which means "a house" and "*nomos*", which means "to manage". Therefore, this word was used to mean "home management" with limited funds available in the most economical manner possible (Deepashree, 2018).

Many definitions have been given to clarify the meaning of Economics. For example, (L. Robbins) defined Economics as "*the science which studies human behavior as a relationship between ends and scarce means which have alternative uses*" (Robbins, 1932, p. 15). (P. A. Samuelson) defined it as "*the study of how men and society choose, with or without the use of money, to employ scarce productive resources which could have alternative uses, to produce various commodities over time and distribute them for consumption now and in future among various people and groups of society*" (Samuelson, 1948). For, (R. A. Arnold) economics is "*the science of scarcity; or how individuals and societies deal with the fact that wants are greater than the limited resources available to satisfy those wants*" (Arnold, 2019, p. 3).

From the definitions presented above, we can say that: "economics is the social science that studies the choices that individuals, businesses, governments, and entire societies make as they cope with scarcity of resources."

1.2. Economic Problem

Economic problem is the problem of choice, or the problem of fuller and efficient utilization of the limited resources to satisfy maximum number of unlimited wants. Human beings have wants which are unlimited. But the economic resources to satisfy these unlimited wants are limited. In other words, resources or factors of production (land, labor, capital and entrepreneurship) are scarce. They are available in limited quantities in relation to the demand. Resources are not only scarce but they also have alternative uses. All this necessitates a choice between which goods and services to produce first. So, the basic economic problem is the problem of choice which is created by the scarcity of resources.

There are three fundamental questions that summarize the scope of economic problem: —*what, how* and *for whom* to produce—which are grouped under **the allocation of resources**, which means *how much of each resource is devoted to the production of goods and services* (Deepashree, 2018, pp. 21-22).

- **What to produce?** Due to the limited resources, every economy must decide what goods and services to produce and in what quantities. It has to make a choice of the wants which are important for the society as a whole. Thus, an economy has to decide what goods or services it would produce on the basis of availability of technology, cost of production, cost of supplying and demand for the commodity.
- **How to produce?** It relates to the choice of the most efficient technique of production using the least quantity of scarce resources. That is, choosing that technique of production which will maximize production or minimize costs. Generally, there are two types of techniques of production: labor-intensive and capital-intensive techniques. In labor-intensive technique, more labor and less capital are used. In capital-intensive technique, more capital and less labor are used.
- **For whom to produce?** Goods and services are produced in the economy for those people who have the ability to buy them, which depends on the incomes they earn. More income means more capacity to buy and vice versa. Each person in the society cannot get sufficient income to satisfy all his/her wants. This raises the problem of distribution of national product (the total output generated by the firms within an economy) among different sections of the society in such a way that all of them get a minimum level of consumption.

2. Microeconomics as a Branch of Economics

2.1. What is Microeconomics?

The prefix *micro*- in microeconomics is derived from the Greek word "*mikros*", meaning "*small*." (Browning & Zupan , 2015, p. 2). It is one of the two branches of economics; the other branch is called "macroeconomics." **Microeconomics** *deals with the behavior of individual economic units—consumers, firms, workers, and investors—as well as the markets that these units comprise.* It explains how and why these units make economic decisions. Microeconomics is also known as **Price Theory** since its major subject-matter deals with the determination of price of commodities and factors. **Macroeconomics**, by contrast, *deals with aggregate economic variables, such as the level and growth rate of national output, interest rates, unemployment, and inflation* (Pindyck & Rubinfeld, 2013, p. 4).

Notice that the boundary between macroeconomics and microeconomics has become less and less distinct in recent years. The reason is that macroeconomics also involves the analysis of markets. To understand how the aggregate markets operate, we must first understand the behavior of the individual economic units that constitute them. Therefore, macroeconomists have become increasingly concerned with the microeconomic foundations of aggregate economic phenomena, and much of macroeconomics is actually an extension of microeconomic analysis.

2.2.The Themes of Microeconomics

Much of microeconomics is about explaining how individual economic units can best allocate their scarce resources. It explains how consumers can best allocate their limited incomes to the various goods and services. How workers can best allocate their limited time to labor or leisure. And how firms can best allocate limited financial and technical resources to produce products. The most important themes of microeconomics are the following (Pindyck & Rubinfeld, 2013, pp. 4-6):

- **Trade-Offs:** Microeconomics describes the optimal trade-offs that consumers, workers, and firms face, and shows how these trade-offs are best made.
  - ***Consumers*** can maximize their well-being by trading off the purchase of more of some goods for the purchase of less of others. They can also decide how much of their incomes to save, thereby trading off current consumption for future consumption;
  - ***Workers*** must trade off working now and earning an immediate income for continued education and earning a higher future income. They also face trade-offs in their choice of employment, such as choosing to work for large companies or small ones. In addition, workers must sometimes decide how many hours per period they wish to work, thereby trading off labor for leisure.
  - ***Firms*** must decide what types of products that they can produce and how many of each type to produce. They must also decide whether to hire more workers, build new factories, or do both.
- **Prices and Markets:** All the trade-offs described above are based on the *prices* faced by consumers, workers, or firms. A consumer trades off one good for another based in part on his or her preferences for each one, but also on their prices. Workers trade off labor for leisure based partly on the price (the wage) that they can earn for their labor. Similarly, firms decide whether to hire more workers or purchase more machines based in part on wage rates and machine prices. Microeconomics also describes how prices are determined. In a market economy, prices are determined by the interactions of consumers, workers, and firms in *markets*—collections of sellers and buyers that together determine the price of a good or a service.
- **Theories and Models:** Microeconomics is concerned with the explanation of observed phenomena and making predictions, which are based on *theories*. For example, the theory of the firm explains, based on an assumption that firms try to maximize their profits, how firms choose the amounts of inputs such as labor, capital, and raw materials that they use for production and the amount of output they produce. It also explains how these choices depend on input prices, and the prices that firms can receive for their outputs. The theory of the firm

can also predict whether a firm's output level will change in response to a change in wage rates or the price of raw materials.

With the application of statistical and econometric techniques, theories can be used to construct *models* from which quantitative predictions can be made. For example, a model of a particular firm can be developed and used to predict by how much the firm's output level will change as a result of the change in the price of raw materials.

- **Positive versus Normative Economics:** Positive economics deals with what is, or how an economic problem facing a society is actually solved. It involves facts which can be verified using available data over time. Examples of positive statements are the following:
  - A fall in the price of a good or a service leads to a rise in its quantity demanded;
  - An increase in real per capita income increases the standard of living of people;
  - Minimum-wage laws cause unemployment.

  Positive statements are descriptive. They make a claim about how the world is. Normative Economics deals with what ought to be, or how an economic problem should be solved. It involves values as well as facts which cannot be judged using data alone. It also involves our views on ethics, religion, and political philosophy. Examples of normative statements are the following:
  - Algerian government should spend more money on research and development;
  - Governments should guarantee a minimum wage for every worker;
  - All poor people should get free education and healthcare.

Note that much of economics is positive; it just tries to explain how the economy works. Yet those who use economics often have normative goals. They want to learn how to improve the economy. When you hear economists making normative statements, you know they are speaking as policy advisers not as scientists.

3. Key Concepts in Microeconomics

3.1. The 'Ceteris Paribus' Assumption

Economic analysis often involves variables and how they affect one another. *A variable is a measure of something that can take on different values* (O'Sullivan, Sheffrin, & Perez, 2018). Economists are interested in exploring relationships between two variables— like the relationship between the price of a given good and the quantity purchased of it by the consumers. Of course, the quantity purchased depends on many other variables, including the consumer's income, prices of related goods, tastes and so on. To examine the relationship between the quantity and price of the good, we must assume that all other variables that influence the good's purchases don't change during the time period we're considering. Economists refer to this as "partial" analysis—where everything in the economic environment is held constant except for one thing. Partial analysis rests on the assumption that "all else remains unchanged" or what is called ***ceteris paribus***, *the Latin phrase meaning* ***"****holding all else unchanged*." (Feigenbaum & Hafer, 2012, p. 10).

3.2. Opportunity Cost

We are often forced to make choices, due to the scarcity of resources. When we make a choice, we select from the available alternatives. For example, you can spend some time studying for your next microeconomics test, or having fun with your friends, but you can't do both of these activities at the same time. You must choose how much time to devote to each.

You can think about your choices as *tradeoffs*—giving up one thing to get something else (Parkin, 2012). So, when you choose how to spend your time, you face a tradeoff between studying and hanging out with your friends. *The value of the next-best forgone alternative that was not chosen because something else was chosen* is called **the opportunity cost** (Taylor & Weerapana , 2009, p. 4). For instance, if you choose to study, the opportunity is the fun you miss with your friends. While, if you go out with your friends, the opportunity cost is the knowledge and possibly higher grade from studying (We will also discuss the concept opportunity cost in the last chapter 8, see page 105).

3.3. Rationality

Economists view the choices that economic agents (consumers and firms) make as *rational*, meaning that these agents act in their own *self-interest*. Rationality is about comparing *costs* and *benefits* to achieve the greatest benefit over cost for those making the choice (Parkin, 2012, p. 8). In the neoclassical economy, decisions made by economic agents led to the assumption of rationality of economic entities (*homo-economicus*). Generally, it can be said that homo-economicus attempts to maximize utility as a consumer and profit as a producer (Bernat & Gąsior, 2020, p. 61). A rational consumer must choose what and how much of goods and services to consume to maximize utility given his/her limited income. A rational firm decides the quantity of output to be produced to maximize its profit given the price of inputs or the price to set in a market where it competes with other firms.

3.4. Decisions Made at the Margin

Economists often consider how a small, one-unit change in one variable affects another variable and what impact that has on people's decision making. For example, you might ask: "If I spend one more unit of my time or money on option 1, on option 2, or on option 3, which will give me the greatest net benefit?" This approach is called *marginal analysis*, which requires us to compare the *marginal benefit* (*MB*) to the *marginal cost* (*MC*) of spending an additional unit of a scarce resource in a particular way. MB is the increase in benefit that results when an additional unit of a scarce resource spent on a particular activity. For example, your MB from one more hour studying for your next microeconomics test is the boost it gives to your grade. MC is the increase in cost that results when an additional unit of a scarce resource spent on a particular activity. Your MC of studying one more hour is the cost of not spending it on your favorite leisure activity (Feigenbaum & Hafer, 2012, pp. 37-38).

To make your decisions, you should compare MB and MC. If the MB from an extra hour of study exceeds its MC, you study the extra hour. If the MC exceeds the MB, you don't spend this extra hour studying microeconomics.

3.5. Incentives

The central idea of economics is that the self-interested choices that economic agent make can be predicted by looking at the *incentives* they face. An incentive is something that encourages or motivates a person to undertake an action. Or, undertaking the actions that make a person better off (Arnold, 2019, p. 12). For example, if you have an incentive to study for the upcoming microeconomics exam, we imply that, by studying, you can make yourself better off, probably in terms of receiving a higher grade on the exam than if you didn't study. Incentives are closely related to benefits and costs. People have incentives to undertake actions for which they expect to receive net benefits (the MB exceeds MC).

3.6. Efficiency (Equilibrium or Benefit Maximization)

Efficiency is achieved when the MB equal the MC (Arnold, 2019, p. 9). Suppose you are studying for microeconomics test, your study is worthwhile as long as the MB are greater than the MC, and there will be a net benefit to studying. But you must stop studying when the MB equal the MC. Suppose you reach this equality at four hours of studying. So, four hours are the *efficient* or *optimal* amount of time to study in this situation. At less than four hours, the MB of studying are greater than the MC; thus, at all these hours, studying has net benefits. At more than four hours, the MC of studying are greater than the MB, so studying beyond four hours is not worthwhile. By studying exactly four hours (where MB = MC), you can maximize the net benefit, meaning that efficiency is achieved or you are at the equilibrium point.

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Problems

1. Match each term with the correct definition?

Economics, scarcity, opportunity cost, rationality, economic problem, utility, marginal analysis

a. The satisfaction, pleasure, or happiness obtained from consuming a good or service.

b. The social science that studies the choices that individuals, businesses, governments, and societies make under conditions of scarcity.

c. Making choices based on comparing marginal benefits with marginal costs.

d. The value of the next-best forgone alternative that was not chosen because something else was chosen.

e. The problem of fuller and efficient utilization of the limited resources to satisfy maximum number of unlimited wants.

f. The resources used to produce goods and services are limited.

g. People act in their own self-interest; they try to maximize utility with a given cost or minimize costs of achieving a given satisfaction.

2. Economics is divided into two main branches: microeconomics and macroeconomics. Both branches contain elements of positive economics and normative economics.

a. What is the difference between microeconomics and macroeconomics?

b. Indicate whether each of the following statements applies to microeconomics or macroeconomics?

- The unemployment rate in Algeria was 12.7 percent in 2024.
- An unexpected heavy hailstorm damaged the grape crop and caused its price to rise.
- Air Algérie lowered ticket prices on domestic flights to attract more passengers.
- The European central bank raised interest rates to curb inflation in the Euro zone.
- A clothing manufacturer hired more workers to increase production and meet the growing customer demand.
- Algeria’s non-hydrocarbon exports increased by 6 percent in 2025.

c. What is the difference between positive economics and normative economics?

d. Indicate whether each of the following statements applies to positive or normative economics?

- The government should restrict imports.
- A fall in input prices leads to a rise in quantity supplied of output.
- All firms that produce cigarettes should be taxed by the government.
- Inflation in Algeria has increased during the recent years.
- Algeria should support national production.
- Granting subsidies for local farmers increases domestic food production.

3. A student has three hours in which he can either study or play football.

a. What is the opportunity cost of studying?

Suppose that this student has **DZD 2,000**, and he can eat in a restaurant or buy a book. The student bought the book for **DZD 2,000**.

b. What is the opportunity cost of buying the book?

4. A firm buys a specialized piece of equipment for **$25,000**. What is its opportunity cost if:

a. The firm cannot resell this equipment, because it has no alternative use.

b. The firm could resell this equipment for the same price?

c. The firm could resell this equipment for **$10,000**.

5. Sami has a telephone plan for which the marginal cost (MC) is constant at **DZD 5** per minute for a long-distance call to his mother. (Fractional minutes are billed at the same rate, so a 30-second call would cost him DZD 2.5) The values of marginal benefit (MB) and the total amount of conversation are shown in the following table.

The marginal benefit (DZD)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
The amount of conversation (minutes per month)	**800**	**600**	**500**	**400**	**350**	**300**	**250**	**200**

a. Draw the curves MC and MB in the same set of axes?

b. What is the optimal amount of conversation per month?

c. Explain when should Sami speak longer or less with his mother?

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Chapter 2: The Theory of Consumer Behavior (Cardinal Utility Approach)

Learning Objectives

By the end of this chapter, students will be able to:

- Understand the meaning of utility in general, as well as the meaning of total utility and marginal utility.
- Explain the law of diminishing marginal utility (DMU) and its assumptions.
- Analyze the consumer's equilibrium situation and examine the situations of disequilibrium.
- Understand how a mutually advantageous exchange of utility takes place between two individuals.
- Derive the individual demand curve based on the law of DMU and the principle of the consumer's equilibrium.
- Identify the substitution and income effects associated with a price change of particular good, and understand how they work.

1. The Concept of Utility

1.1. What is Utility

Utility *is the satisfaction, usefulness, or value that a person gets from consuming goods and services (**bundle** of goods and services, or **market basket**).* (Lynham, 2018, p. 88)

It means realized satisfaction to a consumer when s/he is willing to spend money (income) on a commodity which has the capacity to satisfy her/his want.

Utility is a **cardinal concept** *i.e.*, it can be measured. *Frederic Benham* formulated the unit of measurement of utility as **utils** (*i.e.*, say consumption of 2 units of *X* gives 10 utils). According to *Alfred Marshall*, money should be used to measure utility (*i.e.*, say consumption of 2 units of *X* give utility worth DZD 20).

1.2. Total Utility and Marginal Utility

Total Utility (TU) is *the total satisfaction that a person gets from the consumption of all the different goods and services*. **Marginal Utility (MU)** is *the addition to the total utility that results from a one-unit increase in the quantity of a good consumed* (Douglas & Ian, 2017, pp. 127-128). Thus, it measures the additional satisfaction obtained from consuming one additional unit of a good. It is calculated as: $MU_n = TU_n - TU_{n-1}$

Or, $MU = \frac{\Delta TU}{\Delta X}$ ***(X** refers to the quantity of the good**)***

Total utility can also be obtained by the sum of marginal utilities from the consumption of different units of the commodity. $TU_n = MU_1 + MU_2 + ... + MU_n$

Example (1): Suppose that a person consumes units of a given good (X). The table below shows the quantities consumed and the total utility got from the consumption of this good.

Quantity of good (X)	0	1	2	3	4	5	6	7	8	9
TU_x (Utils)	0	8	14	19	23	26	28	29	29	27

a. Calculate MU_X?

b. Draw two graphs showing the curves of TU_X and MU_X?

c. What is the relationship between TU_X and MU_X curves?

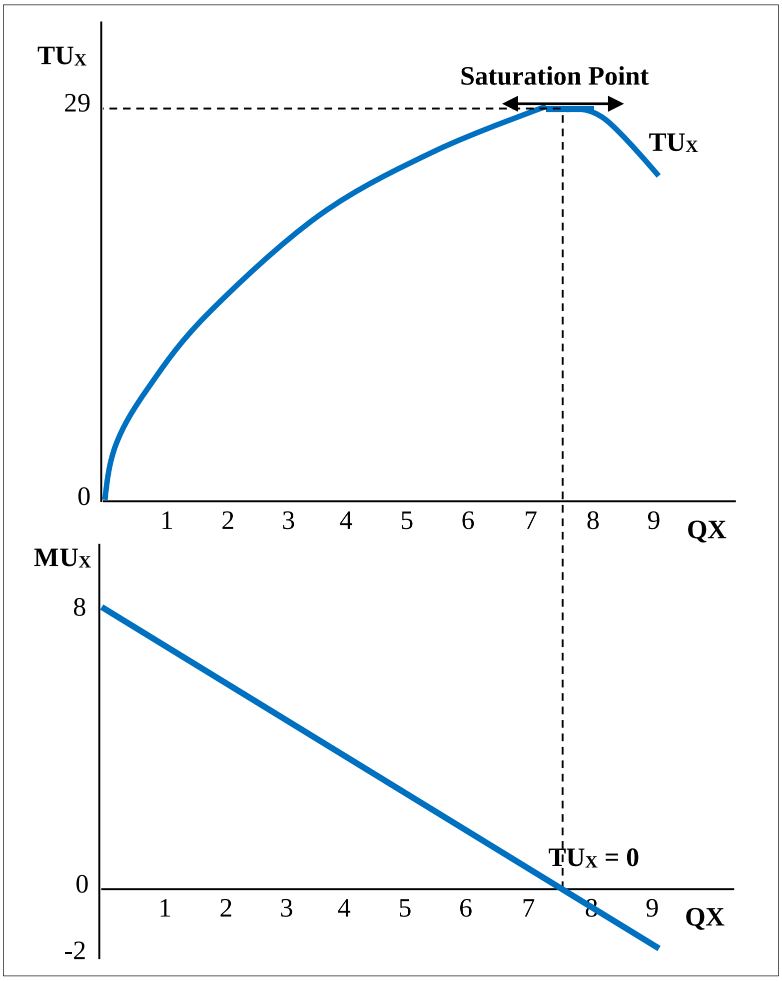
Answer :

a. Calculating MUX : $MU_{X1} = \frac{(8-0)}{(1-0)} = \mathbf{8}$; $MU_{X2} = \frac{(14-8)}{(2-1)} = \mathbf{6}$ **…**

MU_X	—	**8**	**6**	5	4	3	2	1	0	**–2**

b. The graphs of total utility and marginal utility:

Figure 4.1. Total and Marginal Utility Curves



c. The relationship between TUX and MUX curves:

- TU curve starts from the origin, increases at a decreasing rate, reaches a maximum and then starts falling.
- MU curve is the slope of the TU curve, since MU = $\frac{\Delta TU}{\Delta X}$
- When TU is maximum, MU is zero, it is called saturation point. (Since the slope of TU curve at that point is zero). Units of the good are consumed till the saturation point.
- As long as TU curve is concave, MU curve is downward sloping and remains above the x-axis (Q-axis).
- When TU curve is decreasing, MU curve becomes negative.

1.3. The Law of Diminishing Marginal Utility and its Assumptions

The Law of **Diminishing Marginal Utility (DMU)** states that: *"as more of a given good is consumed, the marginal utility associated with the consumption of additional units tends to decline"* (Browning & Zupan, 2015, p. 70).

We can see from Figure 2.1 above that marginal utility declines continuously as the consumer consumes more and more units of the same good. Therefore, the falling MU curve shows the law DMU.

The law of DMU holds good when the following assumptions are satisfied (Deepashree, 2018, p. 52):

- *Using a standard unit of measurement:* Examples of inappropriate units are: rice measured in grams, water in drops, diamonds in kilograms;
- *Homogeneous commodity:* All units of the commodity consumed are homogeneous and perfect substitutes.
- *Continuous consumption:* The consumption of successive units of a good is without a time gap.
- *Mental and social condition of the consumer:* The consumer's mental condition must be normal and good. His income and tastes are unchanged and his behavior is rational.

1.4. Assumptions of the Cardinal Utility Approach

The main assumptions of the cardinal utility approach are as follows (Deepashree, 2018, p. 52):

- Utility can be measured, *i.e.* can be expressed in exact units. It is also measurable in monetary terms;
- Consumer's income is given;
- Prices of commodities are given and remain constant;
- Marginal utility of money, which is addition made to utility of the consumer as he spends one more unit of the income, is assumed to be constant.

2. Consumer's optimal Choice (Consumer's Equilibrium)

2.1. Utility-Maximizing Rule

A **consumer's optimal choice** (**consumer's equilibrium)** is a situation in which a consumer has allocated all of his/her limited income in the way that **maximizes** his/her total utility, given the prices of goods and services in the market.

A consumer's total utility is maximized by satisfying the following two conditions (Parkin, 2012, p. 184):

- ✓ **Equalize the marginal utility per dinar for all goods:** we suppose that the consumer consumes only two goods. So, the consumer buys good (X) up to the point at which marginal utility per dinar spent on it is the same as the marginal utility of a dinar spent on good (Y).
- ✓ **Spend all the available income on goods:** the consumer spends his/her entire income (I) on the two goods (X) and (Y) at given prices P_X and P_Y. Consumer purchases must satisfy the budget constraint.

These two conditions can be written mathematically as:

$$\begin{cases} \dfrac{MU_X}{P_X} = \dfrac{MU_Y}{P_Y} \\ \text{Subject to budget constraint: } I = P_X X + P_Y Y \end{cases}$$

(In case of consuming just one good, the consumer is in equilibrium when he satisfies the following condition: $MU_X = P_X$).

Example (2): Suppose that a consumer buys two goods (X) and (Y), the price of good (X) is P_X = DZD 2, the price of (Y) is P_Y = DZD 1, and his income is I = DZD 12. The table below shows the marginal utility schedule for the two goods.

Q	1	2	**3**	4	5	**6**	7
MU_X	17	14	12	10	8	6	4
MU_Y	12	10	9	8	7	6	5

a. How much of good (X) and good (Y) the consumer should purchase to maximize his utility (the quantities of equilibrium)?

b. Calculate the total utility $TU_{X,Y}$?

Answer:

a. The quantities of equilibrium X_0 and Y_0:

The condition of equilibrium:

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} \Rightarrow \frac{\mathbf{12}}{\mathbf{1}} = \frac{\mathbf{6}}{\mathbf{1}}$$

s. t. $I = P_X X + P_Y Y$ => **12 = 2x3 + 1x6** **X_0 = 3 units ; Y_0 = 6 units**

b. Calculating the total utility $TU_{X,Y}$:

$TU_{X,Y}$ = 17 + 14 + 12 + 12 + 10 + 9 + 8 + 7 + 6 **= 95 utils.**

2.2. Consumer's Disequilibrium Situations

What happens when: $\frac{MU_X}{P_X} \neq \frac{MU_Y}{P_Y}$

There will be two disequilibrium situations which are (Deepashree, 2018, p. 55):

(1) $\frac{MU_X}{P_X} > \frac{MU_Y}{P_Y}$ **:** In this case, the consumer is getting more marginal utility per dinar in case of good (X) as compared to (Y). Thus, he will buy more of (X) and less of (Y). This will lead to fall in MU_X and rise in MU_Y. The consumer will continue to buy more units of (X) till: $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$

(2) $\frac{MU_X}{P_X} < \frac{MU_Y}{P_Y}$ **:** In this case, the consumer is getting more marginal utility per dinar in case of good (Y) as compared to (X). Thus, he will buy more of (Y) and less of (X). This will lead to fall in MU_Y and rise in MU_X. The consumer will continue to buy more units of (Y) till: $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$

3. Exchange

The consumer who is in equilibrium can increase his/her total utility through exchanging goods with other consumers who are also in equilibrium. The condition necessary for mutually advantageous exchange between two individuals (A) and (B) is: $\left(\frac{MU_X}{MU_Y}\right)^A \neq \left(\frac{MU_X}{MU_Y}\right)^B$

The two individuals will gain more total utility from the exchange process. This process will continue until: $\left(\frac{MU_X}{MU_Y}\right)^A = \left(\frac{MU_X}{MU_Y}\right)^B$.

4. From Utility to the Demand Curve

4.1. Deriving the Individual Demand Curve

Based on the law of DMU and the principle of the consumer's equilibrium, we can derive **the individual demand curve** which shows *the quantity of a good demanded by the individual consumer at each different price*. The law of DMU explains why demand curves are generally downward sloping. As we have seen before, according to this law, marginal utility decreases with each successive additional units of the good consumed. Since consuming a good further gives lesser marginal utility, consumers will be willing to pay a lesser price for the good or service. Hence, demand curves are downward sloping because more of a commodity is purchased only at a lower price, consumers will choose to buy other goods that give higher utility if the price of the commodity remains high.

As a result, we find that when the price of the good rises, the quantity demanded of that good falls and vice versa. This means that there is an inverse relationship between the price and the quantity demanded. So, the individual demand curve is normally downward sloping. (We will discuss the topic of demand in more detail in chapter 4).

Example (3): From the same data of example (2), we assume that the price of good (X) falls from 2 to DZD 1, holding the price of good (Y) and income constant.

a. What are the new optimal quantities of equilibrium?

b. Derive the demand curve of the consumer?

Answer:

a. The new optimal quantities of equilibrium:

$$\frac{MU_X}{P_X'} = \frac{MU_Y}{P_Y} \Rightarrow \frac{6}{1} = \frac{6}{1}$$

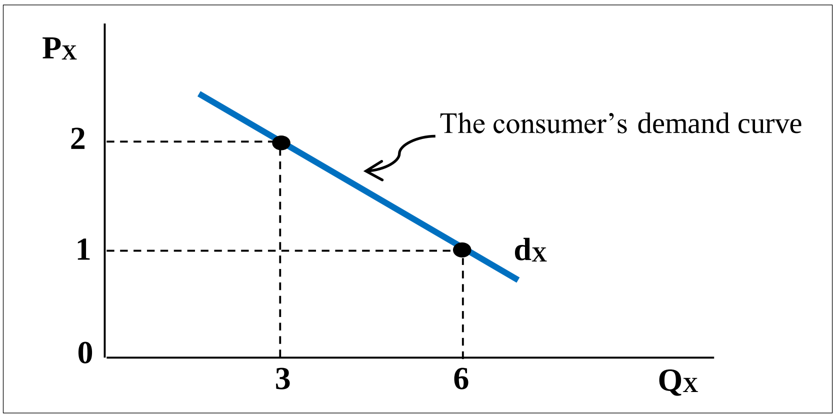
s. t. $I = P_X'X + P_YY \Rightarrow 12 = 1x6 + 1x6$ $X_0' = Y_0' = 6$ **units.** $TU_{X,Y}' = 119$ **utils.**

b. Deriving the consumer's demand curve:

In example (1): When: $P_{X1} = 2_{DZD}$, $Q_X = 3$ **units**

In example (2): When: $P_{X1} = 1_{DZD}$, $Q_X = 6$ **units**

Figure 2. 2. The consumer's Demand Curve



Note that **the price elasticity of demand** for good (X), (we will study the topic of elasticity in chapter 4), determines the demand for good (Y). That is, if:

- The price elasticity of demand for good (X) is ***less than 1*** (the demand curve is ***relatively inelastic***), then the quantity demanded of good (Y) ***increases***;
- The price elasticity of demand for good (X) is ***equal to 1*** (the demand curve is **unit-elastic**), then the quantity demanded of good (Y) ***remains unchanged***;
- The price elasticity of demand for good (X) is ***more than 1*** (the demand curve is ***relatively elastic***), then the quantity demanded of good (Y) **decreases**.

4.2. Substitution Effect and Income Effect

When the price of a good changes, we can identify two separate effects on quantity demanded (substitution effect and income effect). These two effects are also related to the demand curve, as they explain the inverse relationship between the quantity demanded and the price. That is, why the demand curve is downward sloping.

4.2.1. Substitution Effect

The substitution effect of a change in the price of a good is the change in the quantity consumed as the consumer *substitutes* the good that has become relatively cheaper in place of the good that has become relatively more expensive. The substitution effect always moves quantity demanded in the opposite direction to the price change. When price decreases, the substitution effect works to increase quantity demanded; when price increases, the substitution effect works to decrease quantity demanded.

4.2.2. Income Effect

The income effect of a change in the price of a good is the change in the quantity consumed that arises from a change in *the consumer's purchasing power* (the consumer's real income). A drop in price increases purchasing power, while a rise in price decreases purchasing power.

How will a change in purchasing power influence the quantity of a good demanded? That depends on whether the good is **normal** or **inferior**. An *increase* in the real income, due to the fall of price, will *increase* the demand for *normal goods*, and *decrease* the demand for *inferior goods* and vice versa.

4.2.3. Combining Substitution and Income Effects

Now let's look again at the impact of a price change, considering the substitution and income effects together. To help clarify this, we'll consider the total impact of a price change on different types of goods.

- **<u>Normal Goods</u>:** When the price of a normal good ***falls***, the substitution effect ***increases*** quantity demanded. The price drop will also increase the consumer's purchasing power, and—for a normal good—***increases*** quantity demanded even further. The opposite occurs when price ***rises***:

The substitution effect ***decreases*** quantity demanded and the decline in purchasing power (the income effect) further ***decreases*** it. *So, for normal goods, the substitution and income effects for a price change work together (they reinforce each other), causing quantity demanded to move in the opposite direction of the price. Normal goods, therefore, must always obey the law of demand (the demand curve is downward sloping).*

- **Inferior Goods:** When the price of an inferior good ***falls***, the substitution effect would work, as always, to ***increase*** quantity demanded. The price drop will also increase the consumer's purchasing power, and—for an inferior good—***decrease*** quantity demanded. The opposite happens when price ***increases***: The substitution effect ***decreases*** quantity demanded, but the decline in purchasing power will ***increase*** it. *So, for inferior goods, the substitution and income effects of a price change work against each other (they oppose each other). The substitution effect moves quantity demanded in the opposite direction of the price, while the income effect moves it in the same direction as the price. But since the substitution effect virtually always dominates, consumption of inferior goods—like normal goods—will virtually always obey the law of demand (the demand curve is downward sloping).*
- **Giffen goods:** Giffen goods are inferior goods for which the income effect *dominates* the substitution effect. Therefore, demand curves for these goods slope upward.

  Note that if the negative income effect *equals* the positive substitution effect, a change in price leaves the quantity demanded unchanged, the demand curve will be vertical.

******

Problems

1. Suppose that an individual consumes only one commodity (**X**). The table below shows the total utility schedule that the consumer receives at each quantity level.

Q_X	0	1	2	3	4	5	6
TU_X	0	40	70	90	100	100	90

a. Derive the marginal utility (MU_X) schedule?

b. Plot the TU_X and the MU_X schedules, and indicate the saturation point?

c. Explain the relationship between TU_X and MU_X curves?

2. Suppose that a student has only **9** hours left to cram for final exams and he wants to get as high an average mark as possible in three courses: microeconomics, mathematics, and accounting. His mark in each course depends on the time devoted to studying the subjects in the following manner.

Microeconomics		Accounting		Statistics	
Hours of study	mark	Hours of study	mark	Hours of study	mark
0	0	0	14	0	9
1	7	1	15.5	1	13
2	11	2	16.9	2	14.5
3	13.6	3	18.2	3	15.9
4	15.4	4	19.3	4	17.2
5	17	5	19.6	5	18.4
6	18.5	6	19.8	6	18.8
7	18.5	7	19.8	7	19

a. How many hours should this student devote to studying each subject?

b. What is the possible high mark that can be got in each course?

3. Ahmed has **$70** a month to spend, and he can spend as much time as he likes playing golf and tennis. The price of an hour of golf is **$10**, and the price of an hour of tennis is **$5**. The table shows Ahmed's marginal utility from each sport.

Hours per month	Marginal utility from golf (MU_G)	Marginal utility from tennis (MU_T)
1	80	40
2	60	36
3	40	30
4	30	10
5	20	5
6	10	2
7	6	1

a. Indicate the budget constraint of Ahmed?

b. What are the optimal quantities of golf and tennis? And what is the total amount of utility received by Ahmed from these quantities?

c. If "tennis" referred to savings, how would the equilibrium condition be affected?

4. The table below gives the marginal utility schedules of individual (**A**) and individual (**B**) for commodity (**X**) and commodity (**Y**). Suppose that initially individual (**A**) is consuming **6** units of (**X**) and **3** units of (**Y**), while individual (**B**) is consuming **1** unit of (**X**) and **5** units of (**Y**).

	Individual (A)		**Individual (B)**	
Q	**MUx**	**MUy**	**MUx**	**MUy**
1	24	36	**27**	33
2	22	33	22	28
3	20	**30**	18	24
4	18	27	16	20
5	16	24	14	**17**
6	**14**	21	12	15

a. Is there any basis for mutually advantageous exchange between individuals **A** and **B**? If so, determine the direction of exchange between them?

b. To which extent the exchange process continues between the two parties, if the rate of exchange is one unit of good (**X**) against one unit of good (**Y**)?

5. Maria has **$200** to spend on cellphones and sunglasses. Each cellphone costs **$100** and each pair of sunglasses costs **$50**. The table below gives Maria's total utility derived from the consumption of the two goods.

Quantity of cell phones	**Total utility (utils)**	**Quantity of sunglasses**	**Total utility (utils)**
0	**0**	**0**	**0**
1	**500**	**1**	**300**
2	**980**	**2**	**550**
3	**1280**	**3**	**700**
4	**1530**	**4**	**800**

a. Which market baskets of cellphones and sunglasses can Maria consume to maximize her utility?

b. Suppose that the price of cell phones falls to **$50** each, but both the price of sunglasses and income remain constant. Calculate the new market basket of cell phones and sunglasses in this case?

c. Describe the substitution effect and the income effect of this fall in the price of cellphones. (Cellphones are a normal good.)

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Chapter 3: The Theory of Consumer Behavior (Ordinal Utility Approach)

Learning Objectives

By the end of this chapter, students will be able to:

- Understand the meaning of ordinal utility and state its main assumptions as a new approach of studying consumer preferences.
- Identify the concept of indifference curve and describe its key properties.
- Explain the concept of marginal rate of substitution (MRS) and analyze its diminishing nature.
- Understand the budget constraint and illustrate how the changes in prices and income affect the budget line.
- Determine the consumer's equilibrium using indifference curve approach and analyze disequilibrium situations.
- Describe the price-consumption curve (PCC) and income-consumption curve (ICC) and explore how to derive the demand curve from the PCC and Engel curve from the ICC.
- Analyze the substitution and income effects graphically for normal, inferior, and Giffen goods' cases.

1. Ordinal Utility (Indifference Curve) Approach and its Assumptions

1.1. What is Ordinal Utility?

Ordinal utility is the satisfaction that a consumer receives from consuming combinations of goods and services. Unlike cardinal utility which can be measured, ordinal utility can be ranked in order of most to least preferred, and all combinations that give the same level of satisfaction can be represented in form of indifference curve which is a graphic picture of a consumer's tastes and preferences. The consumer is indifferent among all the different combinations of goods on the same indifference curve, but prefers points on a higher indifference curve to points on a lower one.

1.2. Assumptions of Ordinal Utility Approach

The main assumptions of ordinal utility approach are (Nicholson & Snyder, 2012, p. 89; Pindyck & Rubinfeld, 2013, p. 70):

- **Completeness:** Consumers can compare and rank all possible baskets. For any two market baskets (*A)* and *(B)*, a consumer will prefer *(A)* to *(B)*, will prefer *(B)* to *(A)*, or will be *indifferent* between the two.
- **Transitivity:** If good (*X)* is preferred to good (*Y)* and good (*Y)* is preferred to good (*Z)*, then good (*X)* is preferred to good (*Z).*
- **Rationality:** The consumer aims at maximizing his benefits from consumption, given his income and prices of the goods.
- **More is better than less (Non-satiation):** Goods are assumed to be desirable—i.e., to be "***good***"*.* Consequently, consumers are never satisfied or satiated and they always prefer more of a good than less of it.

2. The Concept of Indifference Curve and its Properties

2.1. What is an Indifference Curve?

An indifference curve (or **Iso-utility curve)** is "*a curve that represents all combinations of market baskets that provide a consumer with the same level of satisfaction, so that the consumer is indifferent as to the particular combination s/he consumes"*. (Koutsoyiannis, 1979, p. 18)

We assume that the consumer consumes only two goods (*X*) and (*Y*). So, we define the difference curve, more precisely, as *the curve showing different combinations of two goods that yield the same level of utility or satisfaction to the consumer*.

<u>Example (1):</u> Assume that a person consumes good (X) and good (Y). The table below shows several market baskets consisting of various amounts of the two goods.

Market baskets	A	B	C	D	E	F
Q_X	1	2	1	2	4	3
Q_Y	6	4	10	6	4	8

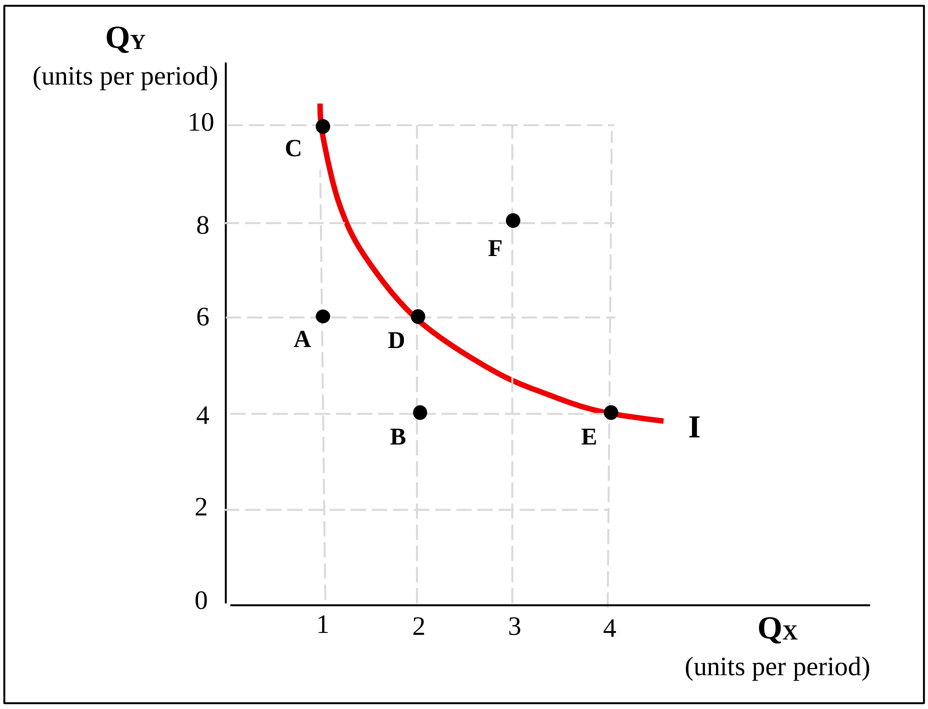
a. Draw the indifference curve **(I)** that includes baskets C, D, and E?

b. Do these three baskets provide the same or a different level of satisfaction to the consumer?

<u>Answer:</u>

a. The indifference curve (I) that includes baskets C, D, and E:

Figure 3.1. An Indifference Curve

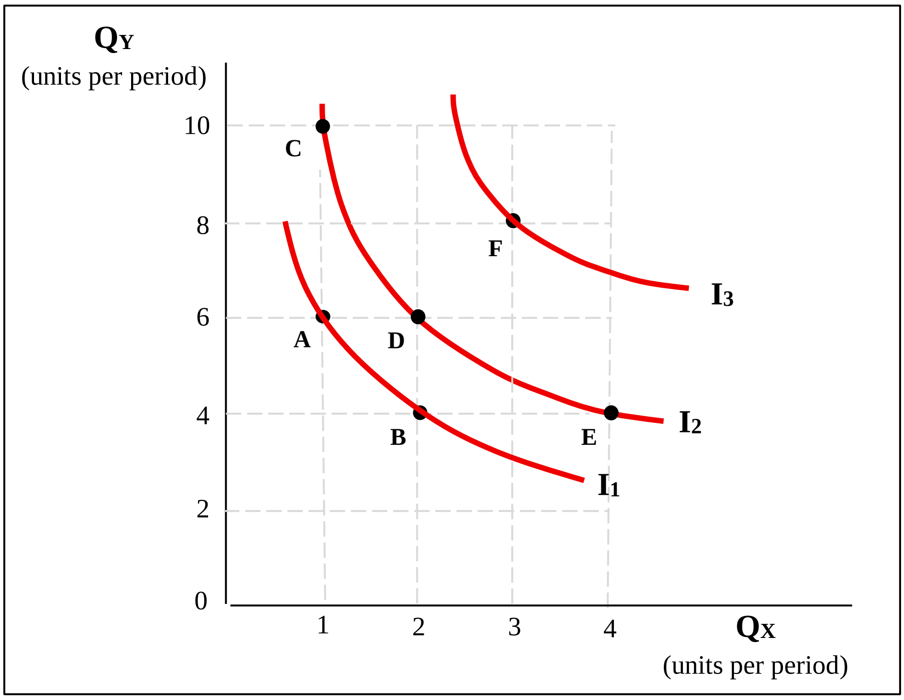


b. Market baskets C, D, and E provide the same level of satisfaction to the consumer, because they lie on the same indifference curve— the consumer views them as being equally desirable. Those baskets are preferred to both baskets A and B, but baskets F is preferred to all other baskets.

2.2. Indifference Map and its Properties

An **indifference map** is *a set of indifference curves which rank the preferences of the consumers* (Koutsoyiannis, 1979, p. 18). Figure 3.2 shows three indifference curves that form part of an indifference map.

Figure 3.2. Indifference Map



All ordinary indifference curves must have four important properties (Deepashree, 2018, pp. 57-59):

- **Downward Sloping to the Right:** This is because if the quantity of one good is reduced then the quantity of the other good is increased;
- **Convex to the Origin:** Because of diminishing the slope of an indifference curve (diminishing Marginal Rate of Substitution of X for Y ($MRS_{X,Y}$) that we will explain it next).
- **Indifference curves cannot cross each other:** This follows from the assumptions of transitivity and non-satiation.
- **A higher indifference curve represents a higher level of satisfaction:** Baskets on indifference curves farther from the origin are preferred to those on indifference curves closer to the origin.

3. The Marginal Rate of Substitution

3.1.The Concept of Marginal Rate of Substitution (MRS)

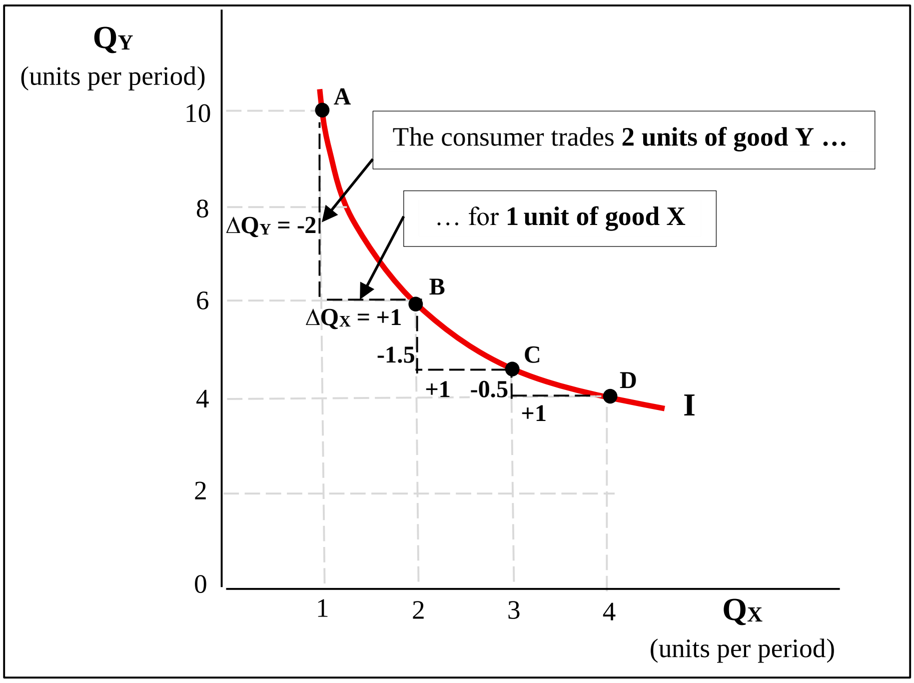
The marginal rate of substitution ($MRS_{X,Y}$) is *the rate at which a consumer will give up one good (Y) to get one additional unit of another good (X), while maintaining the same level of satisfaction (remaining on the same indifference curve)* (Besanko & Braeutigam, 2020).

This definition means that the consumer can substitute one of the goods to another with keeping the same level of total utility. The total gain in utility associated with the increase in Q_X must balance the loss due to the lower consumption of Q_Y, so, mathematically, the total change in satisfaction must equal zero. $MU_X(\Delta Q_X) + MU_Y(\Delta Q_Y) = 0$

We can rearrange this equation to find $MRS_{X,Y}$ equation: $MRS_{X,Y} = -\frac{\Delta Q_Y}{\Delta Q_X} = \frac{MU_X}{MU_Y}$

As the consumer gives up more and more of *Y* to obtain more of *X*, the marginal utility of *X* falls and that of *Y* increases, so MRS decreases. The magnitude of the slope of the indifference curve along any one of its segments measures the MRS between two goods, as shown in the Figure 3.3.

Figure 3.3. The marginal rate of substitution



The slope of the indifference curve between A and B is: $\frac{\Delta Q_Y}{\Delta Q_X}$ = **-2**, between B and C is **-1.5**, and between C and D is **-0.5**. So, the indifference curve has a negative slope and gets flatter as we move down it to the right—that is, it is downward sloping and convex to the origin. (The two first properties of an indifference curve for ordinary goods.)

3.2. Diminishing Marginal Rate of Substitution

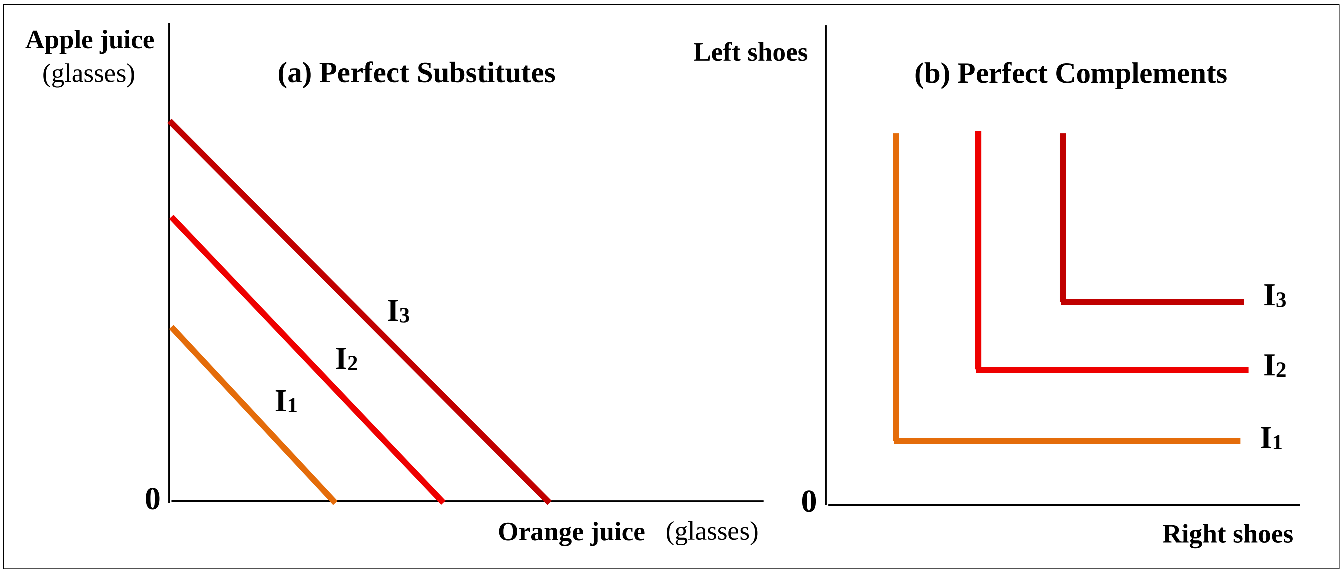
The MRS varies along a typical indifference curve that is convex to the origin. The term *convex* means that the slope or MRS of the indifference curve becomes smaller in absolute value: the consumer will give up fewer of good (Y) to obtain one more unit of good (X) as we move down and to the right along the curve. This willingness to trade fewer of good (Y) to obtain one more unit of good (X) reflects **a diminishing MRS:** The *MRS* approaches zero—becomes flatter or less sloped—as we move from basket A to basket D in the Figure 3.3.

3.3. Perfect Substitutes and Perfect Complements

The shape of an indifference curve tells us about the consumers' willingness to trade one good for the other and remain equally well off. An indifference curve with a different shape implies a different willingness to substitute. To see this principle, look at the two extreme cases: perfect substitutes and perfect complements (Pindyck & Rubinfeld, 2013, pp. 75-76)

- **Perfect Substitutes:** *goods that can be used interchangeably, so that a consumer is completely indifferent as to which to consume*. For example: **apple juice** and **orange juice**.
- **Perfect Complements:** *goods that go together and must be jointly consumed in a precise combination*. For example: **left shoes** and **right shoes**.

Figure 3.4. Perfect Substitutes and Perfect Complements



In panel (a), the consumer views orange juice and apple juice as perfect substitutes. S/he is always indifferent between a glass of one and a glass of the other. In panel (b), the consumer views left shoes and right shoes as perfect complements. An additional left shoe gives him/her no extra satisfaction unless s/he also obtains the matching right shoe.

4. The Budget Constraint

After examining the consumer's preferences for various goods. Now, we turn to understanding the budget constraint, or how consumer's income and the prices that must be paid for various goods limit choices.

4.1. The budget line

The **budget line** indicates *all combinations of good (X) and good (Y) for which the total amount of money spent is equal to income* (Pindyck & Rubinfeld, 2013, p. 82). Because we are considering only two goods (and ignoring the possibility of saving), the consumer will spend his entire income on the two goods. As a result, the combinations of those goods that s/he can buy will all lie on this line: $\mathbf{I = P_X X + P_Y Y}$

With income and prices are fixed. The quantity of good (Y) can be expressed as a function of the quantity of good (X): $\mathbf{Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} X}$

Taking the derivative of Y with respect to X, then yields: $\mathbf{\frac{dY}{dX} = -\frac{P_X}{P_Y}}$

This shows the slope of the budget line equals the negative of the price ratio of the two goods, which is called **the Marginal Rate of Transformation (MRT).** So, **the budget line's slope** is:

$$\mathbf{MRT = \frac{dY}{dX} = -\frac{P_X}{P_Y}}$$

The slope of the budget line indicates *how many units of the good (Y) a consumer must give up to get one more unit of the good (X) with spending the same amount of income*.

<u>Example (2):</u> Suppose that a consumer has an income of DZD 400, the price of good (X) is $5 per unit, and the price of good (Y) is $10 per unit. The following table shows various combinations of the two goods that s/he can purchase with her/his income.

Market basket	A	B	C	D	E
Good (X)	0	20	40	60	80
Good (Y)	40	30	20	10	0

a. Draw the budget line associated with the market baskets given in table above?

b. Can the consumer purchase the baskets: **F** (40, 10), and **G** (100, 10) with the same income?

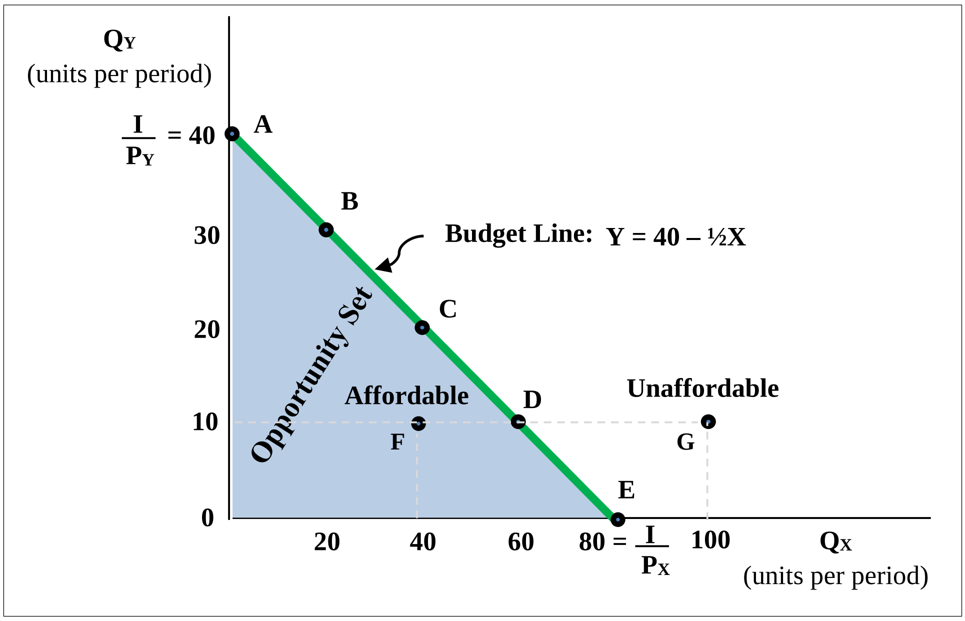
c. Find the budget line equation?

d. What is the maximum amount of one good that can be purchased if none of the other is bought?

<u>Answer:</u>

a. The budget line associated with the market baskets A, B, C, D, and E:

Figure 5.5. The Budget Line



The budget line (AE) is a straight-line and the consumer can purchase any basket that lies on it, such as baskets A, B, C, D, and E.

b. The consumer can purchase the basket **F** (40, 10), because it lies inside the line. Thus, it requires an outlay less than the available income (I = 5x40 + 10x10 = DZD **300**). While, this consumer cannot purchase the basket **G** (100, 10), because it lies outside the line. This means that it requires an outlay larger than the available income (I = 5x100 + 10x10 = DZD **600**) and is therefore beyond reach (unaffordable). Any basket on, or inside the budget line can be purchased (affordable). All the baskets a consumer can buy, including all those inside and on the budget line are called **the opportunity set** (the blue-shaded area in the Figure).

c. The budget line equation is: $\mathbf{I = P_X X + P_Y Y \Rightarrow 400 = 5X + 10Y \Rightarrow \Rightarrow Y = 40 - \frac{1}{2}X}$

The slope of the line is constant at – ½, (**MRT** = $\frac{\Delta Y}{\Delta X} = \frac{P_X}{P_Y} = -\frac{1}{2}$), indicating that a move from basket to another involves sacrificing ½ unit of good (X) to gain one additional unit of good (X) without changing the total amount of money spent.

d. The maximum amount of one good that can be purchased if none of the other is bought is shown in the intercepts with the two axes. Point A at the vertical axis indicates that **40** units ($\frac{I}{P_Y} = \frac{400}{10}$) of good (Y) can be bought at a price of DZD 10 per unit. Similarly, point G at the horizontal axis indicates that **80** units ($\frac{I}{P_X} = \frac{400}{5}$) of good (X) can be bought at a price of DZD 5 per unit.

4.2. Shifts in Budget Line

The budget line, as we have seen, depends both on income and on the prices of the goods. But prices and income often change. Let's see how such changes affect the budget line (Browning & Zupan, 2015, pp. 54-55):

- **Income Changes:** A change in income (with prices unchanged) causes the budget line to shift parallel to the original line (L_1), but **doesn't change its slope** because prices have remained unchanged.

Figure 3.6. Effects of a Change in Income on the Budget Line

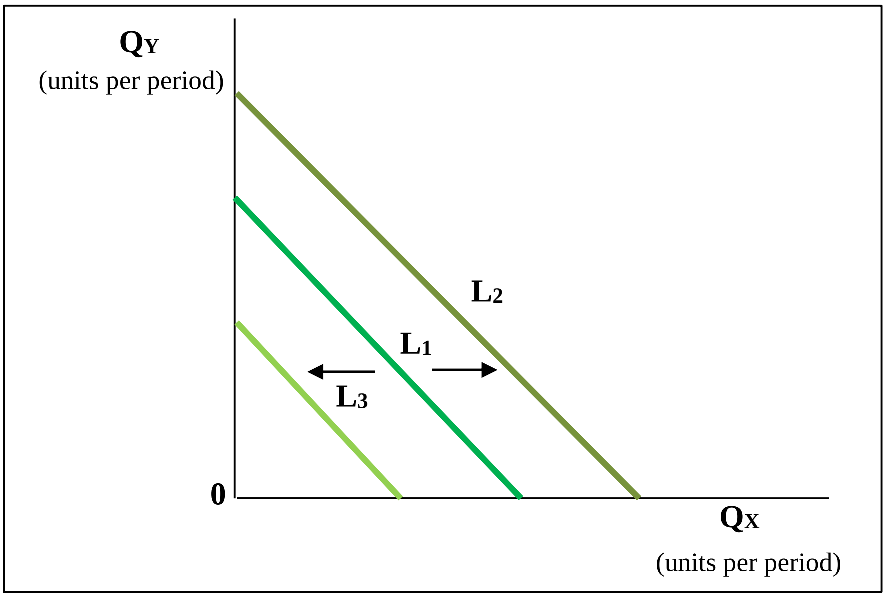


Figure 3.6 shows that if income increases, the budget line **shifts outward**, from budget line L_1 to budget line L_2. Likewise, if income decreases, the budget line **shifts inward**, from L_1 to L_3.

- **Price Changes:** A change in the price of one good (with the price of the other good and income unchanged) causes the budget line to rotate about one intercept, but with **a change in its slope** because the price has changed.

Figure 3.7. Effects of a Change in Price on the Budget Line

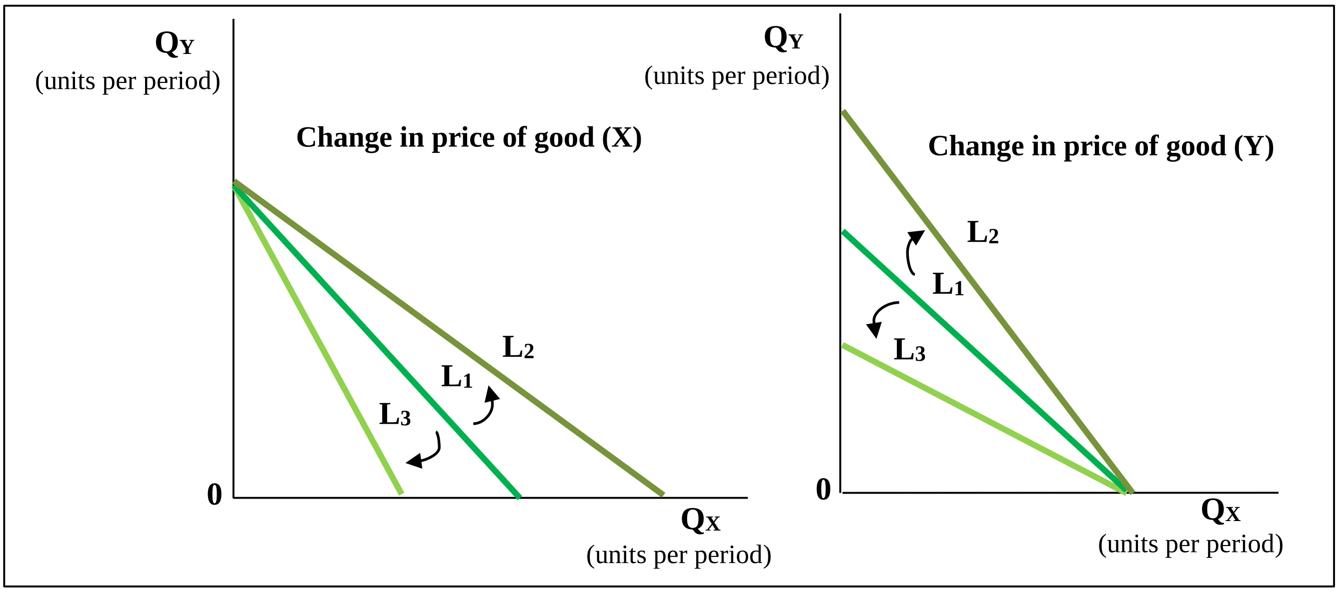


Figure 3.7 shows that if the price of good (X) falls, we obtain the new budget line L_2 by **rotating the original budget line L_1 outward**, pivoting from the C-intercept and become *flatter*. On the other hand, when the price of good (X) rises, the budget line **rotates inward**, to line L_3 because the consumer's purchasing power has diminished, pivoting from the C-intercept and becomes *steeper*. In the same way, we can illustrate the changes in the price of good (Y) with the price of good (X) and income held constant.

5. The consumer's Optimal Choice (Consumer's Equilibrium)

Indifference curves represent the consumer's preferences toward various market baskets; the budget line shows what market baskets the consumer can afford. Putting these two together, we can determine what market basket the consumer will actually choose.

5.1.Utility Maximization (Consumer's Equilibrium) Condition

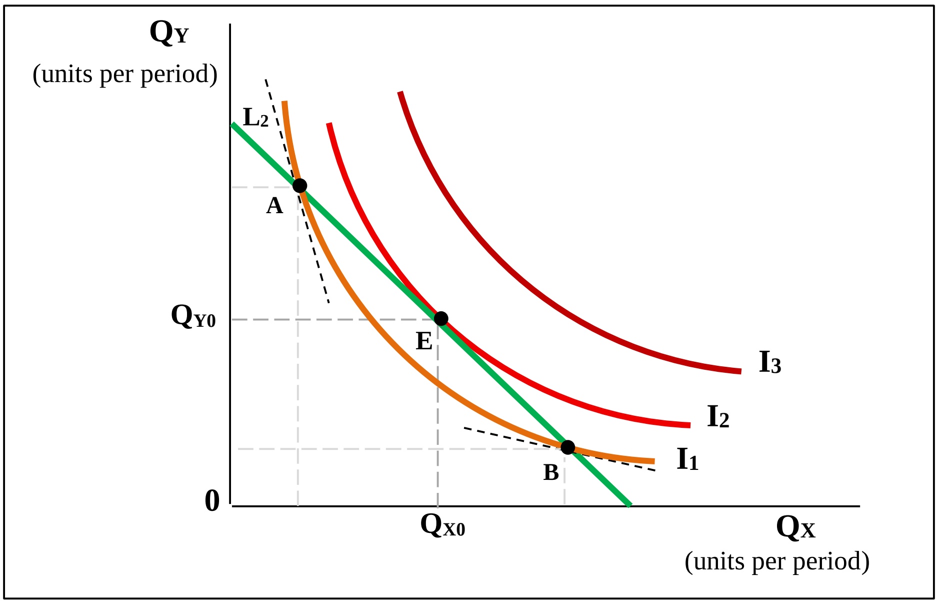
A consumer is in equilibrium when s/he *maximizes* her/his utility (satisfaction), given income and market prices. In other words, *equilibrium is attained when the consumer reaches the highest possible indifference curve given his/her budget constraint* (Salvatore, 2006, p. 67).

The highest indifference curve attainable is the one that just is **tangent to** the budget line. That means the slopes of the two curves must be equal. Thus, the condition of the consumer's optimal choice (given the budget constraint), can be expressed by the following equality:

$$\mathbf{MRS}_{X,Y} = \mathbf{MRT}_{X,Y} \Rightarrow \mathbf{MRS}_{X,Y} = \frac{\mathbf{P_X}}{\mathbf{P_Y}}$$

On the other hand, the MRS (the slope of the indifference curve) measures the **Marginal Benefit**— *the value the consumer derives from consuming one more unit of a good*. The price ratio (the slope of the budget line) measures the **Marginal Cost**— *the cost of consuming one more unit of a good*. Therefore, satisfaction is maximized when **the marginal benefit is equal to the marginal cost** (Pindyck & Rubinfeld, 2013, p. 87)**.**

Figure 3.8. Maximizing Consumer Satisfaction



The consumer will choose basket **E** **(Q_{X0}, Q_{Y0})** on indifference curve **I_2**. Because this indifference curve is tangent to the budget line **L_2** at this point, and the slopes of the two curves are equal.

5.2. Consumer's Disequilibrium Situations

When: $\mathbf{MRS_{X,Y}} \neq \frac{\mathbf{P_X}}{\mathbf{P_Y}}$, there will be two disequilibrium situations which are (Deepashree, 2018, p. 63):

(1) $MRS_{X,Y} > \frac{P_X}{P_Y}$: This case is shown by the point **A** in Figure 3.8. It means that the marginal benefit (the MRS) is greater than the marginal cost (the price ratio), the consumer will be better off consuming more of good (X) and fewer of good (Y). Thus, by moving along the budget line, the consumer will reach a higher indifference curve I_2 and the MRS falls till it becomes equal to the ratio of prices and the equilibrium is established at the point E.

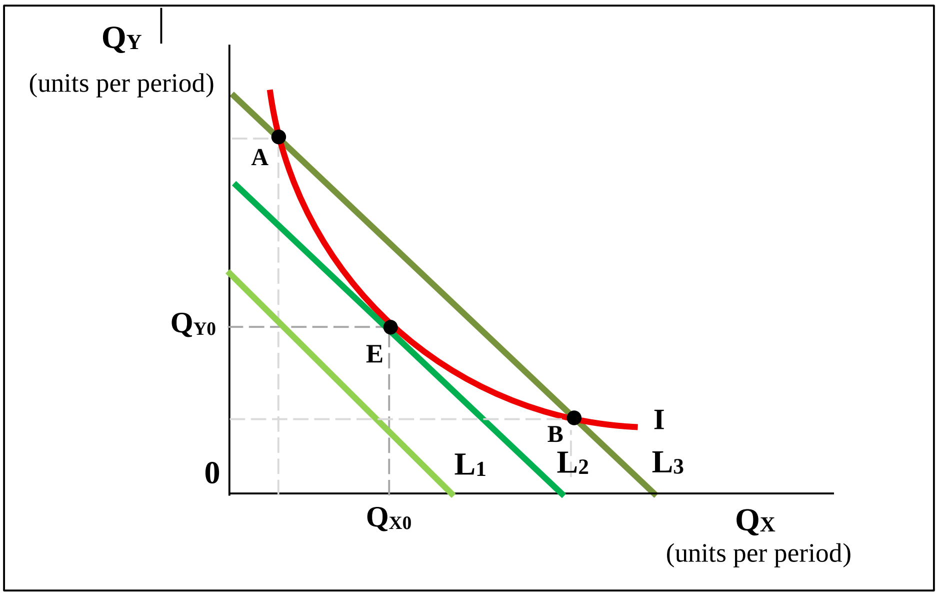
(2) $MRS_{X,Y} < \frac{P_X}{P_Y}$: This case is shown by the point **B** in figure 3.8. It means that the marginal benefit (the MRS) is less than the marginal cost (the price ratio), the consumer will be better off consuming fewer of good (X) and more of good (Y). Thus, by moving back along the budget line, the consumer will reach a higher indifference curve I_2 and the MRS rises till it becomes equal to the ratio of prices and the equilibrium is established at the point E.

5.3. Minimizing Expenditure

In Figure 3.8, we showed that the consumer maximizes his/her utility at the point E, where the indifference curve touches the budget line. In a ***dual*** constrained minimization problem, the consumer wants to find that combination of goods that achieves a particular level of utility U* for the lowest possible expenditure (income) (Nicholson & Snyder, 2012, p. 130).

Figure 3.9 shows three possible budget lines L_1, L_2, and L_3. The lowest of these budget lines L_1 lies below the indifference curve I, so the consumer cannot achieve the level of utility on I for such a small expenditure. The budget line L_3 crosses I at baskets A and B, so these two baskets are not ones that minimize consumer's expenditure. The budget line L_2 is tangent to the indifference curve I, and this is the rule for minimizing expenditure while achieving a given level of utility. Therefore, the point of tangency E is the same as in Figure 3.8.

Figure 3.9. Minimizing the Expenditure



We conclude that solving either of the two problems—maximizing utility subject to a budget constraint, or minimizing the expenditure subject to maintaining a given level of utility—yields the same optimal values for this problem.

5.4. The Mathematics behind Consumer's Optimal Choice

In previous sections, we have just shown how to use a graphical approach to choose the optimal basket that maximizes the consumer's satisfaction or minimizes his/her expenditure. We now express this choice problem mathematically, using calculus to find this optimal basket.

- **Maximizing Utility:**

The consumer's preferences which are represented graphically by a set of indifference curves can also be represented mathematically by a function which is called, an **ordinal utility function:** $TU_{x,y} = f(X,Y)$

The budget constraint is given by the following equation: $I = P_X X + P_Y Y$

There are **two methods** that we can use to solve the problem of the consumer's choice in order to find the optimal quantities of the two goods that maximize the consumer's total utility given the limitation posed by a fixed income.

1) The Condition of Tangency Method:

We have found that the consumer maximizes his/her utility at the basket where the indifference curve is tangent to the budget line (the slopes of the two curves are equal). We can express this mathematically by the following condition:

$$\begin{cases} MRS_{X,Y} = MRT_{X,Y} \Rightarrow \dfrac{MU_X}{MU_Y} = \dfrac{P_X}{P_Y} \quad \ldots(1) \quad \left(MU_X = \dfrac{dTU}{dX} \text{ and } MU_Y = \dfrac{dTU}{dY}\right) \\ \text{s. t. } I = P_X X + P_Y Y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2) \end{cases}$$

From equation (1) we find one of the quantities of goods as a function of the other (new equation 3). Then we substitute it into the equation (2) to find the optimal quantity of good one. By plugging this quantity into equation (3) we find the optimal quantity of good two.

2) The Lagrangian Multiplier Method

The consumer problem is to choose the quantities of the two goods that yield the greatest utility possible given the limitation posed by a fixed income. Mathematically, this can be expressed as a problem of constrained maximization: maximize utility subject to the budget constraint. **The Lagrangian multiplier technique** is the most straightforward way to solve this problem.

We begin by forming the Lagrangian expression (L): $L = f(X,Y) + \lambda(I - PxX - PyY)$

The first order conditions for a maximum are that the partial derivatives with respect to X, Y, and λ, and then equating them to zero. The resulting equations are:

$$\frac{dL}{dX} = \frac{df}{dX} - \lambda P_X = 0; \quad \left(\frac{dL}{dX} = MU_X - \lambda P_X = 0\right)$$

$$\frac{dL}{dY} = \frac{df}{dY} - \lambda P_Y = 0; \quad \left(\frac{dL}{dY} = MU_Y - \lambda P_Y = 0\right)$$

$$\frac{dL}{d\lambda} = I - PxX - Py.Y = 0;$$

The three equations can be rewritten as:

$$\frac{MU_X}{P_X} = \lambda \qquad \frac{MU_Y}{P_Y} = \lambda \quad => \quad \frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} \quad => \quad \frac{\mathbf{MU_X}}{\mathbf{MU_Y}} = \frac{\mathbf{P_X}}{\mathbf{P_Y}}$$

$$\mathbf{I = P_xX + P_yY}$$

The solution to the consumer's optimal choice will be the same as the condition of tangency method.

Note: We can derive **the consumer's demand functions for good (X) and good (Y),** which give quantities as a function of the two unknowns, prices and the consumer income, by following the same way in finding the optimal quantities that maximize the total utility of the consumer.

$$X = f(P,I)$$

$$Y = u(P,I)$$

- **Minimizing Expenditure:**

We can use calculus to solve the expenditure-minimizing problem. The consumer's objective is to minimize his expenditure E, subject to the constraint that holds his/her utility constant at U^*:

$$\begin{cases} \text{Min } E_{X,Y} = P_xX + P_yY \\ \text{s. t. } U^* = U(X,Y) \end{cases}$$

The solution of this problem is an expression of the minimum expenditure as a function of the prices and the specified utility level: $E = f(P_X, P_Y, U^*)$

We call this expression the **expenditure function**: the relationship showing the minimal expenditures necessary to achieve a specific utility level for a given set of prices.

Note that we can use the Lagrangian multiplier method, but we must form the expression as follows:

$$L = P_xX + P_yY + \lambda[U^* - f(X,Y)]$$

Then, we solve for the optimal values that minimize the consumer's expenditure at a given utility, with the same way as we have seen before.

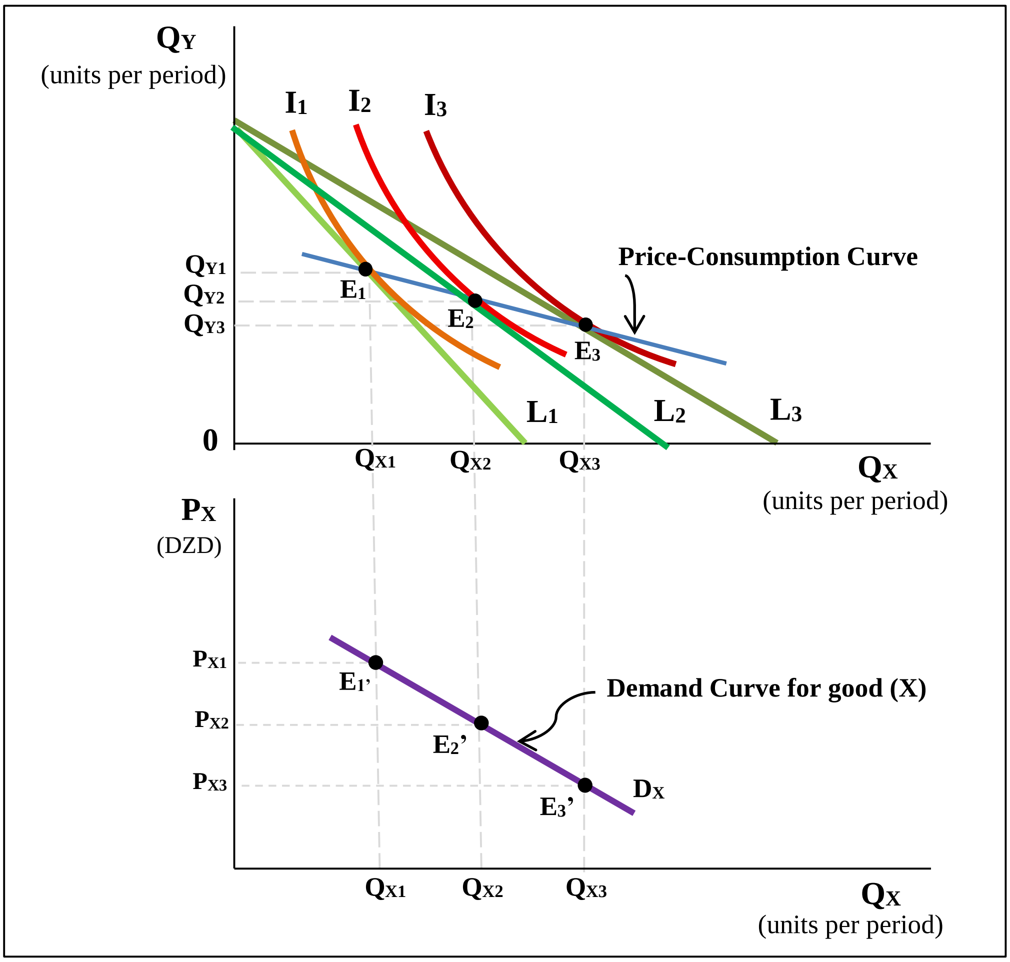
6. Price-Consumption Curve and Income-Consumption Curve

6.1. Price-Consumption Curve

The price-consumption curve (PCC) is *the curve that traces all the optimal consumption points generated by the change in the price of one of goods consumed with other variables (the price of the other good, income, and tastes) remain constant* (Frank, 2008, p. 96).

Figure 3.10 shows the PCC, which is the line that passes through points E_1, E_2, E_3, assuming that the price of good (X) *falls,* from P_{X1} to P_{X2} then to P_{X3}. By varying the price of the good, we can also trace out the consumer's **demand curve** for that good. For each possible price of good (X) on the vertical axis, we record on the horizontal axis the quantity of good (X) demanded by the consumer. Points E_1', E_2', and E_3' on the demand curve correspond to baskets E_1, E_2, and E_3 on the PCC. The demand curve for good (X) is downward sloping, as the law of demand predicts.

Figure 3.10. Price-Consumption Curve and Deriving the Consumer's Demand Curve



Note that, in Figure 3.10, the PCC is downward sloping. But this curve may also slope upward, horizontal, or it may be U-shaped. On the other hand, the slope of a consumer's PCC can provide important information about the elasticity of demand and the relationship between the two goods (Browning & Zupan, 2015, p. 95; Chyrak, 2018, p. 58):

- When the PCC is *negatively sloped*, demand is *elastic* and the two goods are *substitutes*.
- When the PCC is *positively sloped*, demand is *inelastic* and the two goods are *complements*.
- When the PCC has *a slope of zero (horizontal)*, demand is *unit elastic*;
- When the PCC is *U-shaped*, demand is *elastic at high prices and inelastic at low prices.*

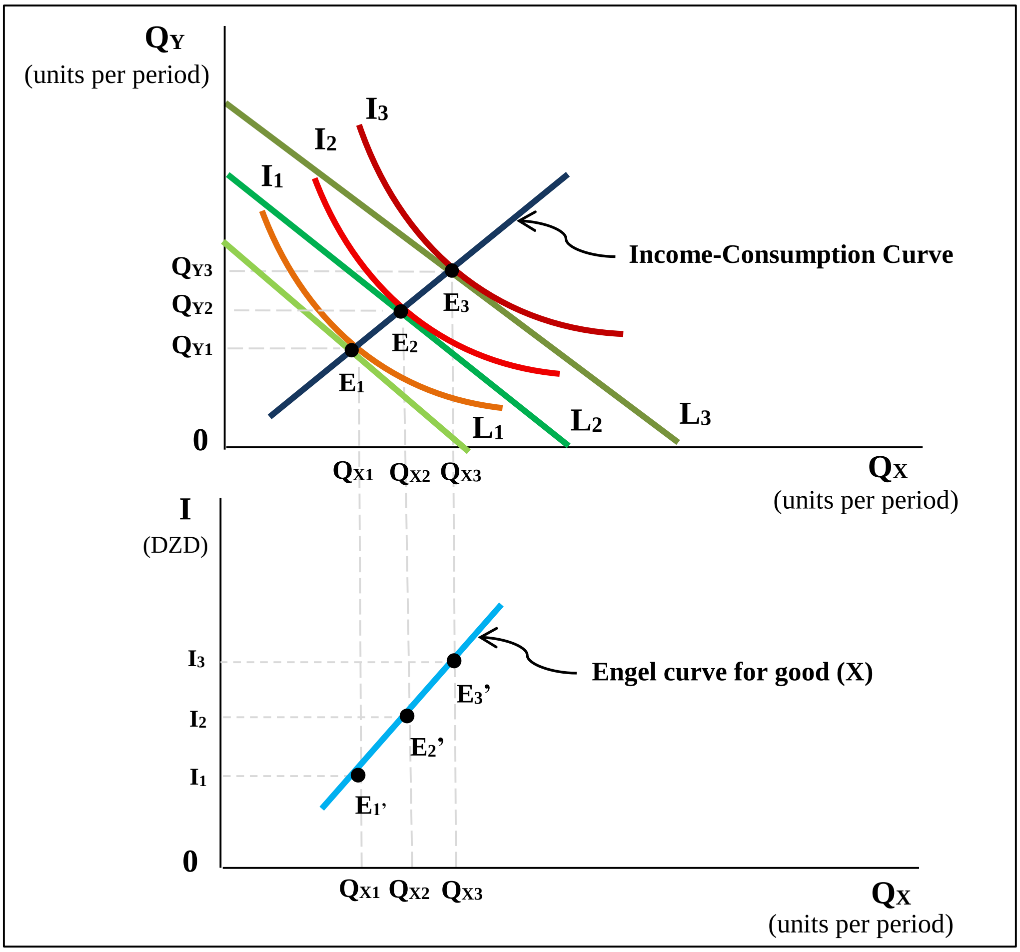
6.2. Income-Consumption Curve

The income-consumption curve (ICC) is *the curve that traces all the optimal consumption points generated by the change in income with other variables (the prices of goods, and tastes) remain constant* (Frank, 2008, p. 98).

The effect of a change in income on the good differs according to the type of that good; whether it is a *normal good*, or an *inferior good*.

- **Normal goods:** for a normal good, if the consumer's income increases, for example from I_1 to I_2 and then to I_3, holding prices and tastes constant, the consumer buys more of that good. Figure 3.11 shows the ICC which is the line that passes through points E_1, E_2, E_3. We can also use the ICC to construct **Engel curve,** which *shows the relationship between the quantity demanded of a single good (X) and income, holding prices and tastes constant* (Frank, 2008, p. 99).

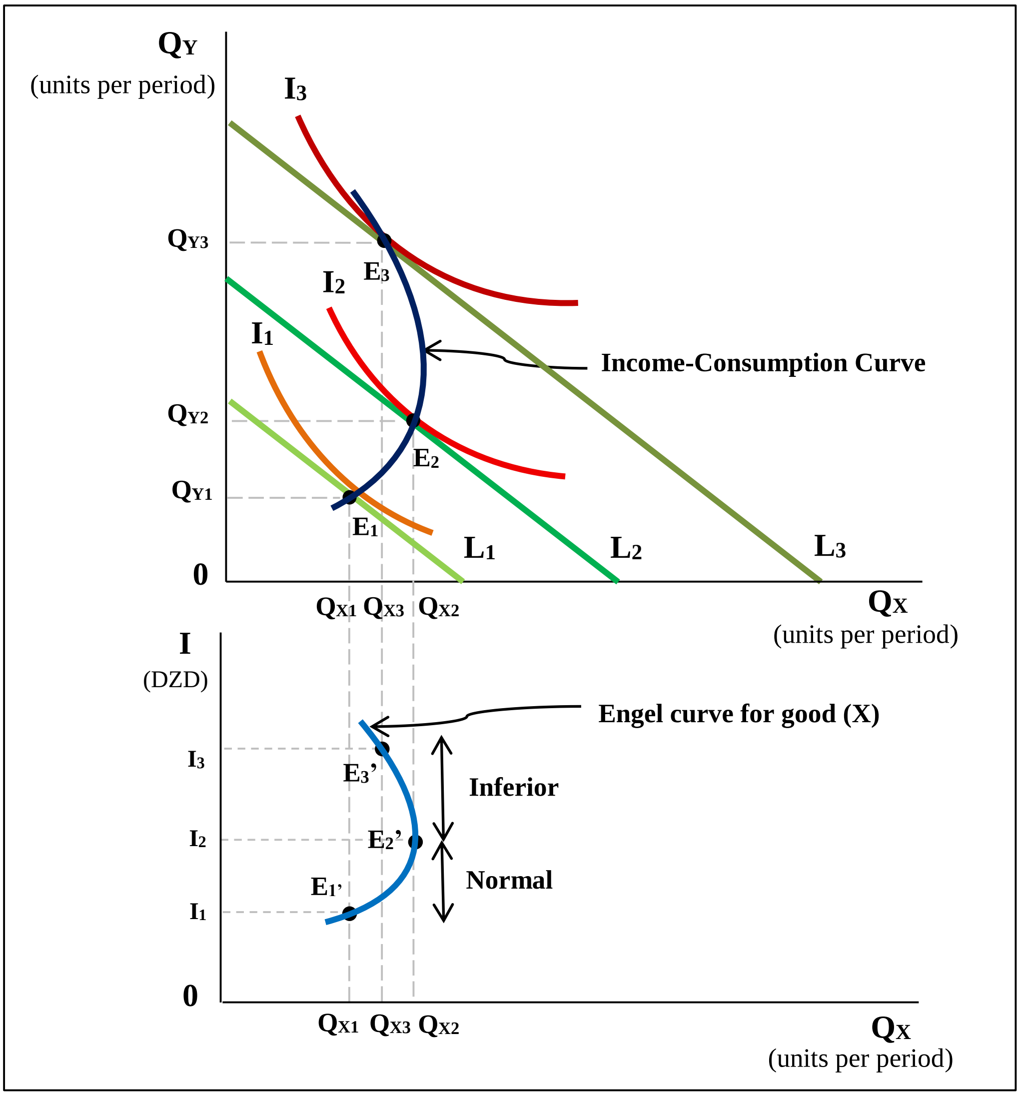
Figure 3.11. Income-Consumption Curve and Deriving Engel Curve of *a Normal Good*



The upward-sloping Engle curve implies that an increase in income leads to an increase in quantity demand of the good which, in turn, means that good (X) in this case, is a *normal* good.

- **Inferior goods:** we consider that good (X) is an inferior good, like "fast-food meals". We will see what happens if the consumer's income rises from I_1 to I_2 and then to I_3, holding prices and tastes constant.

Figure 3.12. Income-Consumption Curve and Deriving Engel Curve of *an Inferior Good*



If the income rises from I_1 to I_2, then the consumption of good (X) will increase from Q_{X1} to Q_{X2} units. The ICC connecting the optimal points E_1, E_2 is positively sloped, implying that consumption increases when income rises. Then, if the income continues to rise to I_3, then the consumption of good (X) drops to Q_{X3} units. The ICC connecting the optimal points E_2, E_3 is now negatively sloped after bending backward, implying that consumption decreases when income rises.

The corresponding Engel curve for good (X) is upward sloping when the income rises from I_1 to I_2, which means that good (X) is a *normal good* in this range. But for income greater than I_2, Engel curve is downward sloping which means that good (X) has become an *inferior good*.

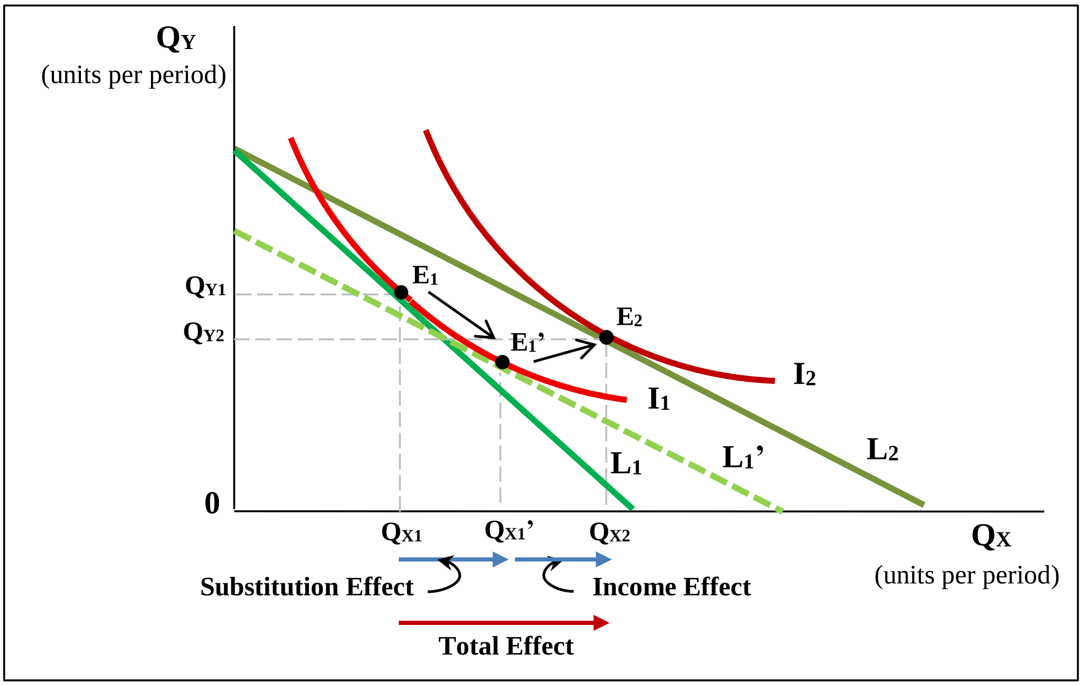
7. Substitution Effect and Income Effect

In the last section of chapter 2, we have discussed the substitution effect and income effect of a price change for both normal and inferior goods' cases. In this section we will illustrate these two effects graphically using budget lines and indifference curves.

7.1. Substitution and Income Effects: The Normal Good Case

To illustrate this case, we assume that the price of good (X) decreases, holding the price of good (Y) and the consumer's income constant.

Figure 3.13. Substitution and Income Effects of a Price Reduction: *Normal Goods*



When the price of good (X) falls, the consumption of this good increases from Q_{X1} to Q_{X2} units, and the consumer moves from basket E_1 on budget line L_1 and indifference curve I_1 to the new basket E_2 on budget line L_2 and indifference curve I_2. This increase in consumption represents **the total effect (TE)**, which can be decomposed conceptually into two effects—**the substitution effect (SE)** and **the income effect (IE)** (Mankiw, 2015; Besanko & Braeutigam, 2020).

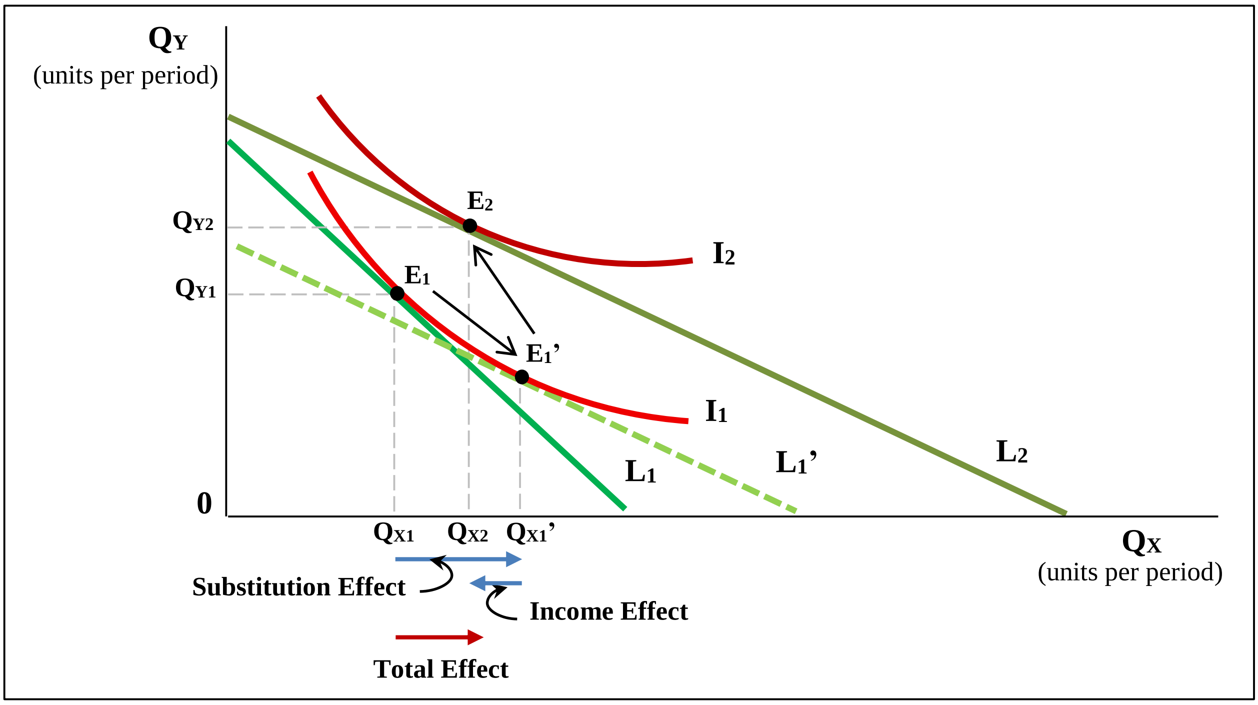
The SE can be obtained by drawing an imaginary budget line L_1' which is parallel to L_2, and is just tangent to I_1. This substitution is marked by *a movement along the original indifference curve I_1* from basket E_1 to E_1', and leads to *an increase* in consumption of good (X) from Q_{X1} to Q_{X1}' and reduces consumption of good (Y), holding the level of satisfaction constant.

The IE is shown by the change in consumption when the consumer moves from point E_1' to E_2. This change involves a parallel shift outward from L_1' to L_2 which means that there is an increase in the consumer's income (an increase in the consumer's purchasing power due to the reduction in price). Thus, the IE causes consumption of good (X) to *rise* from Q_{X1}' to Q_{X2} (a *positive effect* because it is *a normal good*), and this effect reflects *a shift from one indifference curve to a higher one*, with increasing the level of satisfaction. Both SE and IE reinforce each other to increase quantity demanded of the good. In this case the consumer's demand curve for good (X) slopes downward.

7.2. Substitution and Income Effects: Inferior Good Case

We assume that the price of good (X), as an inferior good, falls, holding the price of good (Y) and the consumer's income constant.

Figure 3.14. Substitution and Income Effects of a Price Reduction: *Inferior Goods*

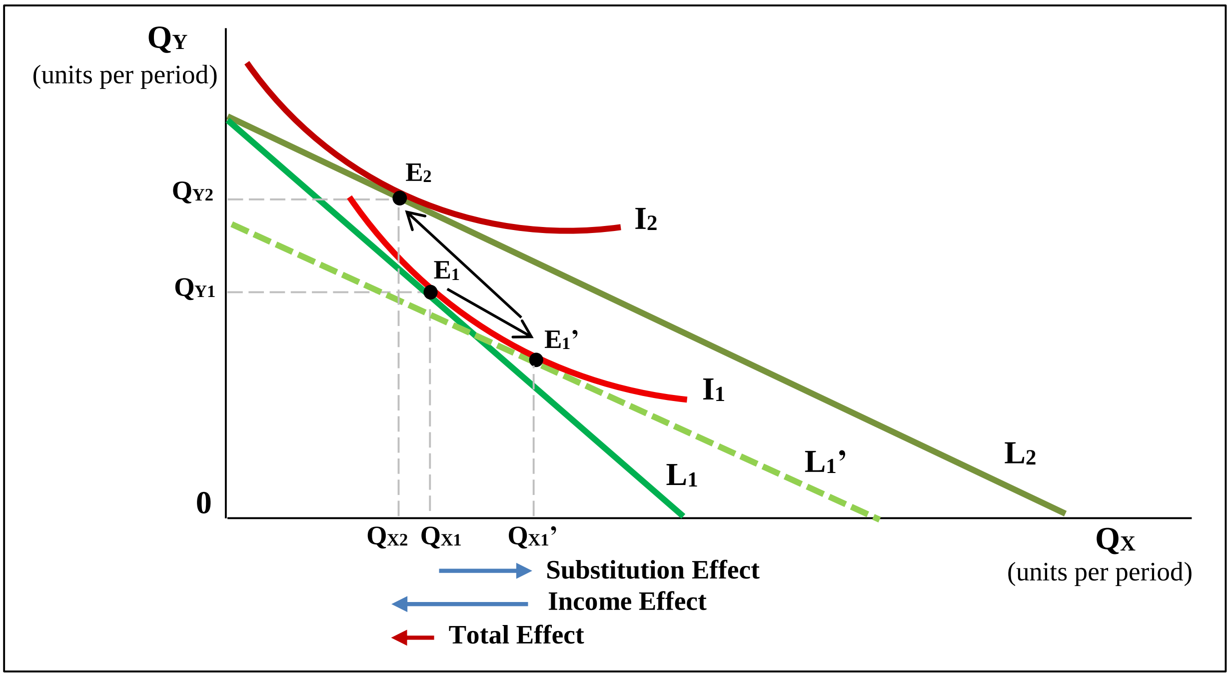


The same analysis applies here, as explained in the case of a normal good. However, since the good (X) is *an inferior good*, the IE *reduces* the consumption of this good from Q_{X1}' to Q_{X2}, (*negative effect*). Overall, the TE increases the quantity demanded of the good, even though the SE and IE oppose each other, this is because for most inferior goods, the SE is *larger than* the negative IE. In this case *the consumer's demand curve for good (X) also slopes downward*.

7.3. Substitution and Income Effects: Giffen Good Case

For an inferior good there is another special case: the Giffen good, illustrated in the following figure.

Figure 3.15. Substitution and Income Effects of a Price Reduction: *The Giffen Good*



As we mentioned in chapter 2, Giffen good is an inferior good, thus, a reduction in its price causes the same effects as in the case of inferior goods. However, the IE for Giffen good not only works in the opposite direction, but also *dominates* the SE. Because the IE *more* than offsets the SE. In this case *the consumer's demand curve for Giffen good will slope upward*.

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Problems

1. Suppose that 'Adam' is a rational consumer, and he has ranked many combinations of two goods (**X**) and (**Y**). His preferences are represented by three indifference curves I_1, I_2, and I_3, as shown in the table below.

Combinations	I_1		I_2		I_3	
	X	**Y**	**X**	**Y**	**X**	**Y**
A	**2**	**13**	**3**	**12**	**5**	**12**
B	**3**	**6**	**4**	**8**	**5.5**	**9**
C	**4**	**4.5**	**5**	**6.3**	**6**	**8.3**
D	**5**	**3.5**	**6**	**5**	**7**	**7**
E	**6**	**3**	**7**	**4.4**	**8**	**6**
F	**7**	**2.7**	**8**	**4**	**9**	**5.4**

The price of good (**X**) is **$2** per unit, the price of good (**Y**) is **$1** per unit, and the individual's money income is **$12** per time period which is all spent on the two goods.

a. Draw the three indifference curves and the budget constraint line on the same set of axes?

b. Find, geometrically, the point at which Adam will be in equilibrium?

c. Suppose that Adam's money income rises from **$12** to **$16** and then to **$20** per time period, and the price of the two goods remain unchanged. Derive the income-consumption curve and the Engel curve for this consumer?

d. What is the nature of good (**X**)?

2. Sarah receives her utility from consuming food (***F***) and clothing (***C***) as given by the utility function:

$$TU_{(F,C)} = FC$$

Suppose that Sarah's income is $**1200** and that the prices of food and clothing are $**2** and $**10** per unit respectively.

a. What are the optimal quantities of food and clothing that maximize the satisfaction of Sarah? Calculate the value of total utility $TU_{F,C}$?

b. What is Sarah's Marginal Rate of Substitution ($MRS_{F,C}$) when utility is maximized? Explain it?

c. Find the demand equations for food and clothing as a function of the two prices and income?

d. Suppose that Sarah's income rises to **1500**, and the prices remain unchanged. What combination of food and clothing should she buy to maximize her utility in this case? What is the type of each good?

3. A consumer spends his entire income on two goods (***X***) and (***Y***). His preferences are represented by the utility function: $TU_{(X,Y)} = 3X^2Y$

In addition, the price of good (***X***) is **DZD 10** per unit, the price of good (***Y***) is **DZD 5** per unit.

a. What is the amount of minimum income spent on the two goods to receive a total utility of **3000**?

b. Suppose that (***I***) is the consumer's income, P_X and P_Y are the prices of good (***X***) and good (***Y***) respectively. Derive the demand equations for the two goods as a function of prices and income? What is the relationship between the two goods?

4. A woman receives her utility from consuming two goods (***X***) and (***Y***) as given by the following utility function:

$$TU_{(X,Y)} = \sqrt{XY} = X^{1/2}Y^{1/2}$$

The price of good (***X***) is $**60** per unit, and the price of good (***Y***) is $**40** per unit.

a. Find the marginal utility functions (MU_X) and (MU_Y)?

b. Is the market basket (**X = 5, Y = 10**) her optimal basket or not? Justify your answer?

c. Suppose that the consumer's income is $**1200**. Calculate the optimal basket that maximizes her total utility? What is the level of total utility?

d. Calculate (MU_X) and (MU_Y) at the point of equilibrium?

5. The utility that 'Ali' receives from consuming good (**X**) and good (**Y**) is given by:

$$TU_{(X,Y)} = X^2Y + 4$$

a. Derive the demand functions for good (**X**) and good (**Y**)?

b. What is the optimal basket of the two goods? Knowing that the price of good (**X**) is £**2** per unit, the price of good (**Y**) is £**4** per unit, and Ali's income is £**24**.

c. Calculate the total utility at the equilibrium point? Find the indifference curve equation at this point?

d. Suppose that the price of good (**X**) *doubled*, while the price of good (**Y**), and income remain constant. What is the new optimal basket of the two goods in this case? What is the curve that traces the utility maximizing combinations of the two goods as the price of good (**X**) increases?

e. Separate the substitution effect and income effect resulting from the rise in the price of good (**X**)? Draw a graph to show these effects?

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Chapter 4: Demand, Supply, and Market Equilibrium

Learning Objectives

By the end of this chapter, students will be able to:

- Understand the main aspects related to the demand topic by defining individual demand and its features, explaining the law of demand, determining the factors affecting demand, identifying individual demand function, schedule, and curve, distinguishing between a change in quantity demanded and a change in demand, and comprehending how to move from individual demand to market demand.
- Understand the main aspects related to the supply topic as well by defining individual supply, explaining the law of supply, determining the factors affecting supply, identifying individual supply function, schedule, and curve, distinguishing between a change in quantity supplied and a change in supply, and comprehending how to move from individual supply to market supply.
- Explain how market equilibrium price and quantity are determined for a good or service, as well as analyze the market disequilibrium situations and the types of equilibria.

1. Demand

1.1. Definition of Individual Demand and its Features

An individual demand refers to *the quantity of a commodity which an individual is willing and able to buy at a particular price during a given period of time.* (Deepashree, 2018, p. 77).

The main **features** of individual demand for a commodity are (Deepashree, 2018, pp. 77-78):

- It always means *effective demand i.e.;* it must be backed by *ability* and *willingness* to spend;
- It depends on *utility* of the commodity; considering that the individual consumer is *rational*;
- It shows individual's *wish* or *need* to buy the commodity.

1.2.The Law of Demand

The law of demand states that: "*Other things remaining the same (ceteris paribus), the higher the price of a good or a service, the smaller is the quantity demanded; and the lower the price of a good or a service, the greater is the quantity demanded"* (Parkin, 2012, p. 57).

This means that, there is a *negative* (an *inverse*) relationship between quantity demanded of a good or a service and its price.

1.3. Factors (determinants) Affecting Individual Demand

The main factors that affect individual demand for a commodity are (Deepashree, 2018, pp. 78-80):

- **Price of the Commodity:** In general, there is an inverse relationship between price and quantity demanded of a commodity, as shown by the law of demand.
- **Prices of related goods:** demand for one good is influenced by the prices of other related good. We distinguish between three types of goods:
  - **Substitutes**: *goods that are alternatives to each other in consumption*, such as: tea and coffee. When the price of a substitute *rises*, buyers switch to buy *more* quantity of the other good, and vice versa. There is a *positive* relationship between the price of a good and the quantity demanded of its substitute.
  - **Complements:** *goods that are jointly used or consumed together*, such as: sugar and coffee. When the price of a complement *rises*, individuals buy *less* quantity of the other good, and vice versa. There is an *inverse* relationship between the price of a good and the quantity demanded of its complement.
  - **Independent goods:** *goods that are not related to each other*, such as: coffee and automobiles. A change in the price of one good does not affect the quantity demanded of the other good.
- **Income:** changes in money income of individuals affect the demand for goods. We distinguish, here, between two types of goods:
  - **Normal goods:** *goods that individuals demand more of them as their income rises, and vice versa*. These goods are *positively* related to money income of individuals.
  - **Inferior goods**: *goods that individuals demand less of them as their income rises, and vice versa*. These goods are *negatively* related to money income of individuals.
- **Tastes and preferences:** when tastes and preferences change in favor of a good, demand for this good increases. When they change against a good, demand for this good decreases.

- **Future Expectations:** especially future changes in a good's price and income. If buyers expect the price of a good to rise, or to earn a higher income in the future, their current demand increases, and vice versa.

1.4.The Individual Demand Function

The individual demand function shows "*the correspondence between the quantity demanded of a commodity (by an individual consumer), price, and other factors that influence purchases"* (Perloff, 2020, p. 12).

It can be expressed as: $q_{dX} = f\,(P_X, P_Y, I, TP, FE)$

Where: q_{dX}: Quantity Demanded for commodity (*X)*

P_X: Price of commodity (*X)*

P_Y: Prices of related goods

I: Income

TP: Taste and Preferences

FE: Future Expectations

If we hold the effects of all other factors constant, other than the price of the good, then we can write the quantity demanded as a function of only the price of the good: $q_{dX} = f\,(P_X)$

In general, there is a *linear* relationship between the quantity demanded and the price. Thus, we can also write the following equation: $q_{dX} = a - bP_X$

a and *b* are constant numbers (parameters). When: $P_X = 0$, $q_{dX} = a$; when: $q_{dX} = 0$, $P_X = a/b$.

1.5. The Individual Demand Schedule and the Individual Demand Curve

An Individual demand schedule is a table showing how much of a good or service a consumer will want to buy at different prices *a table or a list showing the quantities of a commodity that an individual is willing to purchase over a specific time period, with all other variables held constant.* **An Individual demand curve** is *a graphical representation of the demand schedule* (Krugman & Wells, 2009, pp. 63-64).

Example (1): Suppose that a demand function of an individual for good (X) is given by: $q_{dX} = 20 - 2P_X$

a. Form the individual demand schedule at different prices for good (X)?

b. Draw the individual demand curve for this good?

Answer:

a. The individual demand schedule for good (X)

By plugging any particular value for P_X into the demand function, we can determine the corresponding quantities of good (X).

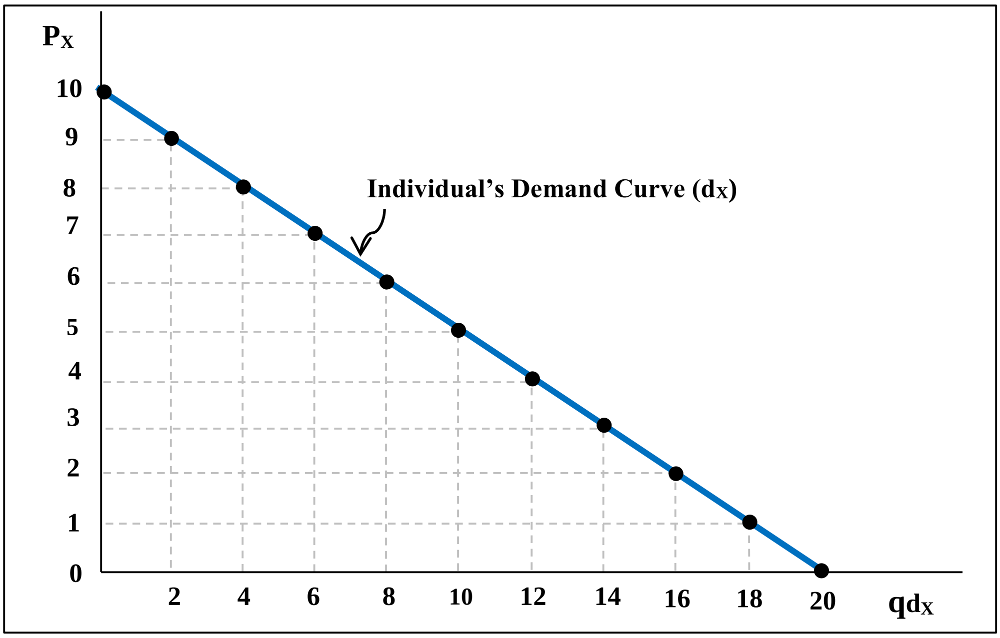
Table 4.1. The Individual's Demand Schedule

P_X	0	1	2	3	4	5	6	7	8	9	10
q_{dX}	20	18	16	14	12	10	8	6	4	2	0

b. Drawing the individual demand curve for this good?

We graph the individual demand schedule as a demand curve, with the quantity demanded, q_{dX}, on the *x*-axis (the horizontal axis), and the price, P_X, on the *y*-axis. (the vertical axis).

Figure 4.1. The Individual's Demand Curve



Graphically, the law of demand tells us that demand curves slope downward to the right, as shown in the figure above.

1.6. Changes in Demand

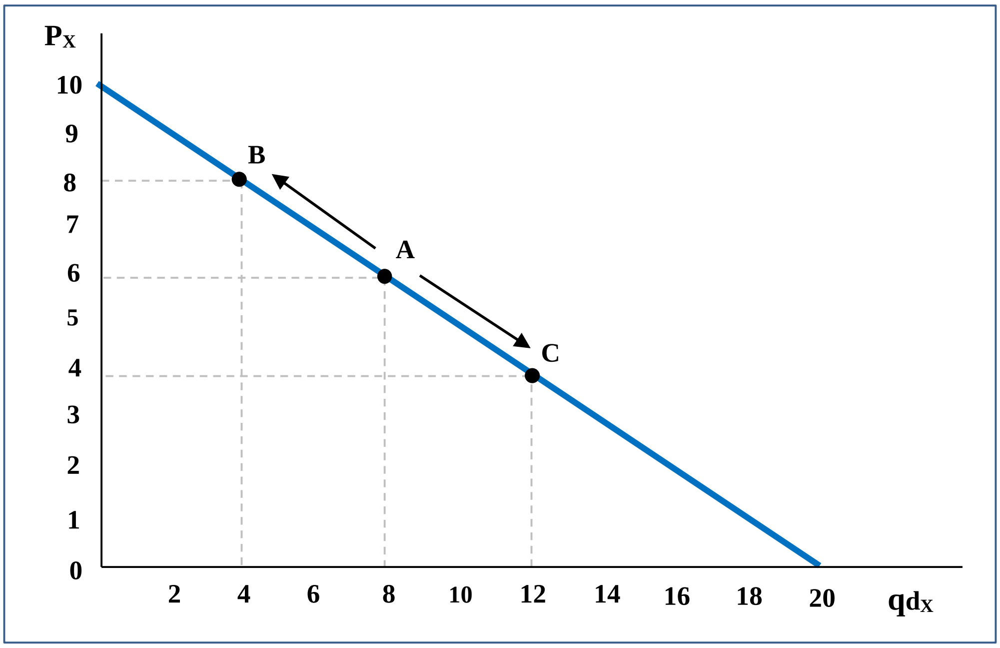
We distinguish, here, between two very different changes in demand: "a change in quantity demanded", and "a change in demand".

1.6.1. Change in Quantity Demanded

A change in the price of the good, other things remaining constant causes ***a movement along the original demand curve.*** We call this ***a change in quantity demanded***. A rise in price causes an *upward movement along the demand curve—a decrease in quantity demanded (contraction of demand)*. A fall in price causes a *downward movement along the demand curve—an increase in quantity demanded (expansion of demand)* (Deepashree, 2018, p. 85).

Assuming that the individual is in the original situation at point A.

Figure 4.2. Movement along the Demand Curve (Change in Quantity Demanded)

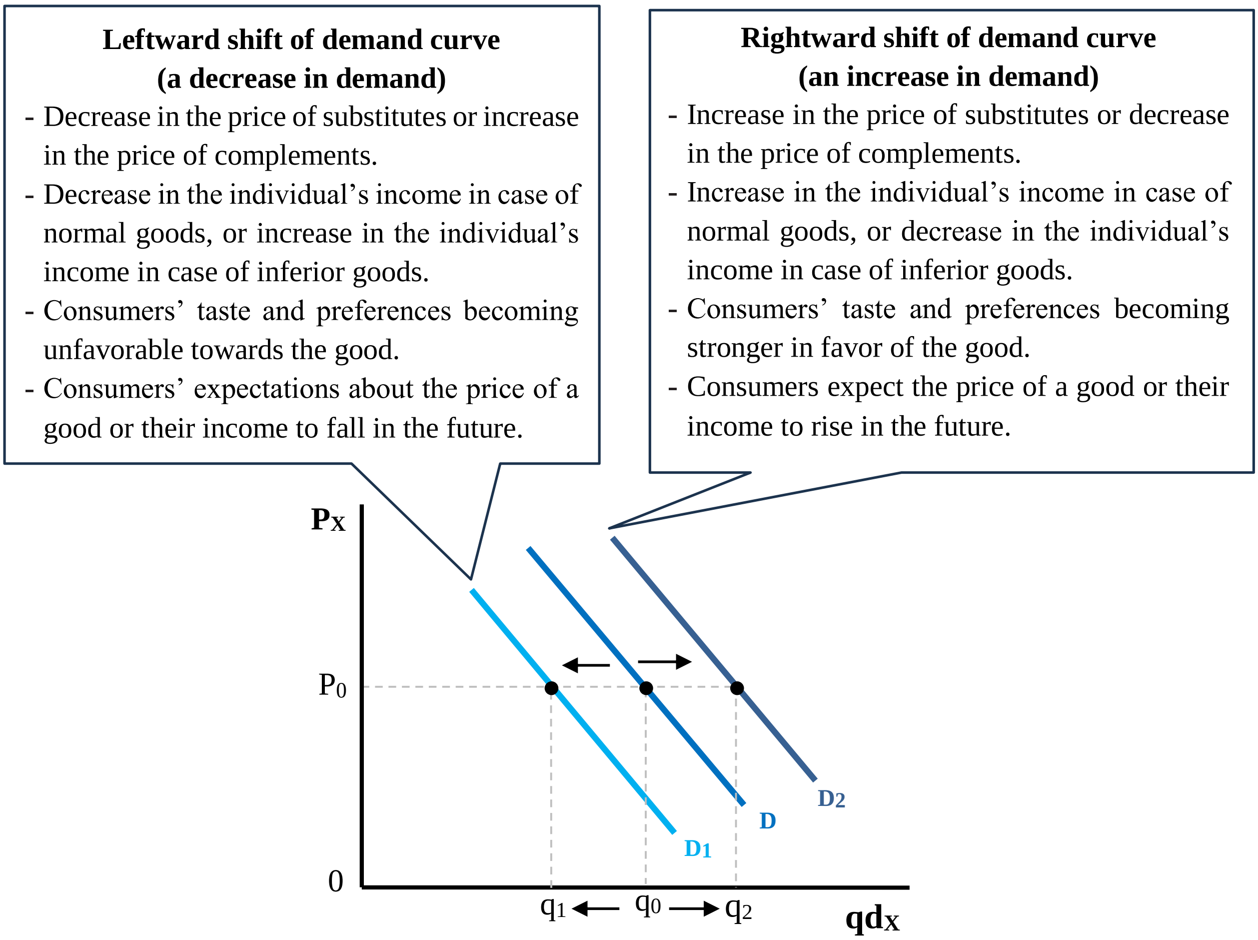


An upward movement from point A to point B shows a lesser quantity demanded at a higher price. A downward movement from point A to point C shows more quantity demanded at a lower price.

1.6.2. Change in Demand

A change in any factor that affects demand—except for the good's price—causes ***a shift in the entire demand curve*** to a new position, denoted by a new demand curve. We call this ***a change in demand***. Any one of the four factors, other than price of the good, that affect the demand, which we have discussed before, can cause a shift in demand curve for a good either *rightward* (*an increase in demand*) or *leftward* (*a decrease in demand*) (Deepashree, 2018, p. 86).

Figure 4.3. Shift in the Demand Curve (A Change in Demand)



1.7. From Individual Demand to Market Demand

1.7.1. Definition and Factors Affecting Market Demand

Market Demand is *the aggregate of the quantities demanded by all consumers in the market at different prices per time period*. The factors affecting individual demand, that we have discussed earlier, affect the market demand as well. But there are some other factors that influence the market demand, which are (Deepashree, 2018, p. 84):

- **Number of consumers in the market:** The larger the number of consumers who buy a good, the greater is the demand for that good.

- **Distribution of income:** More even the distribution of income in a country, more will be the market demand for the commodities.
- **Age and sex of population:** the age group and sex composition of the consumers decide the pattern of market demand.

1.7.2. The Market Demand Schedule and the Market Demand Curve

The market demand schedule shows *the quantities demanded in a market which are the sum of the quantities demanded by all the buyers at each price*. **The market demand curve** is *derived from the individual consumer's demand curves by horizontally summing the individual demand curves* (Browning & Zupan, 2015, p. 88).

Assume that there are only two individuals who purchase the good (X) in the market, and each one has the same

That is, at each price we sum the individual quantities, to obtain the total quantity demanded at that price.

Example (2): Assume that there are only two individuals who purchase the good (X) in the market, and each one has the same demand function, as in the previous example: $q_{dX} = 20 - 2P_X$

a. Find the market demand function for good (X)

b. Form the market demand schedule at different prices for good (X)?

c. Derive the market demand curve for this good?

Answer:

a. The market demand function for good (X): $Q_{DX} = q_{dX1} + q_{dx2} = 2(20 - 2P_X)$ => $Q_{DX} = 40 - 4P_X$

b. The market demand schedule for good (X)

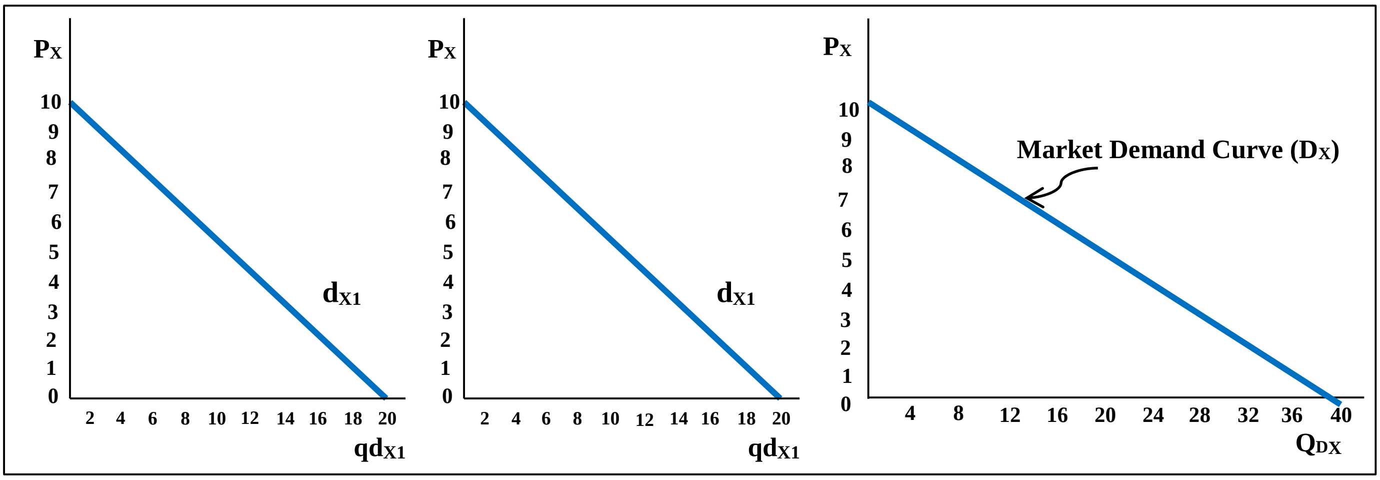
Table 4.3. The Market Demand Schedule

P_X	0	1	2	3	4	5	6	7	8	9	10
q_{dX1}	20	18	16	14	12	10	8	6	4	2	0
q_{dX2}	20	18	16	14	12	10	8	6	4	2	0
Q_{DX}	**40**	**36**	**32**	**28**	**24**	**20**	**16**	**12**	**8**	**4**	**0**

c. The market demand curve for good (X)

The market demand curve is found by horizontally summing of the two individual demand curves.

Figure 4.3. Market Demand Curve as the Sum of Individual Demand Curves



2. Supply

2.1. Definition of Individual Supply

Individual supply refers to *the quantity of a commodity that a single producer (firm) is willing to sell over a specific time period* (Salvatore, 2006, p. 17)

2.2. The Law of Supply

The law of supply states that: "*Other things remaining the same, the higher the price of a good, the greater is the quantity supplied; and the lower the price of a good, the smaller is the quantity supplied"* (Parkin, 2012, p. 62). This means that, there is a *positive* (a *direct*) relationship between quantity supplied of a good or a service and its price.

2.3. Factors Affecting Individual Supply

The most important factors that affect individual supply for a commodity are (Parkin, 2012, pp. 63-64; Deepashree, 2018, pp. 204-205):

- **Price of the commodity produced:** In general, there is a positive relationship between price and supply for a commodity, as shown by the law of supply.
- **Prices of related goods:** supply for one good is influenced by the prices of other related good, whether substitutes or complements:

- **Substitutes**: *goods that can be produced by using the same resources*. When the price of a substitute *rises,* firms switch to produce *more* quantity of it and *less* of the other good, and vice versa. There is an *inverse* relationship between the price of a good and the quantity supplied of its substitute.
- **Complements:** *goods that must be produced together*. When the price of a complement *rises*, the supply of the other good *increases,* and vice versa. There is a *positive* relationship between the price of a good and the quantity supplied of its complement.

- **Prices of factors of production (inputs):** if the price of one or more of the factors of production (such as: raw materials, machines, land, labor, etc.) *rises,* then the cost of production will rise. As a result, supply of the good will *fall*, and vice-versa. Thus, the supply of a good is *negatively* related to the price of inputs used to produce the good.
- **Technology:** A change in technology occurs when a new method or technique is used in production, leading to lower the marginal cost of producing a good. Thus, with advancement in technology supply of goods will *increase*.
- **Future Expectations:** especially future changes in a good's prices— can affect supply. If a firm expects that the price of a good will be higher in the future, then it will supply less to the market today, but if it expects that the price of a good will be lower in the future, then it will supply more to the market today.
- **Government Policy:** if the government imposes *taxes* on commodities, firms' total cost will rise, and the supply of these commodities will *decrease*. Conversely, if it grants *subsidies* on commodities, firms' total cost will fall, and the supply of these commodities will *increase.*

- **The State of nature:** it includes all the natural forces that can influence production such as: the state of weather, earthquakes, hurricanes etc. For example, good weather can increase the supply of many agricultural products and bad weather can decrease their supply.

2.4. The Individual Supply Function

The individual supply function shows *the correspondence between the quantity supplied (by a single firm), price of good produced, and other factors (determinants) that influence sales* (Perloff, 2020, p. 18). It can be written as: $q_{SX} = f\ (P_X, P_Y, P_{FP}, T, FE, GP)$

Where: q_{SX}: Quantity supplied of commodity (*X*)

P_X: Price of commodity (*X*)

P_Y: Prices of related goods

I: Income

P_{FP}: Prices of factors of production

T: Technology

FE: Future expectations

GP: Government policy

Like in demand, economists use statistical techniques to hold the effects of all other factors that affect supply constant other than the price of the good. So, we can write the quantity supplied as a function of only the price of the good: $q_{SX} = f\ (P_X)$

In general, there is a *linear* relationship between the quantity supplied and the price. Thus, we can also write the following equation: $q_{SX} = c + dP_X$

c and *d* are constant numbers (parameters). When: $P_X = 0$, $q_{SX} = c$; when: $q_{SX} = 0$, $P_X = -c/d$.

2.5. The Individual Supply Schedule and the Individual Supply Curve

An Individual supply schedule is *a table showing how much of a good or service a producer will supply at different prices*. **An Individual supply curve** is *a graphical representation of the supply schedule* (Krugman & Wells, 2009, pp. 71-72)

Example (3): Suppose that a supply function of a firm that produces and sells the good (X) is given as follows: $q_{SX} = -4 + 2P_X$

a. Form the firm's supply schedule at different prices for good (X)?

b. Draw the firm's demand curve for this good?

Answer:

a. The firm's supply schedule for good (X):

By substituting any particular value for $P_X > 2$ into the supply function, we can determine the corresponding quantities of good (X).

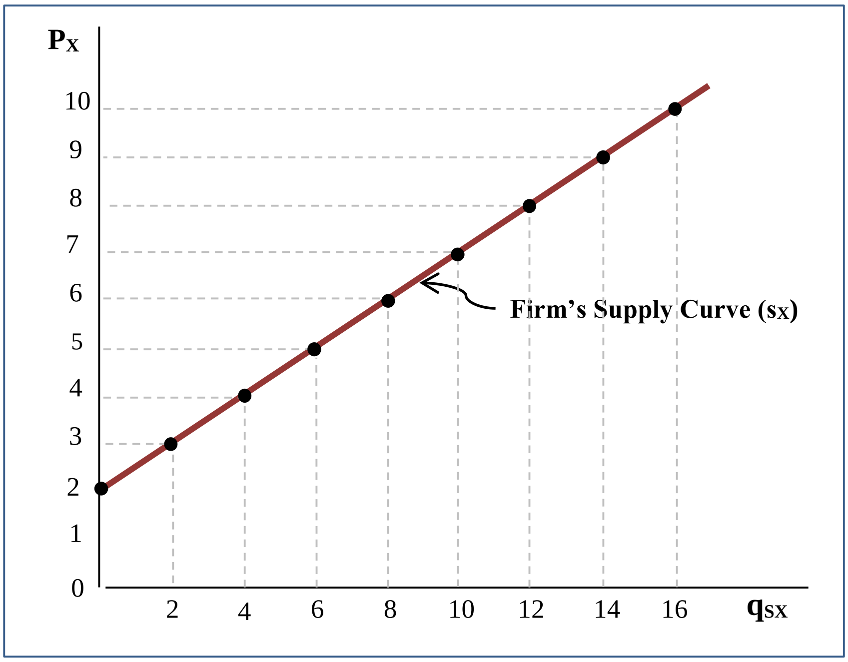
Table 4.4. The Individual's Supply Schedule

P_X	2	3	4	5	6	7	8	9	10
q_{SX}	0	2	4	6	8	10	12	14	16

b. The firm's supply curve for good (X)

We graph the firm's supply schedule as a supply curve, with the quantity supplied, q_{SX}, on the *x*-axis, and the price, P_X, on the *y*-axis.

Figure 4.4. The Firm's Supply Curve



2.6. Changes in Supply

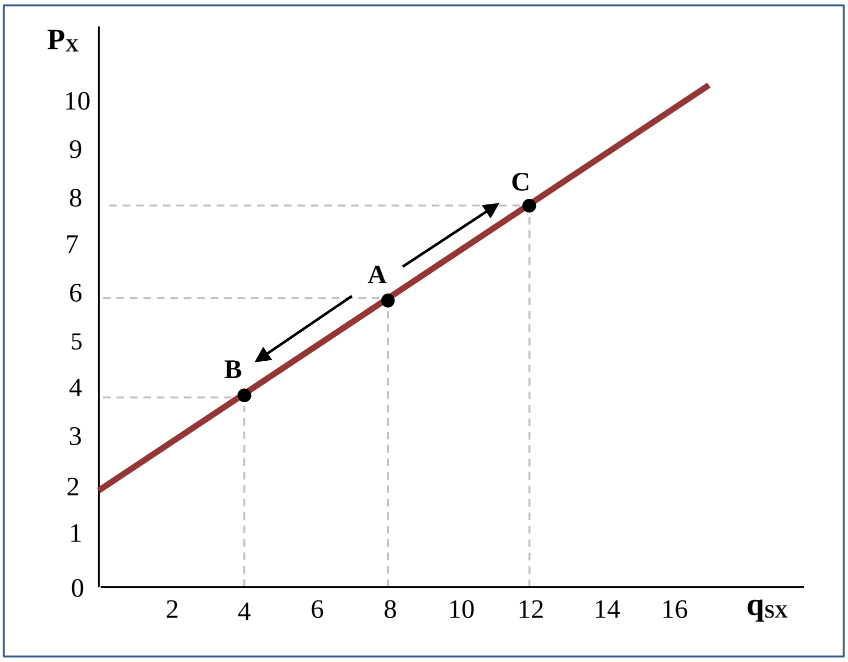
As we have seen with demand, we distinguish between: "a change in quantity supplied" and "a change in supply".

2.6.1. Change in quantity supplied

A change in the price of the good, other things remaining constant causes ***a movement along the original supply curve.*** We call this ***a change in quantity supplied***. A rise in price causes an *upward movement along the demand curve—an increase in quantity supplied (extension of supply).* A fall in price causes a *downward movement along the supply curve—a decrease in quantity supplied (contraction of supply)* (Deepashree, 2018, p. 208).

We assume that the firm is in the original situation at point A.

Figure 4.5. Movement along the Supply Curve (Change in Quantity Supplied)

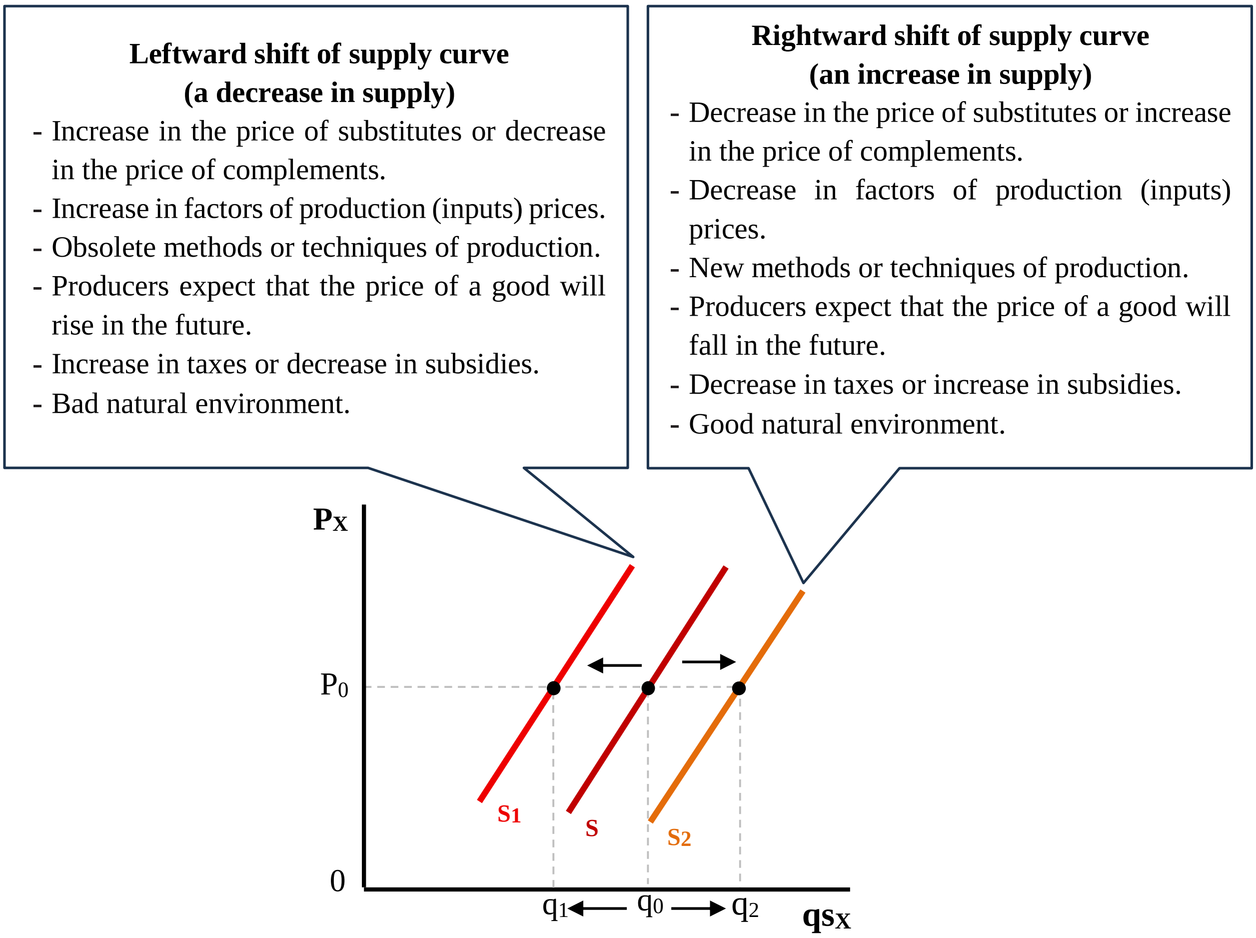


An upward movement from point A to a point such as C shows extension or more quantity supplied at a higher price. A downward movement from point A to a point such as B shows contraction or less quantity supplied at a lower price.

2.6.2. Change in supply

A change in any non-price factor of supply causes ***a shift in the entire supply curve*** to a new position, denoted by a new supply curve. We call this ***a change in supply***. Any one of the factors that affect the supply, except for the good's price, which we have discussed before, can cause a *shift* in supply curve for a good, either *rightward* (*an increase in supply*) or *leftward* (*a decrease in supply*) (Deepashree, 2018, p. 209).

Figure 4.6. Shift in the Supply Curve (A Change in Supply)



2.7. From Individual Supply to Market Supply

2.7.1. Definition and Factors Affecting Market Supply

The market Supply *is the sum of the quantities produced and offered to sale by all producers in the market at different prices in a particular period of time* (Salvatore, 2006, p. 18). The same factors that affect individual supply, which we have discussed earlier, will also affect the market supply. But there are additional factors that influence the market supply, which are:

- **Number of suppliers in the market:** The larger the number of firms that produce a good, the greater is the supply of the good. As new firms enter an industry, the supply in that industry increases. As firms leave an industry, the supply in that industry decreases.

- **Market's Productive Capacity:** It is determined by the number of producers in the market, and the plant and equipment possessed by each firm. Whenever productive capacity increases, the supply increases as well, since sellers would choose to sell a greater total quantity at each price. Similarly, a decrease in productive capacity causes a decrease in supply.

2.7.2. The Market Supply Schedule and the Market Supply Curve

The market supply schedule shows *the quantities supplied in a market which are the sum of the quantities offered by all the sellers at each price*. **The market supply curve** is *derived from the individual supply curves by horizontally summation of all individual supply curves* (Deepashree, 2018, p. 207). That is, at each price we sum the individual quantities to obtain the total quantity supplied at that price.

Example (4): Suppose that there are only two firms that produce and sell the good (X) in the market, and each one has the same supply function, as in the previous example: $q_{SX} = -4 + 2P_X$

a. Find the market supply function for good (X)

b. Form the market supply schedule at different prices for good (X)?

c. Derive the market supply curve for this good?

Answer:

a. The market supply function for good (X): $Q_{SX} = q_{SX1} + q_{SX2} = 2(-4 + 2P_X) \Rightarrow Q_{SX} = -8 + 4P_X$

b. The market supply schedule for good (X):

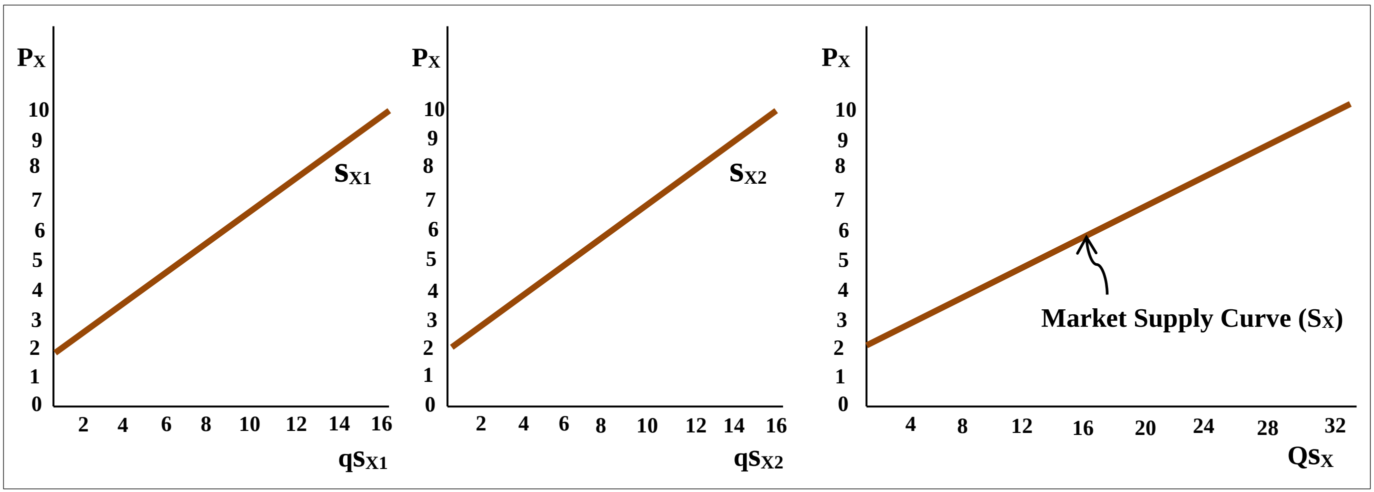
Table 4.5. The Market Supply Schedule

P_X	2	3	4	5	6	7	8	9	10
q_{SX1}	0	2	4	6	8	10	12	14	16
q_{SX2}	0	2	4	6	8	10	12	14	16
Q_{SX}	**0**	**4**	**8**	**12**	**16**	**20**	**24**	**28**	**32**

c. The market supply curve for good (X)

The market supply curve is obtained by horizontally aggregating of the two individual supply curves.

Figure 4.7. Market Supply Curve as the Sum of Individual Supply Curves



3. Market Equilibrium (Putting Market Demand and Supply Together)

3.1. Meaning of Market Equilibrium

An **equilibrium** is *a situation in which quantity demanded equals quantity supplied at the prevailing price* (Browning & Zupan, 2015, p. 21). The price at which this takes place is the **equilibrium price (P_0)**. The quantity of the good bought and sold at that price is the **equilibrium quantity (Q_0)**. In this situation no participant wants to change its behavior.

3.2. Finding the Price and Quantity of Equilibrium

The forces of market demand and supply determine price and quantity of equilibrium at which goods and services are bought and sold. *The condition of market equilibrium* to find the equilibrium price and quantity is determined by *equality between market demand and supply*:

The condition of market equilibrium: **Quantity demanded (Q_{DX}) = Quantity supplied (Q_{SX})**

We can determine the market equilibrium for good (X) either *mathematically* using the demand and supply functions (it is determined by the *equality* between the quantity demanded and quantity supplied), or *geometrically* using the market demand and supply curves (it is identified by *the point of intersection* between the two curves).

Example (5): Let's use the market demand and supply functions of the previous examples (**2**) and (**4**), which are: $Q_{DX} = 40 - 4P_X$; $Q_{SX} = -8 + 4P_X$

a. Find the market equilibrium price and quantity for good (X) mathematically?

b. Find the market demand schedule and the market supply schedule of good (X)?

c. Plot, on one set of axes, the market demand and supply curves for this good, and determine the point of equilibrium?

Answer:

a. The market equilibrium price and quantity for good (X):

The condition of market equilibrium is: $Q_{DX} = Q_{SX}$

$Q_{DX} = Q_{SX} \Rightarrow 40 - 4P_X = -8 + 4P_X \Rightarrow$ **P_0 = DZD 6 ; Q_0 = 16 units**

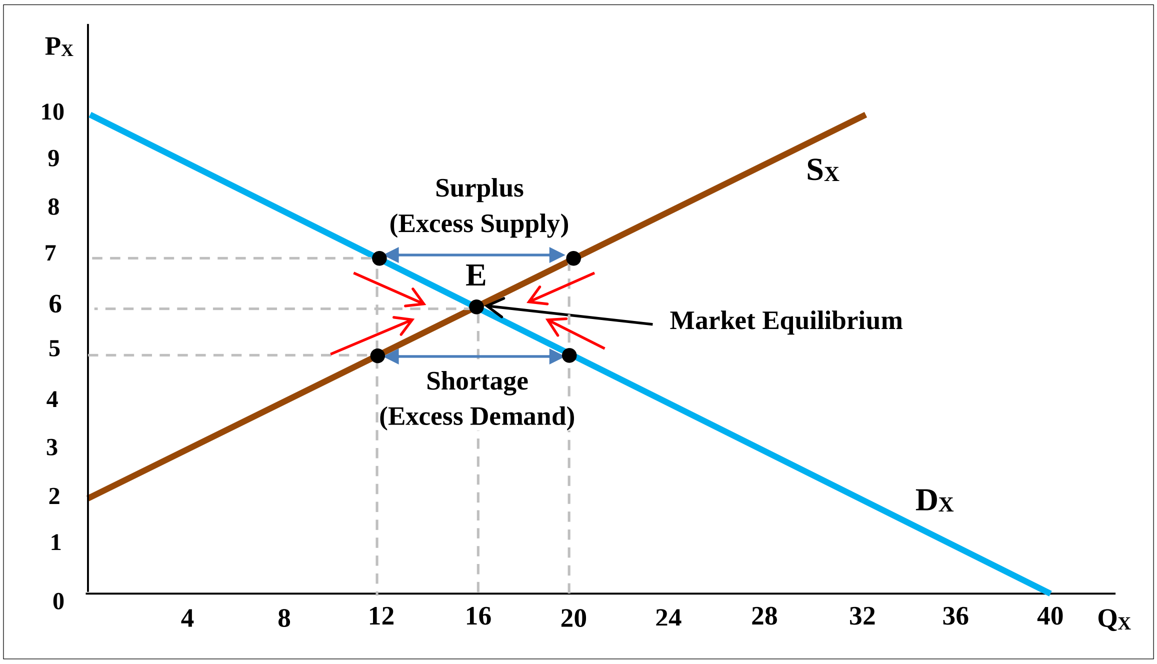
b. The market demand and supply schedules of good (X):

Table 4.6. The Market Supply Schedule

P_X	2	3	4	5	6	7	8	9	10
Q_{DX}	32	28	24	20	16	12	8	4	0
Q_{SX}	0	4	8	12	16	20	24	28	32

c. The market demand and supply curves and the point of equilibrium:

Figure 4.7. Market Equilibrium



The intersection of the market demand curve D_X and the market supply curve S_X for good (X) determines the market equilibrium point E, where $\mathbf{P_0}$ **= DZD 6** per unit and $\mathbf{Q_0}$ **= 20 units**.

3.3. Market Disequilibrium Situations

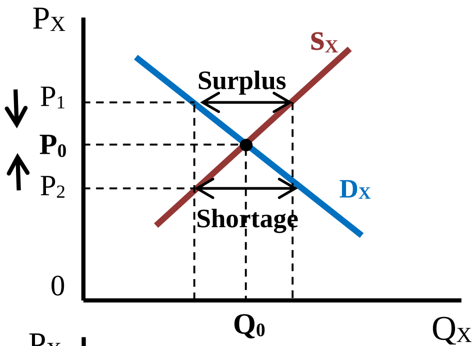
If the price were not at the equilibrium level, then two market disequilibrium situations arise, and consumers or firms would have an incentive to change their behavior in a way that would drive the price to the equilibrium level (Browning & Zupan, 2015, pp. 21-22).

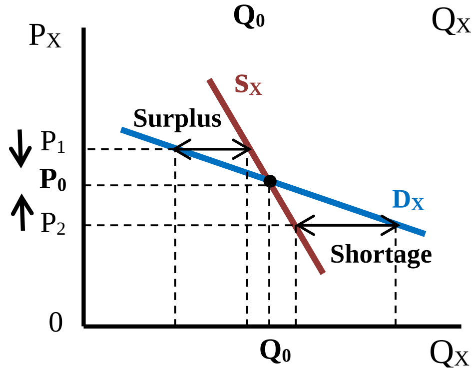
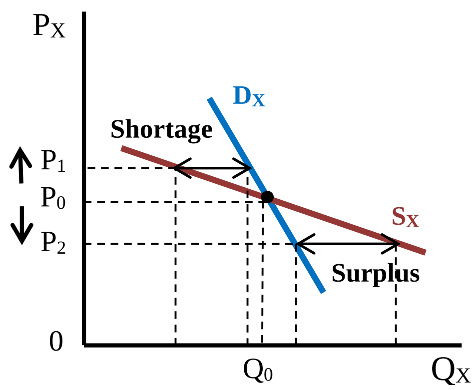
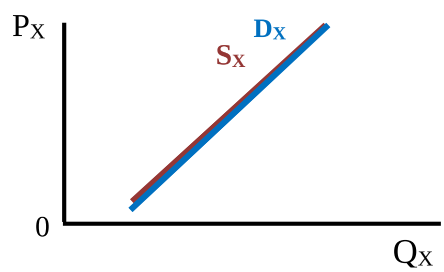
- **Shortage (Excess Demand):** at any price *below* the equilibrium price, there will be a *shortage* or *excess demand (XD)*, meaning that the quantity demanded *exceeds* the quantity supplied (such as the price **5** in figure 4.7, there is a shortage of: **20 – 12 = 8 units**). Buyers will be frustrated and compete with each other to get more of good (X) than is available, offering to pay a higher price (more than **5**). Alternatively, suppliers will be prompted to raise their selling prices. As the price rises, quantity demanded falls (an upward movement along the demand curve), and quantity supplied increases (an upward movement along the supply curve). The process continues until reaching the equilibrium quantity and price level.
- **Surplus (Excess Supply):** at any price *above* the equilibrium price, there will be a *surplus* or *excess supply (XS)*, meaning that the quantity supplied *exceeds* the quantity demanded (such as the price **7** in figure 4.7, there is a surplus of: **20 – 12 = 8 units**). Sellers will be frustrated and compete with each other by offering a lower price to attract additional buyers to sell more of good (X) rather than accumulate unwanted inventories and consumers realize that they do not have to pay as high a price for the good. As the price falls, quantity demanded increases (a downward movement along the demand curve), and quantity supplied decreases (a downward movement along the supply curve). The process continues until reaching the equilibrium quantity and price level.

3.4. Types of Equilibria

There are three types of equilibria, we will explain each type with presenting graphical representations in the table below. (Salvatore, 2006, pp. 19, 33)

Table 1.7. Types of Equilibria

<table>
<tr><th>Types of Equilibria</th><th>Graphical Representation</th></tr>
<tr><td>1. Stable Equilibrium:
The type of market equilibrium where any deviation from equilibrium brings into operation market forces which push us back toward equilibrium. At prices below the equilibrium level, a shortage arises and the market forces—defined as the behavior of buyers and sellers in the market explained before—tend to produce a higher price which is bid up toward the equilibrium level. At prices above the equilibrium price, a surplus results and the market forces tend to produce a lower price which is bid down toward the equilibrium level.
<u>Note:</u> this type of equilibrium occurs when the market demand curve is downward-sloping and the market supply curve is upward-sloping (such as in figure 4.7), or when both curves are downward-sloping, but the market supply curve is steeper than the market demand curve.</td><td>


</td></tr>
<tr><td>1. Unstable Equilibrium:
The type of equilibrium where any deviation from the equilibrium position brings into operation forces which push us further away from equilibrium. At prices below the equilibrium level, a surplus arises and the market forces tend to produce a lower price which moves us further away from the equilibrium level. At prices above the equilibrium price, a shortage arises and the market forces tend to produce a higher price which moves us further away from the equilibrium level.
<u>Note:</u> this type of equilibrium occurs, when both curves are downward-sloping, but the market demand curve is steeper than the market supply curve.</td><td>
</td></tr>
<tr><td>2. Neutral (Metastable) Equilibrium:
This type of equilibrium occurs when the market demand curve and the market supply curve coincide (it is a rare case in the real-world).</td><td>
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Problems

1. Suppose that the individual demand function for a good (X) is: $q_{dx} = 40 - 4P_x$

a. Derive the individual's demand schedule and the individual's demand curve?

b. What is the maximum quantity this individual will ever demand of good (X) per time period?

c. What happens if the price of good (X) falls from **DZD 5** to **DZD 4**, holding all other factors constant?

2. The U.S. supply function for 'corn' is: $Q_{C(US)} = 10 + 10P_C$

If you know that the supply function of the rest of the world for 'corn' is: $Q_{C(RW)} = 5 + 20P_C$

a. What is the world supply function for 'corn'?

b. Explain, by using graphs, how each of the following factors will affect the supply of corn in USA?

***Factor 1*:** The US producers of corn expect that its price will fall in the next few months.

***Factor 2*:** Bad weather destroys the crops of corn in many parts of USA.

3. Suppose that the demand function for a good (X) is given by: $Q_{DX} = 300 - 2P_X + 4I$

Where, Q_{DX}: the quantity demanded of the good (X); P_X: the price of the good; and I: the average income measured in thousands of dollars.

a. If **I = 25**. Write the quantity demanded as a function of only the price of good (X)?

b. What happens if the income is *doubled*, and the price of the good remains constant at DZD **110**?

c. If the supply function for this good is given by: $Q_{SX} = 3P_X - 50$. Find the equilibrium market price and quantity for the good after the income's change?

4. Suppose that there are **10,000** identical individuals in the market for good (X), each with a demand function given by: $q_{dx} = 12 - 2P_x$

There are **1,000** identical sellers of this good, each with a supply function given by: $q_{Sx} = 20P_x$

a. Find the market demand function and the market supply function for good (X)?

b. Calculate the market equilibrium price and quantity?

c. Calculate the quantity bought by each individual, and the quantity sold by each seller?

5. The following table gives hypothetical data for the quantity demanded and supplied of 'gasoline' per month in a city in USA.

P_G ($per gallon)	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**
Q_{DG} (millions of gallons)	**170**	**155**	**140**	**125**	**110**	**95**
Q_{SG} (millions of gallons)	**80**	**110**	**140**	**170**	**200**	**230**

a. Find the function of market demand and the function of market supply, (both are linear functions)?

b. Graph the market demand and supply curves, and illustrate how a rise in the price of 'automobiles' would affect the gasoline market.

6. The estimated world demand function for green (unroasted) coffee beans: $Q_{DC} = 8.56 - P_{Cf} - 0.3P_S + 0.1I$

Where, Q_{DC}: the quantity demanded of coffee in millions of tons per year; P_{Cf}: the price of coffee in dollars per pound (lb); P_S: the price of sugar in dollars per lb, and I: the average annual consumers' income in thousands of dollars.

The estimated world supply function is: $Q_{SC} = 9.6 + 0.5P_{Cf} - 0.2P_{Cc}$

Where, Q_{SC}: the quantity supplied of coffee in millions of tons per year; P_{Cc}: the price of cocoa in dollars per lb.

a. If $P_S = \$0.20$, $I = \$35$, and $P_{Cc} = \$3$. Write the quantity demanded and the quantity supplied as functions of only the price of coffee?

b. Solve for the equilibrium price and quantity? Show the equilibrium point graphically?

c. If the equilibrium price *rises* to **$3.** Will there be a surplus or a shortage in the market, and how large will it be?

7. The following table gives the market demand schedule and the market supply schedule of a good (X).

P_X (DZD)	1	2	3	4	5
Q_{DX}	9,000	8,000	7,000	6,000	5,000
Q_{SX}	13,000	10,000	7,000	4,000	1,000

a. Plot the demand and supply curves using these schedules. Indicate the equilibrium point on your diagram?

b. Is the equilibrium for good (X) 'stable' or' unstable'? Why?

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Chapter 5: The Measurement of Elasticities

Learning Objectives

By the end of this chapter, students will be able to:

- Understand the meaning of an elasticity, and the meaning of price-elasticity of demand and supply.
- Distinguish between arc and point elasticity of demand and supply.
- Determine the factors affecting the price-elasticity of demand and supply.
- Explain the different types (degrees) of price-elasticity of demand and supply.
- Analyze the relationship between total expenditure and price-elasticity of demand.
- Identify other types of demand elasticities (income elasticity and cross-price elasticity of demand).
- Recognize the importance of elasticity for both firms and governments.

1. Elasticity of Demand

1.1. The Concept of Elasticity of Demand

1.1.1. What is an Elasticity?

An **elasticity** measures the sensitivity of one variable to another. Specifically, it is a number that tells us *the percentage change that will occur in one variable (Y) in response to a 1-percent change in another variable (X)* (Pindyck & Rubinfeld, 2013, p. 33).

The elasticity e of (Y) with respect to (X) is:

$$e = \frac{\text{percentage change in } Y}{\text{percentage change in } X} = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}} = \frac{dY}{dX}\frac{X}{Y}$$

Note that the elasticity is a units-free measure, that is, it has no units of measure.

1.1.2. What is the Price-Elasticity of Demand?

The price-elasticity of demand is *the percentage change in quantity demanded of a good (X), Q_{DX}, resulting from a 1-percent change in the price, P_X. Holding constant all other variables that affect the quantity demanded* (Pindyck & Rubinfeld, 2013, p. 33).

$$e_{DX} = - \frac{\text{percentage change in } \textit{quantity demanded}}{\text{percentage change in } \textit{price}}$$

$$= - \frac{\frac{\Delta Q_{DX}}{Q_{DX(ave)}}}{\frac{\Delta P_X}{P_{X(ave)}}} = - \frac{dQ_{DX}}{dP_X}\frac{P_X}{Q_{DX}} = - b\,\frac{P_X}{Q_{DX}}$$

The coefficient of elasticity of demand. e_{DX}, is negative, because price and quantity demanded are inversely related as the law of demand states. The minus sign is dropped from the numbers and we report the absolute value of the elasticity.

Notice that we use the *average* price and *average* quantity, meaning that the base value used to calculate a percentage change in price or quantity is always *midway* between the initial value and the new value. We do this because it gives the most precise measurement of elasticity—getting the same value for the elasticity regardless of whether the price falls or rises from point to another point on the demand curve.

1.2. Arc and Point Elasticity of Demand

Arc elasticity of demand is *the price elasticity of demand between two points on a demand curve*. **Point elasticity of demand** relates to *price elasticity at a single point on a demand curve* (Salvatore, 2006, p. 45).

Example (1): The table below gives the market demand schedule for a good (X).

point	A	B	C	D	E	F	G	H	I
P_X	8	7	6	5	4	3	2	1	0
Q_{DX}	0	100	200	300	400	500	600	700	800

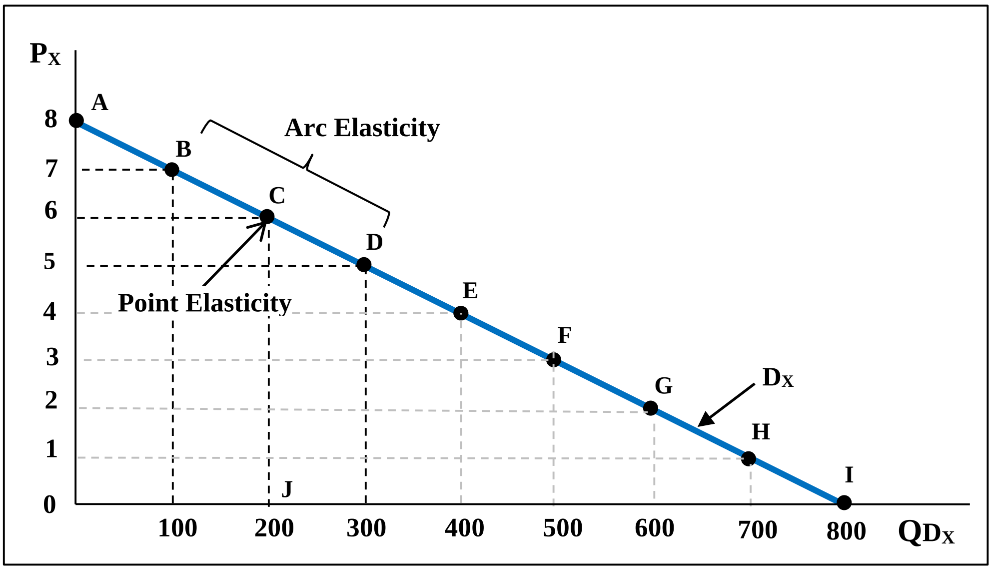
a. Plot the demand curve and then find e_{DX} for a movement from point **B** to point **D**, and from **D** to **B**? and at the point **C** (the point midway between B and D)?

b. Find e_{DX} at the point **C** mathematically? (**$Q_{DX} = 800 - 100P_X$**)

Answer:

a. Finding e_{DX} from point B to D, and from D to B? and at the point C:

Figure 5.1. Arc and Point Elasticity of Demand



- Finding, e_{DX}, from point B to point D, and from D to B:

From B to D, $e_{DX} = -\dfrac{(Q_D - Q_B)}{(P_D - P_B)} \dfrac{P_B}{Q_B} = -\dfrac{200}{-2}\dfrac{7}{100} = 7$

From D to B, $e_{DX} = -\dfrac{(Q_B - Q_D)}{(P_B - P_D)} \dfrac{P_D}{Q_D} = -\dfrac{-200}{2}\dfrac{5}{300} = 1.67$

We get a different value for e_{DX} if we move from B to D than if we move from D to B. This difference results because we used a different base in calculating the percentage changes in each case. We can avoid that by using the average price and the average quantity between the initial value and the new one.

$$e_{DX} = -\frac{\Delta Q}{\Delta P}\,\frac{\dfrac{(P_B + P_D)}{2}}{\dfrac{(Q_B + Q_D)}{2}} = -\frac{\Delta Q}{\Delta P}\,\frac{(P_B + P_D)}{(Q_B + Q_D)}$$

Applying this modified formula to find e_{DX} either for a movement from B to D or for a movement from D to B, we get:

$$e_{DX} = -\left(-\frac{200}{2}\,\frac{12}{400}\right) = 3$$

This is the equivalent of finding, e_{DX}, at the point midway between B and D (i e., at point C).

- Finding e_{DX} at point **C** geometrically:

Expressing each of the values in the mathematical formula for, e_{DX}, in terms of distances, we get:

$$e_{DX} = \frac{\Delta Q_{DX}}{\Delta P_X}\frac{P_X}{Q_{DX}} = \frac{IJ}{JC}\frac{JC}{OJ} = \frac{IJ}{OJ} = \frac{600}{200} = 3$$

b. Finding e_{DX} mathematically at the point C:

The demand function for good (X) is given by: $\mathbf{Q_{DX} = 800 - 100P_X}$

$$e_{DX} = \frac{dQ_{DX}}{dP_X}\frac{P_C}{Q_C} = -b\frac{P_C}{Q_C} = -(-100)\frac{6}{200} = -3\ ;\ |e_{DX}| = 3$$

The negative sign illustrates the law of demand: Less quantity is demanded as the price rises, and vice versa. Although the arc elasticity of demand is sometimes useful, economists generally use the word “elasticity” to refer to a *point* elasticity.

1.3. Factors Affecting Price-Elasticity of Demand

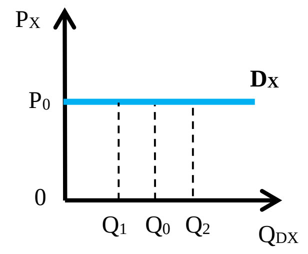
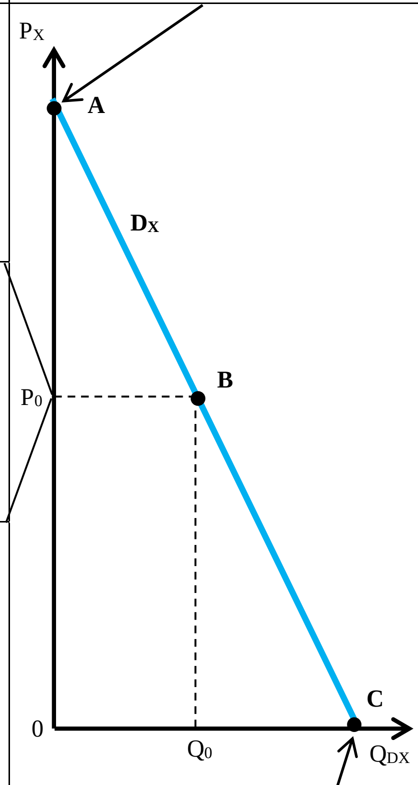
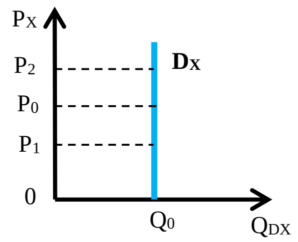
The main factors that can affect the price elasticity of demand are (Deepashree, 2018, pp. 102-103; Case, Fair, & Oster, 2017, pp. 132-134):

- **Availability of close substitute:** a good with close substitutes tend to have more elastic demand, and a good without close substitutes will have an inelastic demand.
- **Income of the consumers:** If the income level of consumers is high, then the elasticity of demand is less. But in low-income groups, the elasticity of demand is high.
- **Luxuries versus Necessities:** The price elasticity of demand is likely to be low for necessities, and high for luxuries.
- **Total expenditure on goods:** the higher the cost of the good relative to total income of the consumer, the more will be the price elasticity of demand, and vice versa.
- **Time Horizon:** the long-run elasticity of demand is often higher than the short-run elasticity, because consumers can make adjustments over time and producers develop substitute goods.

1.4. The Types of Price-Elasticity of Demand

The absolute value of the coefficient of elasticity of demand ranges from **zero to infinity** $(0 \leq |e_{DX}| \leq \infty)$. There are five different magnitudes of elasticity of demand. Accordingly, we can determine five types or degrees of elasticity of demand as shown in the following table.

Table 5.1. Types of Price-Elasticity of Demand

Coefficient of e_{DX}	Type of e_{DX}	Description	Examples of goods	Graphical representation
$\lvert e_{DX}\rvert = \infty$	Perfectly (infinitely) elastic demand	The quantity demanded changes by an *infinitely large* percentage in response to a tiny percentage change in price	Perfect substitute goods under perfect competition	 **Horizontal Demand Curve**
$1 < \lvert e_{DX}\rvert < \infty$	Elastic demand	The percentage change in quantity demanded is *greater* than the percentage change in price.	Luxury goods	 **Straight-Line Demand Curve**
$\lvert e_{DX}\rvert = 1$	Unitary elastic (unit-elastic) demand	The percentage change in quantity demanded is *exactly equal* to the percentage change in price.	Normal goods	
$0 < \lvert e_{DX}\rvert < 1$	Inelastic demand	The percentage change in the quantity demanded is *less* than the percentage change in the price.	Necessities like food, shelter, fuel etc.	
$\lvert e_{DX}\rvert = 0$	Perfectly inelastic demand	The percentage change in price causes absolutely *no change* in quantity demanded.	Essentials like lifesaving drugs such as: Insulin	 **Vertical Demand Curve**

The price-elasticity of demand is determined according to the shape of the demand curve. There are two extreme cases: if demand curve takes a horizontal line, then the curve is *perfectly elastic* ($\lvert e_{DX}\rvert = \infty$), and if demand curve takes a vertical line, then the curve is *perfectly inelastic* ($\lvert e_{DX}\rvert = 0$). However, the price-elasticity of demand varies along a downward-sloping linear demand curve. The demand curve is *perfectly inelastic* ($\lvert e_{DX}\rvert = 0$) where the demand curve hits the quantity axis (point C), and it is *perfectly elastic* ($\lvert e_{DX}\rvert = \infty$) where the demand curve hits the price axis (point

A), and it has *unitary elasticity* ($|e_{DX}| = 1$) at the midpoint of the demand curve (point B). For the range between B and A, the demand curve is *elastic* ($1 < |e_{DX}| < \infty$). For the range between B and C, the demand curve is *inelastic* ($0 < |e_{DX}| < 1$).

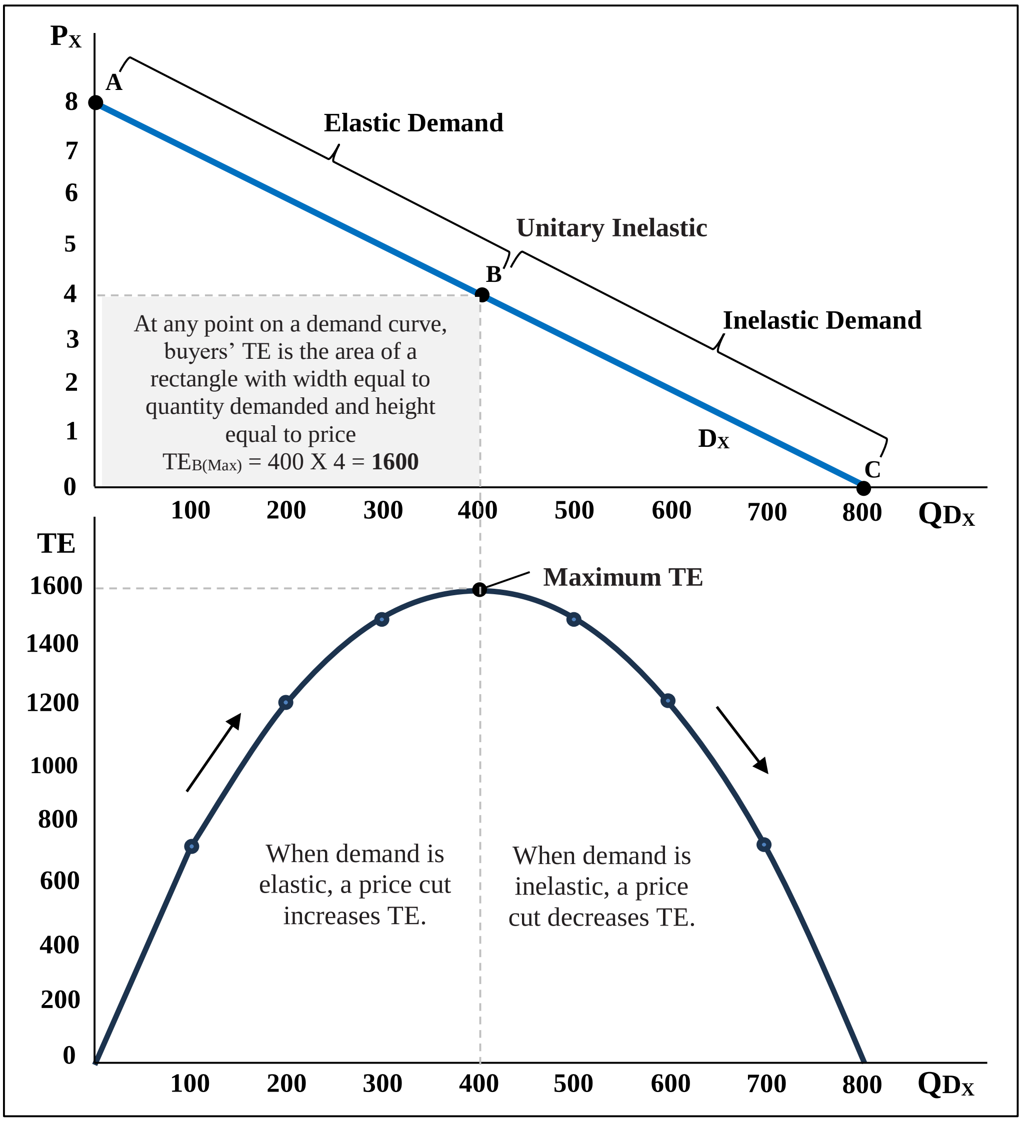
1.5. Total Expenditure and Price-Elasticity of Demand

The total expenditure (TE) on a good (X) equals the price of the good multiplied by the quantity purchased (***TE = $P_X.Q_X$***). When a price changes, TE also changes. But a cut in the price does not always decrease TE. The change in TE depends entirely on the elasticity of demand for the good, which tells us what happens to TE when price changes: its size determines which effect—the price effect or the quantity effect—is stronger (Krugman & Wells, 2009, p. 151).

- If demand is ***inelastic*** *($0 < |e_{DX}| < 1$)*, a 1-percent ***rise*** in price will cause quantity demanded to *fall* by *less* than 1 percent, and vice versa (the price effect is stronger than the quantity effect). So, TE ***rises***. Thus, *the TE moves in the same direction as price*
- If demand is ***unitary elastic*** *($|e_{DX}| = 1$)*, a 1-percent *rise* in price will cause quantity demanded to fall by 1 percent, and vice versa (the quantity effect and the price effect exactly offset each other). So, TE ***does not change***. Thus, *the TE remains the same as price changes.*
- If demand is ***elastic*** *($1 < |e_{DX}| < \infty$)*, a 1-percent ***rise*** in price will cause quantity demanded to fall by *more* than 1 percent, and vice versa (the quantity effect is stronger than the price effect). So, TE ***falls***. Thus, *the TE moves in the opposite direction of price.*

We can see how a change in price changes the TE of buyers, using a graph of the demand curve, as shown in Figure 5.2.

Figure 5.2. Total Expenditure and Elasticity



Note that the analysis will be in the same way, if we want to know the relationship between Total Revenue (TR) and the price elasticity of demand, since the TE of consumers on a good is the TR earned by producers from the sale of that good.

1.6. Other Types of Demand Elasticities

There are two other types of demand elasticities: income elasticity of demand, and corss-price elasticity of demand.

1.6.1. Income Elasticity of Demand

The income elasticity of demand is *the percentage change in the quantity demanded of a good (X),* Q_{DX}*, resulting fro a 1-percent change in income, I, with all other influences on demand, including the price of the good, remaining unchanged* (Pindyck & Rubinfeld, 2013, p. 35).

$$e_I = \frac{\text{percentage change in } \textit{quantity demanded of (X)}}{\text{percentage change in } \textit{income (I)}}$$

$$= \frac{\frac{\Delta Q_{DX}}{Q_{DX}}}{\frac{\Delta I}{I}} = \frac{dQ_{DX}}{dI}\frac{I}{Q_{DX}}$$

An income elasticity can be positive or negative, and the sign gives us valuable information about the *types* of different goods:

- If ($e_I < 0$), then (X) is ***an inferior*** good such as: *bus travel*; as household income rises, travelers are likely to shift from cheaper bus travel to more expensive car or airline travel.
- If ($e_I > 0$), then (X) is ***a normal*** good, which can be divided into two categories: necessity —something that people need, or luxury —something desirable but not really necessary.
  - If ($0 < e_I < 1$), then the good is ***a necessity***; a given percentage increase in income causes a *smaller* percentage increase in quantity demanded. The broad category of food, medicines, and housing are necessities.
  - If ($e_I > 1$), then the good is ***a luxury***; a given percentage increase in income causes a *greater* percentage increase in quantity demanded. For example, most of us would regard trips to Europe, yachts and caviar as luxuries.

1.6.2. Cross-Price Elasticity of Demand

The cross-price elasticity of demand between two goods (X) and (Y) is *the percentage change in the quantity demanded of one good (X), Q_{DX}, resulting from a 1-percent change in the price of the other good (Y), P_Y, holding constant all other variables that affect the quantity demanded, including the price of the good*. (Pindyck & Rubinfeld, 2013, p. 35)

$$e_{X,Y} = \frac{\textbf{percentage change in } \textit{quantity demanded of (X)}}{\textbf{percentage change in } \textit{price of (Y)}}$$

$$= \frac{\frac{\Delta Q_{DX}}{Q_{DX}}}{\frac{\Delta P_Y}{P_Y}} = \frac{dQ_{DX}}{dP_Y}\frac{P_Y}{Q_{DX}}$$

A cross-price elasticity can also be positive or negative, and the sign gives us valuable information about the *relationship* between the two goods:

- If ($e_{X,Y} < 0$), the two goods are ***complements*** such as: coffee and sugar; *an increase* in the price of good Y (sugar) causes *a decrease* in quantity demanded for good X (coffee). Thus, there is an *inverse relationship* between the quantity demanded of the good and the price of its complement.
- If ($e_{X,Y} > 0$), the two goods are ***substitutes*** such as: coffee and tea; *an increase* in the price of good Y (tea) causes *an increase* in quantity demanded for good X (coffee). Thus, there is a *positive relationship* between the quantity demanded of the good and the price of its substitute.
- If ($e_{X,Y} = 0$), the two goods are ***unrelated*** to each other such as: coffee and gasoline; *an increase* in the price of good Y (gasoline) doesn't cause *any change* in quantity demanded for good X (coffee). Thus, there is *no relationship* between the two goods.

2. Elasticity of Supply

2.1. Definition of Price Elasticity of Supply

The price elasticity of supply is *the percentage change in the quantity supplied of a good (X), Q_{SX}, resulting from a 1-percent change in the price, P_X. Holding constant other variables that affect the quantity supplied* (Pindyck & Rubinfeld, 2013, p. 36).

$$e_{SX} = \frac{\textbf{percentage change in } \textit{quantity supplied}}{\textbf{percentage change in } \textit{price}}$$

$$= \frac{\frac{\Delta Q_{SX}}{Q_{SX(ave)}}}{\frac{\Delta P_X}{P_{X(ave)}}} = \frac{dQ_{SX}}{dP_X}\frac{P_X}{Q_{SX}} = +d\,\frac{P_X}{Q_{SX}}$$

The price elasticity of supply is usually *positive* because, when the price of the good rises the quantity supplied increases, and vice versa, as the law of supply states.

2.2. Arc and Point Elasticity of Supply

As we have seen concerning the concept of arc and point elasticity of demand, the same thing is applied on supply.

Arc elasticity of supply is *the price elasticity of supply between two points on a supply curve*. **Point elasticity** of supply is *the price elasticity at a single point on a supply curve*.

Example (2): The table below gives the market supply schedule for a good (X).

point	A	B	C	D	E
P_X	6	5	4	3	2
Q_{SX}	800	600	400	200	0

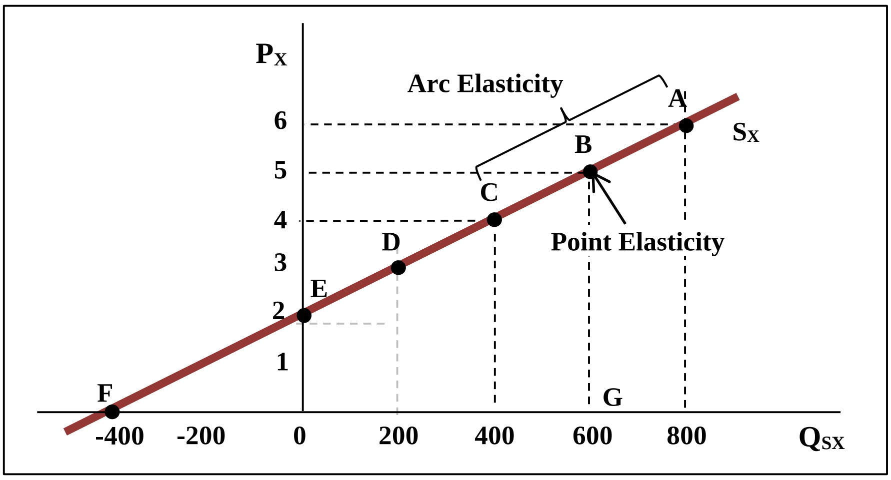
a. Plot the supply curve and then find e_{SX} for a movement from point **A** to point **C**, and from **C** to **A**? and at the point **B** (the point midway between A and C)?

b. Find e_{SX} at the point **B** mathematically? ($\mathbf{Q_{SX} = -400 + 200P_X}$)

Answer:

a. Finding e_{SX} from point A to C, and from C to, and at the point B:

Figure 5.3. Arc and Point Elasticity of Supply



-From A to C, $\mathbf{e_{SX}} = \frac{(Q_C - Q_A)}{(P_C - P_A)} \frac{P_A}{Q_A} = \frac{-400}{-2} \frac{6}{800} = \mathbf{1.5}$

-From C to A, $\mathbf{e_{SX}} = \frac{(Q_A - Q_C)}{(P_A - P_C)} \frac{P_C}{Q_C} = \frac{400}{2} \frac{4}{400} = \mathbf{2}$

We get a different value for e_{SX} if we move from A to C than if we move from C to A. This difference results because we used a different base in calculating the percentage changes in each case. We can avoid that by using the average price and the average quantity between the initial value and the new one.

$$\mathbf{e_{SX}} = \frac{\Delta Q}{\Delta P} \frac{\frac{(P_A + P_C)}{2}}{\frac{(Q_A + Q_C)}{2}} = \frac{\Delta Q}{\Delta P} \frac{(P_A + P_C)}{(Q_A + Q_C)}$$

Applying this modified formula to find e_{SX} either for a movement from A to C or for a movement from C to A, we get:

$$e_{SX} = \frac{400}{2} \frac{10}{1200} = \mathbf{1.67}$$

This is the equivalent of finding, e_{SX}, at the point midway between A and C (i e., at point B).

- Finding, e_{SX}, at the point **B** geometrically:

$$e_{SX} = \frac{\Delta Q}{\Delta P} \frac{P_B}{Q_B} = \frac{FG}{GB} \frac{GB}{OG} = \frac{FG}{OG} = \frac{1000}{600} = \mathbf{1.67}$$

b. Finding, e_{SX}, at the point B mathematically:

The supply function for good (X) is given by: $\mathbf{Q_{SX} = -400 + 200P_X}$

$$e_{SX} = \frac{dQ_{SX}}{dP_X} \frac{P_B}{Q_B} = d \frac{P_B}{Q_B} = 200 \left(\frac{5}{600} \right) = \mathbf{1.67}$$

You can see that using mathematical or geometric method leads to the same value, when calculating e_{SX}.

2.3. Factors Affecting Price-Elasticity of Supply

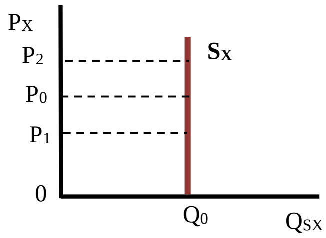
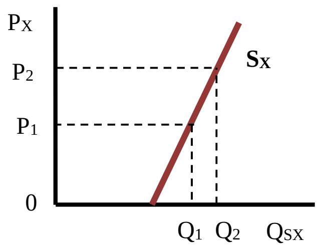
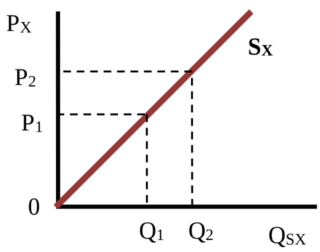
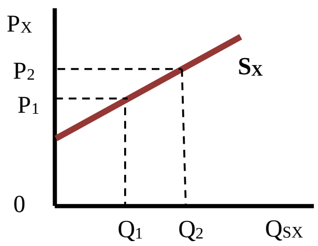
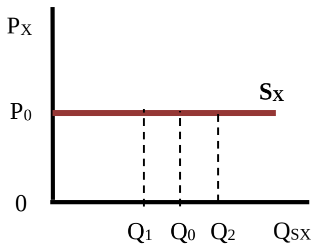
The main factors that influence the price-elasticity of supply are (Deepashree, 2018, pp. 214-215; Parkin, 2012, pp. 95-96):

- **The availability of inputs:** the price-elasticity of supply tends to be large when inputs are easily available, and it tends to be small when inputs are difficult to obtain.
- **Time frame for the supply decision:** the long-run supply is often elastic and perhaps perfectly elastic. While, the short-run supply or momentary supply is often inelastic and perhaps perfectly inelastic.
- **Nature of goods:** perishable goods have an inelastic supply, because their supply cannot be changed within a short period of time. By contrast, durable goods have an elastic supply.
- **Production capacity:** If producers have a big production capacity, then the supply for the good will be elastic. If producers have a limited production capacity, then the supply will be inelastic.
- **Future price expectation:** If producers expect that the price will rise in the future, then they will supply less presently. Thus, supply will become inelastic. If producers expect that the price will fall in the future, supply will be more elastic.

2.4. The Types of Price-Elasticity of Supply

The coefficient of price-elasticity of supply ranges from **zero to infinity (0 ≤ e_{SX} ≤ ∞)**. We can determine **five** types of price-elasticity of supply as shown in the following table.

Table 5.2. Types of Price Elasticity of Supply

Coefficient of e_{SX}	Type of e_{SX}	Description	Examples of goods	Graphical representation
$e_{SX} = 0$	Perfectly inelastic supply	The percentage change in price causes absolutely *no change* in quantity supplied.	Goods produced by using unique or rare productive resources.	 **Vertical Supply Curve**
$0 < e_{SX} < 1$	Inelastic supply	The percentage change in quantity supplied is *less* than the percentage change in price.	Perishable goods.	
$e_{SX} = 1$	Unitary elastic (unit-elastic) supply	The percentage change in quantity supplied is *exactly equal* to the percentage change in price.		
$1 < e_{SX} < \infty$	Elastic supply	The percentage change in the quantity supplied is *greater* than the percentage change in the price.	Durable goods.	
$e_{SX} = \infty$	Perfectly (infinitely) elastic supply	The quantity supplied changes by an *infinitely large* percentage in response to a tiny percentage change in price	Goods produced by using commonly available resources	 **Horizontal Supply Curve**

3. The Importance of Elasticity

The concept of elasticity is of great importance for both firms and governments. It helps them in analyzing, forecasting, and in making decisions. We can highlight this importance through the following points (Shivam, 2025):

- **Determination of Output Level:** since the change in quantity demanded is due to the change in price, knowing the elasticity of demand is necessary for determining the optimal output level.
- **Determination of Price:** if the demand for a good is inelastic, then the producer can charge a high price for it. By contrast, if the demand is elastic, then the producer can charge a low price for it.
- **Price Discrimination by Monopolist:** in a market with elastic demand, the monopolist fixes a low price, and in a market with less elastic demand, the monopolist charges a high price.
- **Price Determination of Factors of Production:** if the demand of a particular factor of production is inelastic, its price will be high, and if it is elastic, its price will be low.
- **Demand Forecasting:** demand forecasting of goods in the future depends on the income elasticity, because managers can know the effect of changing incomes on the demand for their products.
- **Government Policies:** especially protection, subsidy and taxation policies. The government can impose higher taxes on goods with inelastic demand, and lower rates on goods with elastic demand. Subsidy or protection is given to only those industries whose products have an elastic demand.
- **International Trade:** a country will gain more from international trade, by exporting goods with less elasticity of demand charged with higher prices, and importing those goods for which its demand is elastic, which are charged with lower prices.

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Problems

1. Suppose that a demand function for a good (X) is given by:

$$Q_{DX} = P_X^{-0.3} P_Y^{0.1} I^{0.4}$$

Where, $\mathbf{Q_{DX}}$: the quantity demanded of the good (X); $\mathbf{P_X}$: the price of the good; $\mathbf{P_Y}$: the price of other good (Y); and **I**: the consumer's income.

- Calculate the percentage change in quantity demanded for good (X) if the following changes happen:

a. The price of the good (X) increases by **10%**, with all other factors remaining constant? Explain the result?

b. The price of the other good (Y) increases by **5%**, with all other factors remaining constant? Explain the result?

c. The consumer's income increases by **10%**, with all other factors remaining unchanged? What is the type of the good (X)?

2. The table below shows the quantity of "regular cuts of meat" that a family of four would purchase per year at various income levels.

Income (I)	**4,000**	**6,000**	**8,000**	**10,000**	**12,000**	**14,000**	**16,000**	**18,000**
Quantity demanded (Q_{DM})	**100**	**200**	**300**	**350**	**380**	**390**	**350**	**250**

a. Find the income elasticity of demand of this family for regular cuts of meat between the various successive levels of income?

b. Over what range of income are regular cuts of meat a luxury, a necessity, or an inferior good for this family?

c. Plot on a graph the curve that represents the income-quantity relationship? What is it called? Illustrate how we can determine the type of the good graphically? 1

3. The following table shows the demand schedule for 'computer chips' during a period of time.

Price ($per chip)	**500**	**450**	**400**	**350**	**300**	**250**	**200**
Quantity Demanded (millions of chips per year)	**20**	**25**	**30**	**35**	**40**	**45**	**50**

a. Calculate the total expenditure and the price elasticity of demand at each price level?

b. Determine the type (degree) of elasticity of demand at each price level?

c. State and explain the general rule relating total expenditure on computer chips to price elasticity of demand when price falls?

4. A firm produces and sells four types of goods. The elasticities of demand for these goods are shown in the following table.

Goods	**Price Elasticity of Demand***	**Income Elasticity of Demand**	**Cross Elasticity of Demand**
Butter	**0.35**	**+ 0.42**	Butter and margarine: **+ 1.54**
Margarine	**0.27**	**- 0.20**	
Cheese (type 1)	**0.62**	**+ 0.89**	Cheese (type 1) and butter: **- 0.61**
Cheese (type 2)	**1.34**	**+ 1.22**	Cheese (type 1) and cheese (type 2): **+0.82**

*The coefficients of price elasticity of demand are taken in the absolute values.

a. Indicate from the information in the table above if: the demand is elastic or inelastic; if the goods are substitutes or complements; and whether the good is a luxury, a necessity, or an inferior good?

b. What happens to total revenue when the consumers' income rises by **10%**, while the prices of the goods remain the same?

c. If the firm can change the prices of the goods, while the consumers' income remains constant. What is the best price policy (increasing or decreasing prices) for this firm, if it seeks to rise its total revenue?

d. Explain how the price elasticity of demand can help the government in formulating its taxation policy?

5. The supply function of 'Jeans' is given by: $\mathbf{Q_{SJ} = -72 + \frac{4}{5}P_J}$. Where, $\mathbf{Q_{SJ}}$ is the quantity supplied of jeans in millions of pairs per year; $\boldsymbol{P_J}$: the price in dollars per pair. While, the supply function of 'long-distance phone calls' is given by: $\mathbf{Q_{SC} = 20P_C}$. Where, $\mathbf{Q_{SC}}$ is the quantity supplied of long-distance phone calls in millions of minutes per day; $\mathbf{P_C}$: the price in cents per minute.

a. Calculate the price elasticity of supply for 'jeans' at the price **$120** a pair?

b. Calculate the price elasticity of supply for 'long-distance phone calls' at the quantity supplied **600**?

c. Suppose that a big surge in the demand for the two goods occurred on a given day. How will the momentary supply elasticity be for each good?

6. Suppose that a good (X) is produced by only **two** firms (A) and (B) in the market, the supply function for each firm is as follows: **Firm (A):** $\mathbf{q_{SX1} = 15 + 7P_X}$ **Firm (B):** $\mathbf{q_{SX2} = 5 + 3P_X}$

There are **two** groups of consumers who buy this good; each group contains 5 individuals, the demand function of each individual in each group is as follows:

Group (1): $\mathbf{qdx_1 = 15 - 3P_X}$ **Group (2):** $\mathbf{qdx_2 = 7 - P_X}$

a. Find the market demand function and the market supply function of the good (X)?

b. What is the market equilibrium price and quantity?

c. Calculate the price elasticity of demand for each individual consumer, and for the market?

d. Calculate the price elasticity of supply for each firm, and for the market?

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Chapter 6: Market Equilibrium Applications

Learning Objectives

By the end of this chapter, students will be able to:

- Define the concept of consumer surplus and producer surplus and learn how to calculate them.
- Identify how the government can regulate the market through the imposition of price controls, including price ceilings and price floors, and analyze their effects on market outcomes.
- Understand how the government intervenes to regulate the market through fiscal policy, including imposing taxes and granting subsides, and analyze their effects on markets.

1. Consumer Surplus and Producer Surplus

1.1. Consumer Surplus

1.1.1. The Concept of Consumer Surplus

Consumer surplus (CS) relates to the *demand* side of the market. **Individual consumer surplus** is *the net benefit or value that a consumer receives beyond what s/he pays for a good. It is equal to the difference between the consumer's willingness to pay and the price actually paid* (Krugman & Wells, 2009, p. 96).

The sum of the individual CSs achieved by all the buyers of a good is known as the **total consumer surplus** achieved in the market.

1.1.2. Calculating Consumer Surplus

There are two methods that we can use to calculate the CS, whether mathematically or geometrically:

- **Mathematical Method:** by the application of integration technique.

$$CS = \int_0^{Q_0} (f(Q_{Dx})dQ) - P_0Q_0$$

- **Geometric Method:** by calculating the area below the demand curve but above the price.

To illustrate how to calculate the CS using these two methods, we will take the same example of individual and market demand for a good (X) used in chapter 4.

Example (1): The demand function of an individual for good (X) is given as follows: $q_{dX} = 20 - 2P_X$. The market demand function (assuming that there only two individuals who purchase the good in the market) is: $Q_{DX} = 40 - 4P_X$

a. Calculate the individual and market CS, mathematically, at the price: P_0 **= DZD 6**?

b. Calculate the individual and market CS, geometrically, at the price: P_0 **= DZD 6**?

Answer:

a. Calculating the individual and market CS, mathematically, at the price: P_0 = DZD 6.

- **The individual CS:**

We use the integration technique: $CS_1 = \int_0^{q_0} (f(q_{dx})dq) - P_0q_0$

The quantity demanded by the individual at price P_0 **= DZD 6** is: $q_0 = 20 - 2(6)$ => $q_0 = 8$ **units**

The inverse individual demand function is: $q_{dX} = 20 - 2P_X$ => $P_X = 10 - 0.5q_{dX}$

$$CS_1 = \int_0^{8} [(10 - 0.4q_X)dq] - P_0q_0$$

$$CS_1 = {}_0^{8}[(10q - 0.25q_X^2)] - P_0q_0$$

$$CS_1 = [(10(8) - 0.25(8)^2)] - [(10(0) - 0.25(0)^2)] - (6)(8)$$

CS_1 **= DZD 16** the CS of the first individual.

Following the same way, we find the CS of the second individual: CS_2 **= DZD 16**.

- **The market CS:**

$$CS = \int_{0}^{Q_0}(f(Q_{Dx})dQ) - P_0Q_0$$

$Q_0 = 40 - 4(6) \Rightarrow$ **Q_0 = 16 units**

The inverse market demand function is: $Q_{DX} = 40 - 4P_X \Rightarrow$ **$P_X = 10 - 0.25Q_{DX}$**

$$CS = \int_{0}^{16}[(10 - 0.25Q_{DX})dQ] - P_0Q_0$$

$$CS = \left[(10Q_{DX} - 0.125Q_{DX}^2)\right]_0^{16} - P_0Q_0$$

$$CS = [(10(16) - 0.125(16)^2)] - [(10(0) - 0.125(0)^2)] - (6)(16)$$

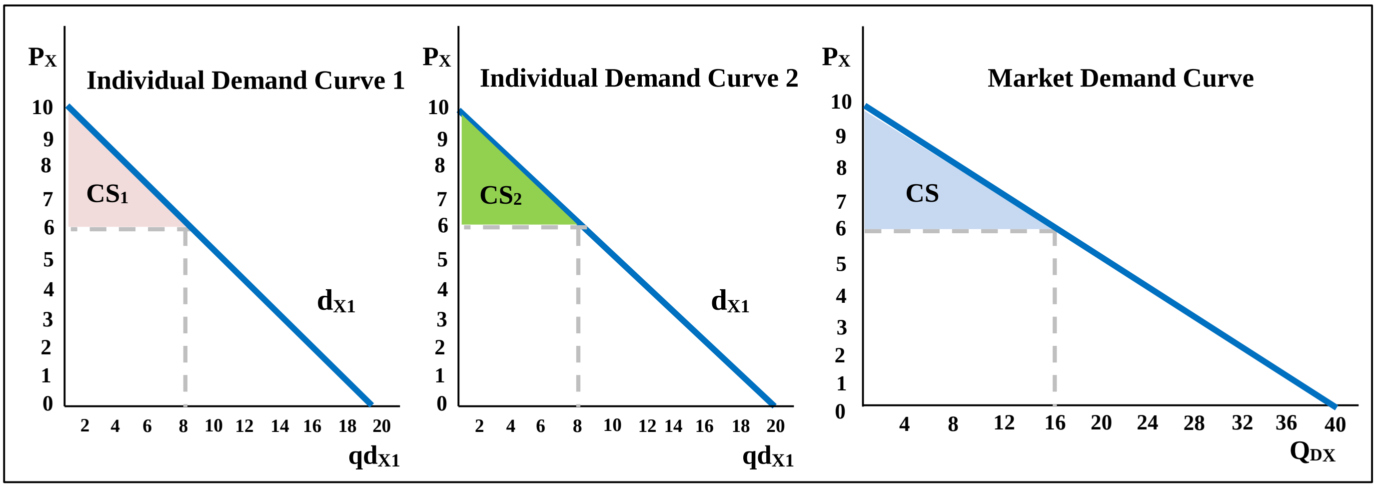
CS = DZD 32

CS = $CS_1 + CS_2$ = 16 + 16 **= DZD 32**

b. Calculating the individual and market CS, geometrically, at the price: P_0 = DZD 6

We calculate the area below the demand curve and above the price line. Figure 6.1 reproduces the individual and market demand curves from Figure 4.3 of chapter 4.

Figure 6.1. Individual and Market Consumer Surplus



Individual CS_1 = the area of the pink-shaded triangle

CS_1 = (the base x the height)/2 = [8 x (10-6)]/2 = **DZD 16**

Individual CS_2 = the area of the green-shaded triangle

CS_2 = (the base x the height)/2 = [8 x (10-6)]/2 = **DZD 16**

Market CS = the area of the blue-shaded triangle

CS = (the base x the height)/2 = [16 x (10-6)]/2 = **DZD 32**

CS = $CS_1 + CS_2$ = 20 + 20 **= DZD 40**

1.2. Producer Surplus

1.2.1. The Concept of Producer Surplus

Producer surplus (PS) relates to the *supply* side of the market. **Individual PS** is *the net profit or gain to a seller from selling a good. It is equal to the difference between the price a seller actually receives and the marginal cost of production—the minimum price at which the seller would* *have been willing to sell the good* (Krugman & Wells, 2009, p. 101). The sum of the individual PSs gained by all the sellers of a good is known as the **total PS** achieved in the market.

1.2.2. Calculating Producer Surplus

As with CS, there are two methods that we can use to calculate the PS, whether mathematically or geometrically:

- **Mathematical Method:** by the application of integration technique.

$$PS = P_0Q_0 - \int_0^{Q_0} (f(Q_{Sx})dQ)$$

- **Geometric Method:** by calculating the area above the supply curve but below the price.

To illustrate how to calculate the PS using these two methods, we will take the same example of individual and market supply for a good (X) used in chapter 4.

Example (2): The supply function of a firm that produces and sells a good (X) is given as follows: $q_{SX} = -4 + 2P_X$. The market supply function (assuming that there only two firms that produce and sell the good (X) in the market) is: $Q_{SX} = -8 + 4P_X$

a. Calculate the individual and market PS, mathematically, at the price: P_0 = **DZD 6**?

b. Calculate the individual and market PS, geometrically, at the price: P_0 = **DZD 6**?

Answer:

a. Calculating the individual and market PS, mathematically, at the price: P_0 = DZD 6.

- **The individual PS**

We use the integration technique: $PS_1 = P_0q_0 - \int_0^{q_0} (f(q_{Sx})dq)$

The quantity supplied by one firm at price P_0 = **DZD 6** is: $q_0 = -4 + 2(6)$ => $q_0 = 8$ **units**

The inverse individual supply function is: $q_{SX} = -4 + 2P_X$ => $P_X = 2 + 0.5q_{SX}$

$$PS_1 = P_0q_0 - \int_0^{8} (2 + 0.5q_{SX})dq$$

$$PS_1 = P_0q_0 - {}_0^8[(2q + 0.25q_X^2)]$$

$$PS_1 = (6)(8) - [(2(8) + 0.25(8)^2)] - [(2(0) + 0.25(0)^2)]$$

PS_1 = **DZD 16** the PS of the first firm.

Following the same way, we find the PS of the second firm: PS_2 = **DZD 16**.

- **The market PS:**

$$PS = P_0Q_0 - \int_0^{Q_0} (f(Q_{Sx})dQ)$$

The market quantity supplied at price P_0 = **DZD 6** is: $Q_0 = -8 + 4(6)$ => $Q_0 = 16$ **units**

The inverse market supply function is: $Q_{SX} = -8 + 4P_X$ => $P_X = 2 + 0.25Q_{SX}$

$$PS = P_0Q_0 - \int_0^{16} (2 + 0.25Q_{SX})dQ$$

$$PS = P_0Q_0 - {}_0^{20}[(2Q + 0.125Q_X^2)]$$

$$PS = (6)(16) - [(2(16) + 0.125(16)^2)] - [(2(0) + 0.125(0)^2)]$$

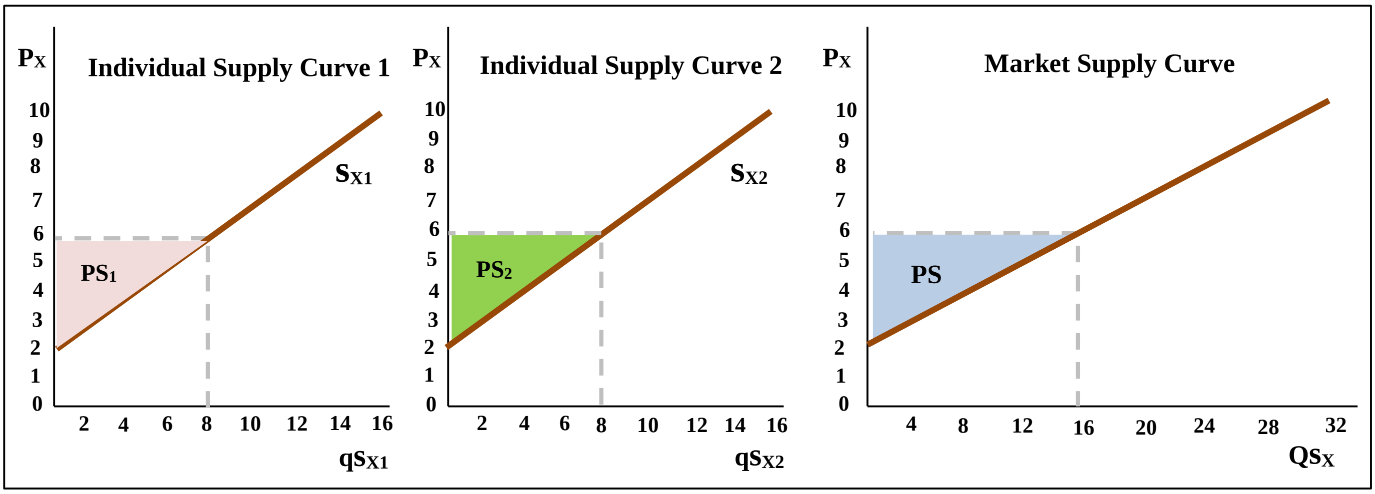
PS = DZD 32

PS = $PS_1 + PS_2$ = 16 + 16 = **DZD 32**

b. Calculating the individual and market PS, geometrically, at the price: P_0 = DZD 6

We calculate the area above the supply curve and below the price line. Figure 6.2 reproduces the individual and market supply curves from Figure 4.7 of chapter 4.

Figure 6.2. Individual and Market Producer Surplus



Individual PS_1 = the area of the pink-shaded triangle

PS_1 = (the base x the height)/2 = [8 x (6-2)]/2 = **DZD 16**

Individual PS_2 = the area of the green-shaded triangle

PS_2 = (the base x the height)/2 = [8 x (6-2)]/2 = **DZD 16**

Market PS = the area of the blue-shaded triangle

= (the base x the height)/2 = [16 x (6-2)]/2 = **DZD 32**

PS = PS_1 + PS_2 = 16 + 16 **= DZD 32**

2. Government Intervention in Markets

2.1. Price Policy (price Controls)

When a government intervenes to regulate prices, we say that it imposes **price controls** which *are legal restrictions on how high or low a market price may go*. These controls typically take the form either of a price ceiling, or a price floor.

2.1.1. Price Ceiling

2.1.1.1. What is a Price Ceiling?

A price ceiling is *a legislated maximum price imposed by the government at which a good can be sold in a market* (Browning & Zupan, 2015, p. 25). Under this price it is illegal to charge a price higher than the ceiling.

2.1.1.2. The purpose of price ceiling policy

Price ceiling is typically imposed in the case of rent control (rent ceiling) to make housing affordable for everyone, or in order to provide necessities such as: milk, bread, medicines, gasoline, etc. for the poor people or those who have limited incomes at lower prices. It is also imposed during crises—wars, harvest failures, natural disasters—because these events often lead to sudden price increases that hurt many people but produce big gains for a lucky few.

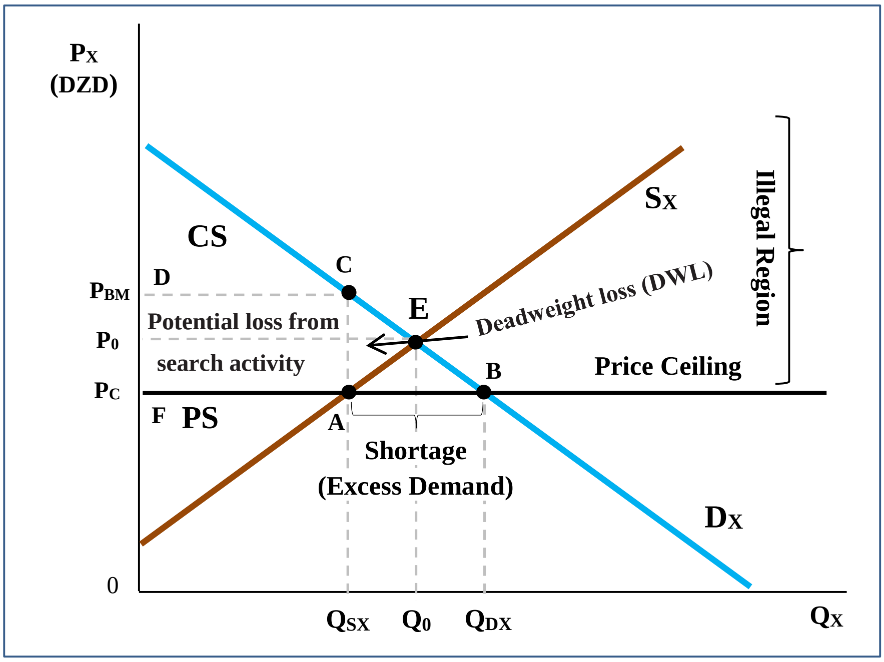
2.1.1.3. How Price Ceilings Affect Market Outcomes

The effects of a price ceiling on a market depend crucially on whether the ceiling is imposed at a level that is above or below the equilibrium price.

A price ceiling set *above the equilibrium price* has no effect. The reason is that it does not constrain the market forces. But a price ceiling set *below the equilibrium price* has powerful effects on a market. The reason is that it attempts to prevent the price from regulating the quantities demanded and supplied. Therefore, for a price ceiling to be *effective* and has effect on the price or the quantity sold, it has to be *below* the equilibrium price. In this case; the ceiling will be a *binding constraint* on the market (Mankiw, 2015, p. 114).

Figure 6.3 shows the market for a good (X), which is necessary for consumers. Suppose that the government imposes a price ceiling (**P_C**) below the market equilibrium price.

Figure 6.3. A Price Ceiling in the Market for a Good (X)



Without a price ceiling, the market clears at equilibrium point E. If a price ceiling is set at P_C, then there will be *a shortage— a persistent excess demand* for the good of: $\Delta Q_{DX} = Q_{DX} - Q_{SX}$. The excess demand would ordinarily force the price back up to the equilibrium price. But now the price ceiling prevents this from occurring and leads to an equilibrium with an excess demand.

The shortage resulting from a price ceiling set below the equilibrium price creates unintended negative side effects in the market, which are (Krugman & Wells, 2009, pp. 121-125; Parkin, 2012, pp. 128-129):

- **Increased search activity:** Buyers may have to go from store to store, and use resources such as phone calls, automobiles, and gasoline, searching for scarce goods.

- **Rapid disappearance of the good:** All the quantities produced of the good will quickly disappear from stores shelves, and only lucky consumers get to buy it at the low price.
- **Supply rationing:** A method of allocating the limited supply of the good among consumers, where one cannot buy more than the quota fixed.
- **Sale discrimination (Favoritism):** Sellers may use criteria other than price to sell the scarce good, selling it only to their friends, relatives, or long-term customers etc.
- **Inefficiency:** A decrease in the quantity supplied (underproduction), CS and PS shrink, and a deadweight loss (the triangle ACE) arises. The full loss is the sum of the deadweight loss and the increased cost of search (the rectangle ACDF).
- **Wasted resources:** Buyers will spend more time waiting in lines or shopping around. That is, there is *an opportunity cost* to the buyers—the leisure or income they had to forgo.
- **Inefficiently low quality:** Suppliers produce goods that are of inefficiently low quality at a low price and they refuse to improve the quality of their products because the price ceiling prevents them being compensated for doing so.
- **The emergence of black market:** A market in which goods are sold illegally at a price higher than the legal ceiling. The scalpers can sell the good at the highest price $\mathbf{P_{BM}}$ **(**at point **C**).

2.1.2. Price Floor

2.1.2.1. What is a Price Floor?

A price floor is *a legislated minimum price imposed by the government at which a good can be sold in a market* (Browning & Zupan, 2015, p. 25). Under this price it is illegal to charge a price lower than the floor.

2.1.2.2. The purpose of price floor policy

Price floors have been widely legislated for agricultural products, such as cotton, wheat, rice, corn, peanuts, dairy products, and many other farm goods, as a way to support the incomes of farmers. This policy is commonly called *price support programs*. Historically, there were also price floors on services such as trucking and air travel, although these were phased out by the USA in the 1970s. Many countries in the world maintain a lower limit on the wage rate—that is, a floor on the price of labor, called *the minimum wage* to help people escape poverty.

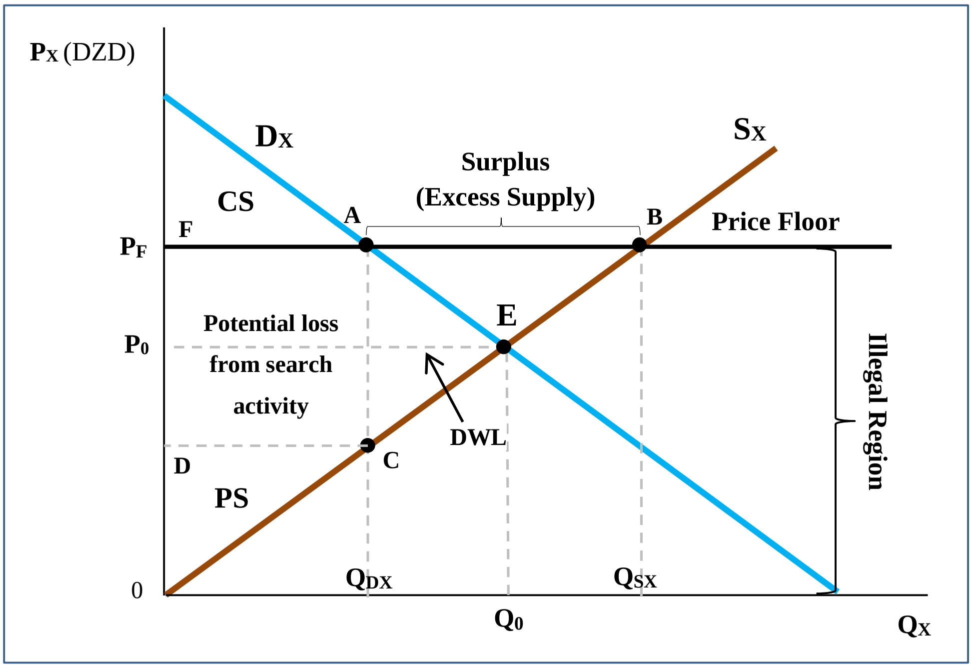
2.1.2.3. How Price Floors Affect Market Outcomes

The effects of a price floor on a market also depend on whether the floor is imposed at a level that is above or below the equilibrium price.

A price floor set *below the equilibrium price* has no effect. The reason is that it does not constrain the market forces. But a price floor set *above the equilibrium price* has powerful effects on a market. The reason is that it attempts to prevent the price from regulating the quantities demanded and supplied. Therefore, for a price floor to be *effective* and has effect on the price or the quantity sold, it has to be *above* the equilibrium price. In this case; the floor will be a *binding constraint* on the market (Mankiw, 2015, p. 119).

Figure 6.4 shows the market for a farm good (X). Suppose that the government imposes a price floor (**P_F**) above the market equilibrium price.

Figure 6.4. A Price Floor in the Market for a Good (X)



Before any price floor is imposed—the market is in equilibrium at point E. If a price floor is regulated at **P_F**, then there would be *a surplus— a persistent excess supply* (*unemployment* in the case of minimum wage) for the good of: **$\Delta Q_{SX} = Q_{SX} - Q_{DX}$**. The excess supply would ordinarily push the market price down to the equilibrium price. But now the price floor prevents this from occurring and leads to an equilibrium with an excess supply.

In order to maintain the new equilibrium situation, the government must *buy up* the entire excess supply at the floor price. But, in many cases, it finds itself warehousing thousands of tons of goods which increases its costs. For that, the government should find ways to dispose of these unwanted goods, and then limit the costs. Some of these ways are the following:

- Developing effective systems to control the production and the sale of goods;
- Giving surplus goods to the poor or as international aids, and in some cases, governments destroy the surplus production;
- Paying subsidies to exporters to sell excess products outside the country at a low price;
- Imposing strict limits on imports of some products from abroad.

Besides the surplus, a price floor set above the equilibrium price creates other unintended negative effects, which are (Krugman & Wells, 2009, pp. 130-131; Parkin, 2012, pp. 131-132; Mankiw, 2015, pp. 117-119):

- **Increased search activity:** This is especially for labor market. Frustrated unemployed workers spend more time and other resources searching for hard-to-find jobs, or waiting in lines in the hope of getting jobs.
- **Rationing mechanisms:** Sellers who appeal to the personal biases of the buyers, perhaps due to racial or familial ties, may be better able to sell their goods than those who do not. By contrast, in a free market, the price serves as the rationing mechanism, and sellers can sell all they want at the equilibrium price.
- **Teenage labor market:** The minimum wage affects the least skilled and least experienced workers including teenagers. It encourages them to drop out of schools, and prevents some unskilled workers from getting the on-the-job training they need.
- **Inefficiency:** An increase in the quantity supplied (overproduction), CS and PS shrink, and a deadweight loss (the triangle ACE) arises. The full loss is the sum of the deadweight loss and the increased cost of search (the rectangle ACDF).
- **Wasted Resources:** The unwanted surplus is sometimes destroyed, and in some cases the stored produce goes "out of condition" and must be thrown away. In addition, the minimum wage also leads to wasted time and effort, when workers spend time and wait in lines searching for jobs.
- **Inefficiently high quality:** Suppliers produce goods that are of inefficiently high quality at a high price. Even though some buyers would prefer lower quality products at a lower price. But suppliers and buyers could make a mutually beneficial deal in which buyers got goods of somewhat lower quality for a much lower price.
- **The emergence of black labor market:** If the minimum wage is far above the equilibrium wage rate, workers desperate for jobs sometimes agree to work off the books for employers who conceal their employment from the government—or bribe the government inspectors.

2.2. Fiscal Policy

2.2.1. Taxes

2.2.1.1. What is a Tax?

A tax is *a certain amount of money that must be paid to the government for each unit of the goods or services sold and bought in the market* (Pindyck & Rubinfeld, 2013, p. 345).

Usually, governments impose taxes to obtain tax revenues, which are the primary source of the funds that keep them operating at the local levels. But in addition to providing revenue, taxes also have important effects on markets: They change the behavior of buyers and sellers, and alter the equilibrium price and quantity of goods exchanged.

2.2.1.2. Types of Taxes

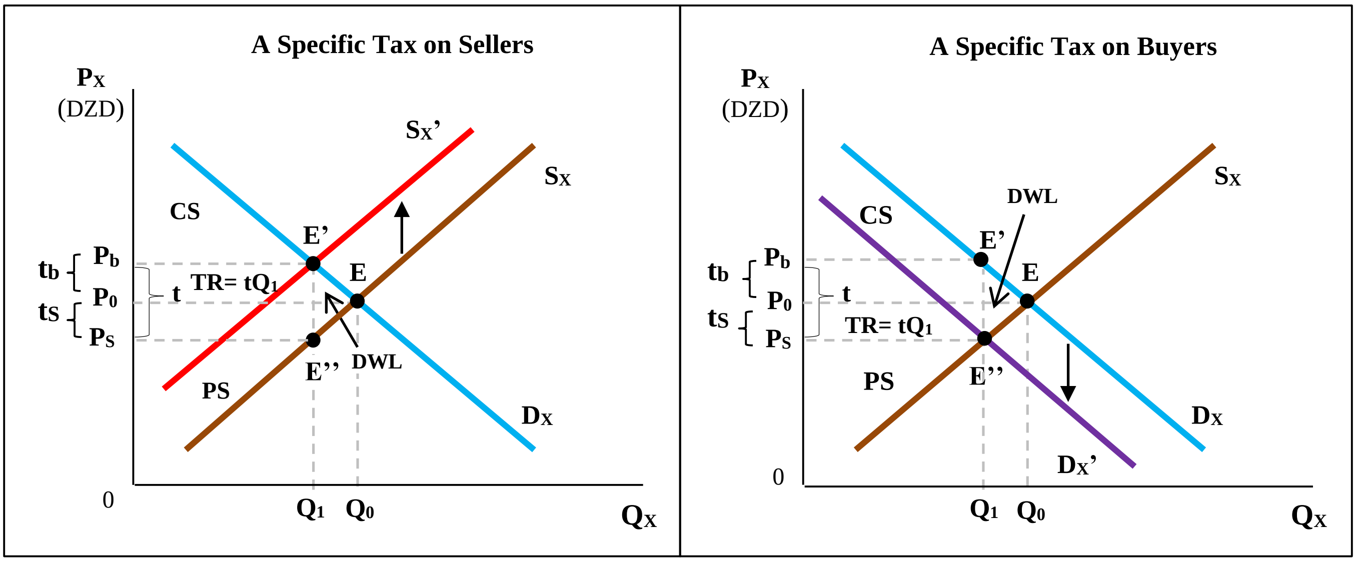
There are *two types* of taxes: a *specific tax* and an *ad valorem tax* (Perloff, 2020, pp. 41-42):

- **Specific tax**: is a tax of a certain amount of money *t* per unit of goods sold. This type is applied to relatively few goods, such as: cigarettes, gasoline, and airline tickets. For example, in the USA, the federal government collects *t* = ¢18.4 on each gallon of gasoline sold.
- **Ad valorem tax:** is a tax on the value—the price—of a good and usually expressed in percentage terms. For example, if the tax is 8 percent, then a good that is priced at DZD 10 will actually sell for DZD 10.8. Governments levy ad valorem taxes on a wide variety of goods and services, exempting only a few staples such as food and medicine.

2.2.1.3. Effects of a Specific Tax on Market

Governments can levy taxes whether on sellers or on buyers. The tax on sellers has the same effects as the tax on buyers. To illustrate these effects, we suppose that the government imposes a specific tax ***t*** per unit on a good (X), as shown in Figure 6.5.

Figure 6.5. Effects of a Specific Tax on Market Equilibrium for a Good (X)



Before the tax is imposed, the market equilibrium occurs at the point **E**. If a specific tax ***t*** per unit is imposed on sellers, then the supply curve shifts upward to the left from S_X to S_X'. The new equilibrium occurs when the new supply curve intersects the original demand curve D_X at point E'. If the tax is imposed on buyers, then the demand curve shifts downward to the left from D_X to D_X'. The new equilibrium occurs when the new demand curve intersects the original supply curve S_X at point **E''**. We can see that the tax on sellers has the same effects as the tax on buyers, in both cases the new equilibrium quantity $\mathbf{Q_1}$ decreases, and there will appear two prices: the buyers' price ($\mathbf{P_b}$), and the sellers' price ($\mathbf{P_S}$)—the tax drives a wedge between the two prices. The burden of the tax that falls on buyers is: $\mathbf{t_b = P_b - P_0}$. The burden of the tax that falls on sellers is: $\mathbf{t_S = P_0 - P_S}$. Thus, both buyers and sellers share the burden of the tax (The division of the burden of a tax between buyers and sellers depends on the elasticities of demand and suppl as we will see later). We can lose an amount of their surplus given by the area of trapezoid $\mathbf{EE''P_SP_0}$, and the government earns tax revenue given by the area of rectangle $E'E''P_SP_b$ which is: $TR = tQ_1$—a *transfer* from the buyers and sellers to the government. The triangle also see that, buyers lose an amount of their surplus given by the area of trapezoid $\mathbf{EE'P_bP_0}$, sellers **EE'E''** represents *the deadweight loss*—a part of **CS** and **PS** that is not transferred to the government which considered to be a net loss of surplus to the whole economy.

2.2.1.4. Finding the After-Tax Equilibrium

As Figure 6.5 shows, market clearing requires *four conditions* to be satisfied after the specific tax is in place (Pindyck & Rubinfeld, 2013, p. 346):

1. The quantity sold and the buyer's price $\mathbf{P_b}$ must lie on the demand curve (because buyers are interested only in the price they must pay).
2. The quantity sold and the seller's price $\mathbf{P_S}$ must lie on the supply curve (because sellers are concerned only with the amount of money they receive net of the tax).
3. The quantity demanded must equal the quantity supplied.
4. The difference between the price buyers pay and the price sellers receive must equal the tax t.

These conditions can be summarized by the following four equations:

$$Q_{DX} = f(P_b)$$

$$Q_{SX} = f(P_s)$$

$$Q_{DX} = Q_{SX}$$

$$P_b - P_s = t$$

If Q_{DX}, Q_{SX}, and t are given, we can solve these equations to find $\mathbf{Q_1}$, $\mathbf{P_b}$, $\mathbf{P_S}$, $\mathbf{t_b}$, $\mathbf{t_S}$, and **TR**.

2.2.1.5. Tax Incidence and Elasticity

Tax incidence is *the division of the burden of a tax between buyers and sellers* (Parkin, 2012, p. 133). When the government imposes a tax on a good, buyers and sellers of the good share the burden of the tax. The division of the burden of a tax between buyers depends on the relative elasticities of demand and supply. If demand is very inelastic relative to supply, buyers will bear the greatest burden of the tax, and if demand is very elastic relative to supply, sellers will bear the greatest burden of it.

In the extreme cases, if demand is perfectly inelastic or supply is perfectly elastic, buyers bear the entire burden of the tax. If demand is perfectly elastic or supply is perfectly inelastic, sellers bear the entire burden of the tax. This leads us to a general rule about how a tax will be divided: *"A tax burden falls more heavily on the side of the market that is less elastic".*

2.2.2. Subsidies

2.2.2.1. What is a Subsidy?

A subsidy is *the opposite of a tax—it is a payment from the government rather than to the government*. It reduces the buyer's price below the seller's price, so we can think of a subsidy as a *negative tax* (Pindyck & Rubinfeld, 2013, p. 348).

Usually, governments grant subsidies to producers, especially farmers, to encourage them to increase and improve domestic production. For example, in the USA, the producers of peanuts, sugar-beets, milk, wheat, and many other farm products receive subsidies. Subsidies may also be given to consumers to help them buying some necessities, such as milk, medicines, etc.

2.2.2.2. Types of Subsidies

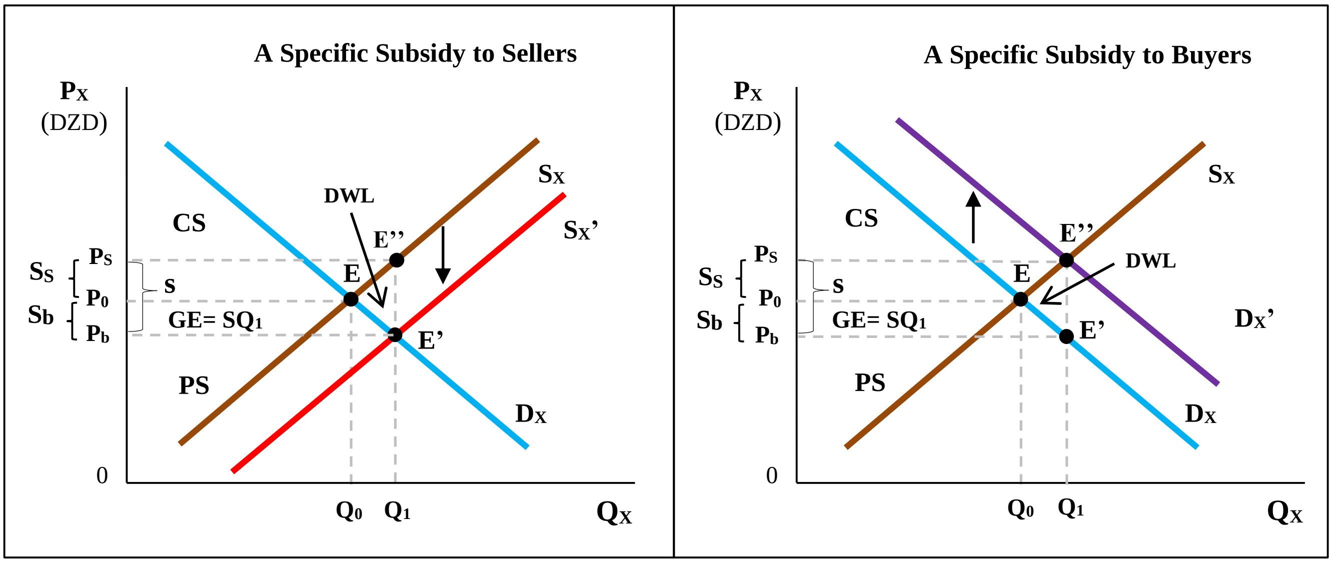
There are also two types of subsidies: a *specific subsidy* and an *ad valorem subsidy*:

- **Specific subsidy**: The government gives a certain amount of money, *S*, per unit of goods sold and bought. If, for example, the consumption of milk is subsidized, the government would pay some amount of money to each consumer of milk depending on the amount that the consumer purchased.
- **Ad valorem subsidy:** It is given based on the price of the good being subsidized. This type is usually expressed in percentage terms. For example, if the price of the good is DZD 80, and the rate of the subsidy is 15%, then the amount of the subsidy granted by the government is, S = 15%(80) = DZD 12.

2.2.2.3. Effects of a Subsidy on Market

The effects of a subsidy are similar to the effects of a tax but they go in the opposite directions. To illustrate these effects, we suppose that the government grants a specific subsidy *S* per unit on the good (X), as shown in Figure 6.6.

Figure 6.6. Effects of a Specific Subsidy on Market Equilibrium for a Good (X)



With no subsidy, the market equilibrium occurs at the point E. If a specific subsidy ***S*** per unit is granted to sellers, then the supply curve shifts downward to the right from S_X to

S_X'. The new equilibrium occurs when the new supply curve intersects the original demand curve D_X at point E'. If the subsidy is granted to buyers, then the demand curve shifts upward to the right from D_X to D_X'. The new equilibrium occurs when the new demand curve intersects the original supply curve S_X at point E''. We can see that the subsidy to sellers has the same effects as the subsidy to buyers, in both cases the new equilibrium quantity $\mathbf{Q_1}$ increases, and there will appear two prices: the sellers' price ($\mathbf{P_S}$), and the buyers' price ($\mathbf{P_b}$)—the subsidy drives a wedge between the two prices. The part of the subsidy that sellers benefit from is: $\mathbf{S_S = P_S - P_0}$. The part of the subsidy that buyers benefit from is: $\mathbf{S_b = P_0 - P_b}$. Thus, both buyers and sellers share the benefit of the subsidy (The division of the subsidy between buyers and sellers depends on the relative elasticities of demand and suppl as we will see later). We can also see that, CS increases by an additional amount given by the area of trapezoid $\mathbf{EE'P_bP_0}$, PS increases by an additional amount given by the area of trapezoid $\mathbf{EE''P_sP_0}$, and the government total expenditure is given by the area of rectangle $\boldsymbol{E'E''P_sP_b}$ which is: $\boldsymbol{TE = SQ_1}$—a *transfer* from the government to the buyers and sellers. The triangle **EE'E''** represents *the deadweight loss* from the subsidy.

2.2.2.4. Finding the After-Subsidy Equilibrium

As with a tax, the same four conditions are needed for the market to clear apply for a subsidy, but now the difference between the sellers' price and the buyers' price is equal to the subsidy. Again, we can write these conditions algebraically:

$$Q_{DX} = f(P_b)$$
$$Q_{SX} = f(P_s)$$
$$Q_{DX} = Q_{SX}$$
$$P_s - P_b = S$$

If we know Q_{DX}, Q_{SX}, and S, we can solve these equations to find $\mathbf{Q_1}$, $\mathbf{P_b}$, $\mathbf{P_S}$, $\mathbf{S_b}$, $\mathbf{S_S}$, and **TE**.

2.2.2.5. Subsidy Incidence and Elasticity

Subsidy incidence is *the division of the benefit of a subsidy between buyers and sellers*. As with a tax, the division of the benefit of a subsidy between buyers and sellers depends on the relative elasticities of demand and supply.

If demand is very inelastic relative to supply, buyers will benefit from the greatest part of the subsidy, and if demand is very elastic relative to supply, sellers will benefit from the greatest part of it. In the extreme cases, if demand is perfectly inelastic or supply is perfectly elastic, buyers benefit from the entire subsidy. If demand is perfectly elastic or supply is perfectly inelastic, sellers benefit from the entire subsidy. This leads us to a general rule about how a subsidy will be divided: *"The benefit of a subsidy accrues mostly to the side of the market that is less elastic"*.

******

Problems

1. The market for rental housing in a city has the following supply and demand schedules.

Rent ($ per unit per month)	150	300	450	600	750
Quantity Demanded (thousands of units per month)	40	30	20	10	0
Quantity Supplied (thousands of units per month)	0	10	20	30	40

a. Plot the demand and supply curves, and indicate the equilibrium point on your diagram?

b. Calculate the consumer surplus and the producer surplus geometrically?

c. If a rent ceiling is set at **$300** a month, what is the quantity of housing demanded and supplied? Is there a shortage or a surplus of housing, and how large will it be?

d. What is the maximum price that someone is willing to pay for the last unit of housing available (the maximum black market rent)?

e. What are the unintended negative effects that can result in the market from setting the rent ceiling?

2. Assume that the market for 'peanuts' in the United States is given by the following functions:

$$P = 1200 - 4Q$$

$$P = 200 + Q$$

Where, Q is the quantity of peanuts in tons per day; P is the price in dollars per ton.

a. Which of the two functions represents the market demand function, and which represents the market supply function? Why?

b. Find the market equilibrium price and quantity?

c. Calculate the consumer surplus and the producer surplus mathematically?

d. To support peanut producers, the government imposes a price floor of **$600** per ton. Does this price result in a shortage or a surplus? Show this graphically?

e. What are the unintended negative effects that can result in the market from imposing the price floor?

3. The market for 'cigarettes' in New York City is given by the following demand and supply functions:

$$Q_{Dc} = 425 - 25P_c$$

$$Q_{Sc} = -70 + 140P_c$$

Where, Q_{Dc} and Q_{Sc} are the quantities demanded and supplied of cigarettes respectively in millions of packs per year; P_c is the price of cigarettes in dollars per pack.

a. Solve for the market equilibrium price and quantity?

b. In order to recover some of the health care costs associated with smoking, the government imposes a specific tax of t = **$1.50** per pack on *sellers* of cigarettes. What are the new equilibrium price and quantity, the buyers' price P_b, the sellers' price S_S, the burden of the tax that falls on buyers t_b, the burden of the tax that falls on sellers t_S, and the total revenue **TR** earned by the government?

c. What is the best amount of the tax that maximizes the government's total revenue?

d. Suppose that instead of taxing sellers, the government taxes cigarette *buyers* (with the same amount of tax, t = **$1.50** a pack). Does the tax on buyers have the same effects as the tax on sellers or not? Explain?

4. The market demand and supply of 'wheat' in the United States is described by the following equations:

$$Q_{Dw} = 3100 - 125P_w$$

$$Q_{Sw} = 1540 + 115P_w$$

Where price is measured in dollars per bushel, and quantities in millions of bushels per year.

a. Solve for the market equilibrium price and quantity?

b. Calculate the price elasticity of demand and the price elasticity of supply at the equilibrium point?

c. Calculate the consumer surplus and producer surplus?

d. In order to encourage wheat producers to increase and improve domestic production, the government grants them a subsidy $s = 20\%$ of the equilibrium price. Who will benefit more from the subsidy? What are the new equilibrium price and quantity, the sellers' price P_S, the buyers' price P_b, the part of the subsidy that sellers benefit from S_S, the part of the subsidy that buyers benefit from S_b, and the government's total expenditure **TE**?

5. Suppose that the market demand function for 'water' from a mineral spring is: $Q_{Dw} = 200 - 2P_w$, and the market supply is constant at $Q_{Sw} = 100$.

Where, Q is the quantity demanded and supplied of water in thousands of bottles per week; P is the price in cents per bottle.

a. What is the market equilibrium price and quantity?

b. Calculate the consumer surplus and the producer surplus?

c. If the government imposes a specific tax of **¢5** a bottle. Find the new equilibrium price and quantity, the buyers' price P_b, and the sellers' price P_S?

d. Who will bear the entire burden of the tax, buyers or sellers? Why?

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Chapter 7: Producer Behavior Analysis (Production Theory)

Learning Objectives

By the end of this chapter, students will be able to:

- Understand the meaning of production and production function.
- Distinguish between production in the short-run and in the long-run.
- Analyze production in the short-run, including distinguishing fixed and variable inputs, defining total, average, and marginal product and understanding the relationships among product curves, as well as explaining the law of diminishing marginal returns.
- Analyze production in the long-run, including understanding the concept of diminishing marginal rate of technical substitution (MRTS), determining the producer's optimal choice of inputs using isoquants and isocosts curves, and exploring returns to scale and homogeneity of the production function.
- Explore Cobb-Douglas production punctions as specific forms of production functions in the long-run.
- Determine the firm's expansion path and the corresponding total cost curves in the short and the long runs.

1. The Production: Basic Concepts

1.1. Meaning of Production

Production is *the process by which inputs are combined, transformed, and turned into outputs* (Case, Fair, & Oster, 2017, p. 176). **Inputs** or **factors of productions** are *the ingredients mixed together by a producer (a firm) through its technology to produce output (product), which* can be a *physical good,* or a *service* (Browning & Zupan, 2015, p. 160).

Firms use many types of inputs, most of which fall into three broad categories (Perloff, 2020, p. 181):

- **Labor Inputs** (***L***): Hours of work provided by managers and skilled workers (such as architects, economists, engineers, and plumbers), or less-skilled workers (such as custodians, construction laborers, and assembly-line workers).
- **Capital Inputs** (***K***): Long-lived inputs such as land, buildings (such as factories and stores), and equipment (such as machines and trucks).
- **Materials** (***M***): Natural resources and raw goods (such as oil, water, and wheat) and processed products (such as plastic, paper, and steel).

1.2. The production function

The production function is *a mathematical representation that shows the maximum quantity of output a firm can produce given the quantities of inputs that it might employ* (Besanko & Braeutigam, 2020, p. 218).

Mathematically, the firm's production function can be written as: $q = f(L_b, K, L_n, M, \ldots)$

Where: q *units of output (total product,* ***TP****),*

L_b *units of labor,*

K *units of capital,*

L_n *units of land,*

M *units of materials.*

Although in practice firms use a wide variety of inputs, we will keep our analysis simple by focusing on only two inputs, labor ***L*** and capital ***K***. We can then write the production function as:

$$q = f(L, K)$$

This function indicates what is **technologically efficient**—*the maximum output the firm can produce from any given combinations of labor and capital inputs* (Browning & Zupan, 2015, p. 160).

2. Production and Time Frames

As the period of time is changed, producers have more opportunities to alter inputs and technology. Generally, two time periods are used in the analysis of production: the *short* and the *long* time periods.

2.1. Production in the Short-Run

2.1.1. Meaning of the Short-Run

The short run is a period of time in which the quantity of at least one input is fixed (Greenlaw & Shapiro, 2018, p. 160). For most firms, capital, land, and entrepreneurship are fixed inputs and labor is the variable input.

The firm can increase or decrease its output in the short run by increasing or decreasing the quantity of a variable input, which is usually labor.

2.1.2. Fixed Inputs and Variable Inputs

As we said above, in the short-run, at least one of the firm's inputs cannot be varied. As a result, the firm will have two types of inputs: fixed and variable.

Fixed inputs are *those whose quantity cannot be changed as output changes*. **Variable inputs** are *those whose quantity whose quantity can be changed as output changes* (Arnold, 2008, p. 172).

In the short-run, firms can only change their variable inputs (labor). For example, a firm might decide this month to increase its production by 10 percent by obtaining additional workers. But it cannot change their fixed inputs (capital) for another year or more, and these inputs are used to refer to the "size of a plant".

2.1.3. The Production Function in the Short-Run

As we have mentioned earlier, a production function indicates the highest output that a firm can produce for every specified combination of inputs. In the short-run, we assume that labor is a variable input and capital is a fixed input, so the firm can increase output only by increasing the amount of labor it uses. In the short-run, the firm's production function is: $q = f(L, \overline{K})$

Where q is total output produced, L is units of labor (workers) employed, and $\overline{K}$ is the fixed number of units of capital used.

2.1.4. Total Product, Average Product, and Marginal Product

We describe the relationship between output and the quantity of labor employed in the short-run by using three related concepts: total product, average product, marginal product of labor. (Note that we can use the term '**productivity**' in place of '**product**', so the three concepts are: total productivity, average productivity, and marginal productivity of labor.)

- **Total Product of Labor (q** or **TP_L):** the maximum amount of output that can be produced by a firm by employing the variable factor which is labor, holding the quantity of other inputs fixed.

- **Average Product of Labor (AP_L):** the amount of output produced per unit of labor used. It measures how much output each worker produces on average. Mathematically: $\mathbf{AP_L = \frac{q}{L}}$
- **Marginal Product of Labor (MP_L):** the additional amount of output produced when one more unit of labor is employed. Mathematically: $\mathbf{MP_L = \frac{\Delta q}{\Delta L}}$

2.1.5. The Relationships Among the Product Curves

The product curves in the short-run are graphs of the relationships between labor and TP_L, AP_L, and MP_L. They show how these three products change as labor changes. They also show the relationships among the three products. To understand these relationships, we take the following example.

Example (1): Suppose that a firm uses only two inputs—L and K—to produce its output. We assume that this firm is stuck with a certain amount of capital at **K = 10 units**, and it can vary only the amount of labor (the number of workers). The table below shows the amount of the two inputs and the total product produced per month.

K	10	10	10	10	10	10	10	10	10	10	10
L	0	1	2	3	4	5	6	7	8	9	10
TP_L	0	10	30	60	80	95	108	112	112	108	100

a. Calculate AP_L and MP_L?

b. Draw the TP_L, AP_L and MP_L curves? And explain how these curves vary with the amount of L employed?

c. Show geometrically how AP_L and MP_L curves can be derived from TP_L curve?

d. What is the relationship between these three curves?

e. Determine the stages of production? What is the optimal stage for the producer? Why?

Answer:

a. Calculating AP_L and MP_L:

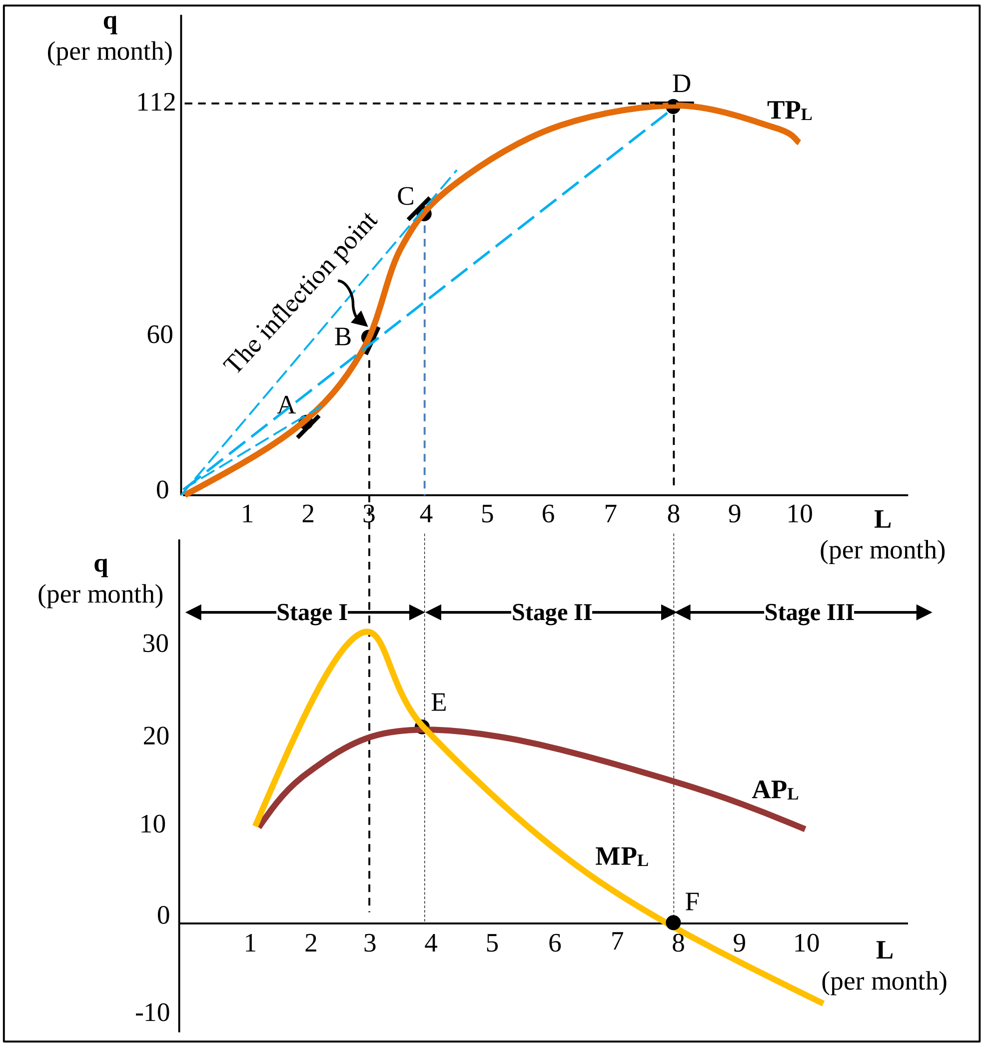
$\mathbf{AP_L = \frac{TP_L}{L}}$; $AP_{L1} = \frac{TP_{L1}}{L_1} = \frac{0}{0}$ = - ; $AP_{L2} = \frac{TP_{L2}}{L_2} = \frac{10}{1}$ = **10** ; $AP_{L3} = \frac{TP_{L3}}{L_3} = \frac{30}{2}$ = **15 …**

$\mathbf{MP_L = \frac{\Delta TP_L}{\Delta L}}$; $MP_{L1} = \frac{\Delta TP_L}{\Delta L} = \frac{10-0}{1-0}$ = **10** ; $MP_{L2} = \frac{\Delta TP_L}{\Delta L} = \frac{30-10}{2-1}$ = **20 …**

AP_L	—	**10**	**15**	**20**	**20**	**19**	**18**	**16**	**14**	**12**	**10**
MP_L	—	**10**	**20**	**30**	**20**	**15**	**13**	**4**	**0**	**- 4**	**- 8**

b. TP_L, AP_L, and MP_L Curves and the Relationship Between Them:

Figure 7.1. Production with one Variable Input



The TP_L curve starts from the origin, increases at *an increasing rate* until the point B (the inflection point), then increases at *a decreasing rate*, reaches a maximum at point D at **112** units, when **8** workers are employed; thereafter, it falls.

The AP_L and MP_L curves are derived from the TP_L curve. As employment of labor increases, MP_L increases at first, reaches a maximum at **30** units, when **3** workers are employed, and then declines. The AP_L also increases at low levels, reaches a maximum at **20** units, when **4** workers are employed, and then declines.

c. The Geometry of Product Curves:

We can use geometrically relationships to derive AP_L and MP_L curves from TP_L curve.

- *AP_L at any point on the TP_L curve is shown geometrically by the slope of the straight-line from the origin to that point*. For example, at point A, AP_L is shown by the slope of the straight-line segment OA drawn from the origin to point A on the TP_L curve, or $\frac{30}{2}$ = 15 units of output per unit of labor.
- *MP_L at any point on the TP_L curve is shown geometrically by the slope of TP_L at that point. The slope of TP_L curve is, in turn, equal to the slope of a line tangent to the curve*. For example, at

point C, we have drawn a line tangent to the TP_L with a slope of $\frac{80}{4}$. Thus, MP_L at this point is equal 20 units of output per unit of labor.

d. The Relationship Between TP_L, AP_L and MP_L Curves:

When TP_L curve initially increases at an increasing rate until the inflection point, both AP_L and MP_L curves increase, MP_L increases at a faster rate than AP_L until reaches its maximum (determined by the inflection point B on TP_L curve) and then declines, but AP_L curve continues to increase until reaches its maximum as it intersects MP_L curve, and MP_L is above AP_L in this range. When TP_L curve continues to rise, beyond the inflection point, at a decreasing rate until reaches its maximum and then falls, both AP_L and MP_L curves are falling but they are positive, and MP_L becomes below AP_L. When TP_L is at maximum, MP_L is zero, and when TP_L falls, MP_L is negative, but AP_L continues to fall and it remains positive as long as the TP_L is positive.

e. The Stages of production:

We can use the relationship between the AP_L and MP_L curves to define *three* stages of production for labor, as shown in Figure 7.1.

- **Stage I**: goes from the origin to the point where the AP_L is maximum (or when AP_L intersects MP_L) at point E.
- **Stage II**: goes from the end of the first stage to the point where the MP_L is zero at point F (or where TP_L is at its maximum at point D).
- **Stage III**: covers the range over which the MP_L is negative (or when TP_L falls).

The producer will not operate in stage I because the producer always has an incentive to expand through this stage because rising AP_L means the average cost decreases as output is increased. In addition, this stage corresponds to stage III for capital, if we leave capital changes and hold labor constant (the MP_K is negative). Similarly, the producer will not operate in stage III, even with free labor; because it would be possible to increase TP_L by using less labor (the MP_L is negative). This leaves stage II as the only optimal (efficient) stage of production for the rational producer who wants to maximize efficiency of scarce variable input (the MP_L and MP_K curves are both positive but declining).

2.1.6. Returns to the Variable Input (Labor)

2.1.6.1.The Law of Variable Proportion and its Assumptions

The law of variable proportion states that: "*as we employ more and more units of the variable input (labor), keeping other inputs fixed, the TP_L increases at increasing rate in the beginning then increases at diminishing rate, and finally starts declining. That is, MP_L initially rises, when the level of employment of labor is low, but after reaching a certain level of employment, it starts falling but is positive and finally continues to fall and becomes negative*". (See figure 7.1)

The main assumptions of this law are (Deepashree, 2018, p. 138):

- State of technology remains unchanged;
- All units of the variable factor (labor) are homogeneous;
- There must always be some fixed input.

2.1.6.2. Marginal Returns to the Variable Input (Labor)

Almost every production process in the short-run has three features (Deepashree, 2018, pp. 139-140):

- **Increasing Marginal Returns to Labor:** the MP_L increases as more labor is employed. It goes from the origin to the point where the MP_L curve is maximum. In this range TP_L curve is increasing at an increasing rate. The reasons for increasing marginal returns are: *underutilization of fixed factor (capital), indivisibility of factors, and specialization and division of labor*.
- **Diminishing marginal returns to labor:** the MP_L decreases as more labor is employed. It ranges from the point where MP_L curve is maximum to the point where it is zero, that is, MP_L curve is positive but declining. But TP_L curve increases at a decreasing rate and reaches a maximum. The reasons for diminishing marginal returns are: *optimal use of fixed factor, and the lack of perfect substitutes between factors*.
- **Negative marginal returns to labor:** the MP_L becomes negative as more labor is employed. It covers the entire range over which MP_L curve is negative (additional worker slows down the production process). While TP_L curve falls but positive. The reason for negative marginal returns is: *over utilization of fixed factor*.

2.1.6.3. The Law of Diminishing Marginal Returns

The law of diminishing marginal returns (DMR) states that: "*As the amount of the variable input is increased in equal increments, while technology and other inputs held constant, the resulting increments in output will eventually begin to decrease. Put more briefly, the law holds that beyond some point the marginal product of the variable input will decline*" (Browning & Zupan, 2015, p. 165).

In figure 7.1 DMR set in when the amount of labor increases beyond *three* workers. Each additional worker beyond this number adds less to total product than the previous one until the *eighth* worker. So, the law of DMR operates as long as the MP_L curve declines and is positive.

2.2. Production in the Long-Run

2.2.1. Meaning of the Long-Run

The long-run is *a time period long enough for a firm to change all of its inputs* (Hall & Lieberman, 2010, p. 191). There are no fixed inputs in the long-run; all inputs become variable inputs. As we have mentioned before, our analysis focuses on only labor (L) and capital (K), thus, in the long run the two inputs are variable.

2.2.2. The Production Function in the Long-Run

We started our analysis of production function by looking at a short-run production function in which one input (labor) is variable and the other (capital) is fixed. However, in the long run, both of these inputs are variable and a firm can produce a given level of output by using a specified combination of both inputs. In the long run, the firm's production function is: $q = f(L, K)$

Where q is total output produced, L is units of labor (workers) employed, and K is the number of units of capital used.

2.2.3. Production Isoquants

An **isoquant** is *a curve that shows all the possible combinations of labor and capital that result in the same quantity of production* (Taylor & Weerapana , 2009, p. 238).

Example (2): Suppose that a firm uses two inputs (L and K) to produce a given level of output. The table below tabulates the output achievable for various combinations of inputs.

	L				
K	**1**	**2**	**3**	**4**	**5**
1	20	40	**55**	65	**75**
2	40	60	**75**	85	**90**
3	**55**	**75**	**90**	100	105
4	65	85	100	110	115
5	**75**	**90**	105	115	120

a. Draw three isoquants that show all combinations of (L, K) that together yield **55**, **75**, and **90** units of output for each isoquant?

b. What are the characteristics of isoquants (compare with those of indifference curves)?

Answer:

a. The three isoquants that yield 55, 75, and 90 units of output:

Figure 7.2. Production Isoquants

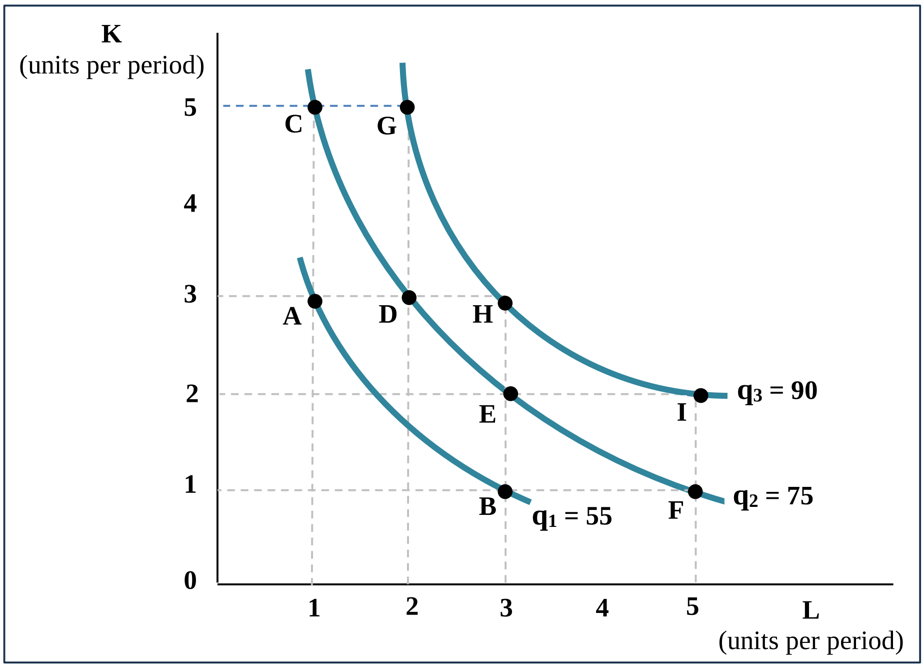


Figure 7.2 shows three isoquants of the many isoquants that can be drawn using the information of the table. Isoquant q_1 shows all combinations of labor and capital (L, K) that together yield 55 units of output per period including points A (1,3) and B (3,1). Isoquant q_2 shows all combinations of inputs that yield 75 units of output including points C (1,5), D (2,3), E (3,2), and F (5,1). Isoquant q_3 shows all combinations that yield 90 units of output including points G (2,5), H (3,3), and I (5,2). A set of isoquants is called **isoquant map**, which describes the firm's production function, just as an *indifference map* that describes the utility function.

b. Characteristics of Isoquants

Isoquants are very similar to *indifference curves* in their characteristics. While indifference curves order levels of a consumer's satisfaction from low to high, isoquants order levels of a producer's output. Isoquants have the same geometric properties as indifference curves: they are downward sloping, convex to the origin, nonintersecting, and a higher isoquant reflects a greater level of output. The main difference is that the quantities associated with isoquants have cardinal properties (each isoquant reflects a measurable output level), while the utilities associated with indifference curves have only ordinal properties (higher indifference curves reflect higher levels of consumer satisfaction).

2.2.4. Substitution Among Inputs (Marginal Rate of Technical Substitution)

2.2.4.1. The Concept of Marginal Rate of Technical Substitution (MRTS)

The marginal rate of technical substitution of labor for capital ($MRTS_{L,K}$) is *the amount of capital that can be substituted for 1 unit of labor, holding output constant* (Landsburg, 2014, p. 146). This is analogous to the $MRS_{X,Y}$ in consumer theory.

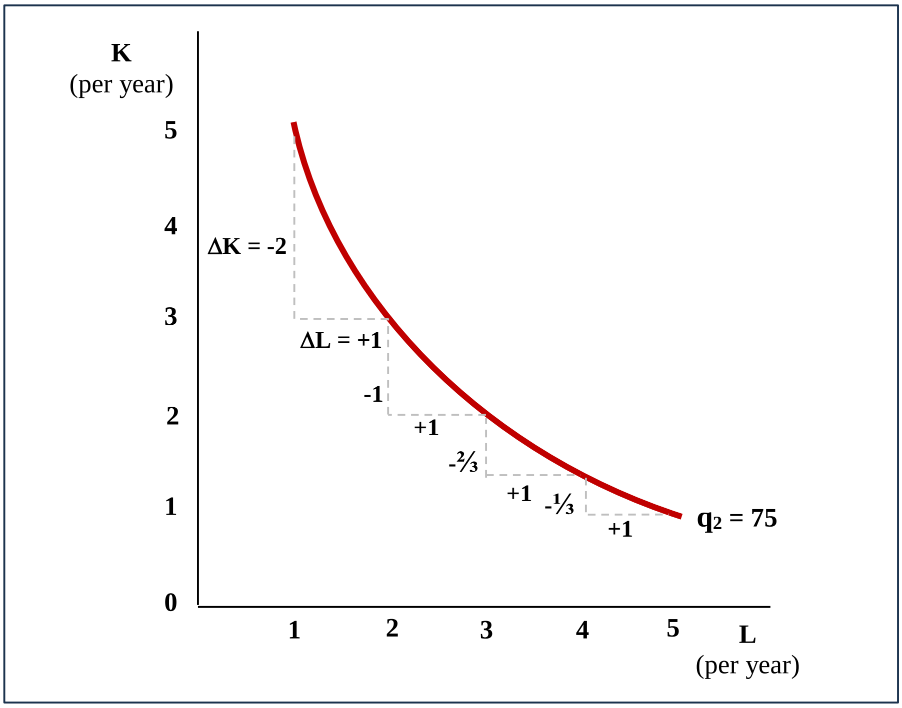
The definition of MRTS means that the producer can substitute one of the inputs to another while keeping output constant. The total gain in output from increasing labor is exactly balanced by the loss in output from decreasing capital, so, mathematically, the total change in output must equal zero. $\boldsymbol{MP_L(\Delta L) + MP_K(\Delta K) = 0}$

We can rearrange this equation to find $MRTS_{L,K}$ equation: $\mathbf{MRTS_{L,K} = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}}$

As the producer replaces more and more of *capital* to use more of *labor*, the MP_L falls and the MP_K increases, so MRTS decreases.

The magnitude of the slope of the isoquant along any one of its segments measures the MRTS between the two inputs, as shown in the Figure 7.3.

Figure 7.3. The Marginal Rate of Technical Substitution



The slope of an isoquant (in absolute value) becomes smaller as we move down the curve from left to right. For example, on isoquant q_2, the MRTS falls from 2 to 1 to ⅔ to ⅓. As more and more labor replaces capital, labor becomes less productive and capital becomes relatively more productive, and the isoquant becomes flatter.

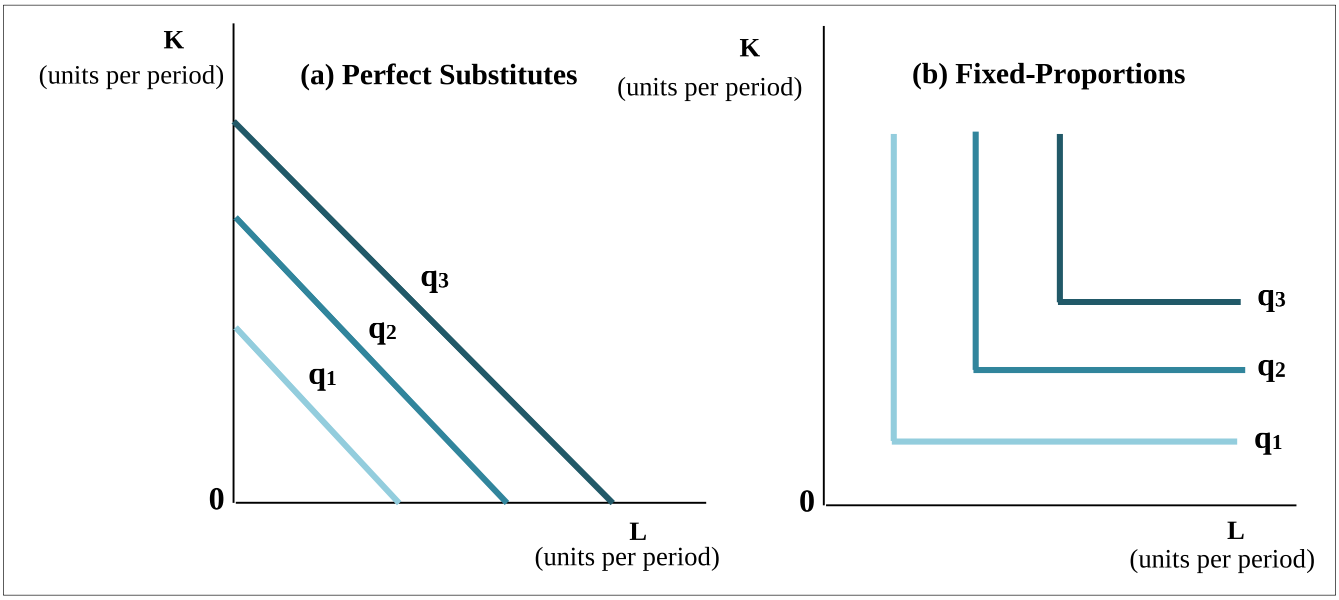
2.2.4.2. Diminishing Marginal Rate of Technical Substitution

The MRTS varies along a typical isoquant that is convex to the origin. *Convexity* of isoquants means that the MRTS diminishes as we move down each isoquant and this reflects **a diminishing MRTS**. The diminishing MRTS tells us that the productivity of any one input is limited. As more and more labor is added in place of capital, the productivity of labor falls. Similarly, when more capital is added in place of labor, the productivity of capital falls. Thus, production needs a balanced mix of both inputs (Pindyck & Rubinfeld, 2013, p. 219).

2.2.4.3. Perfect Substitutes and Perfect Complements

Two extreme cases of production functions show the possible range of input substitution in the production process as shown in Figure 7.4.

Figure 7.4. Perfect Substitutes and Perfect Complements



In panel (a), inputs to production are *perfect substitutes* for one another. The MRTS is constant at all points on an isoquant. In this case isoquants are *straight-lines*. Panel (b) illustrates the opposite extreme, *the fixed-proportions production function* (or *Leontief production function*). In this case, it is impossible to make any substitution among inputs. Each level of output requires a specific combination of labor and capital. As a result, the isoquants are *L-shaped*. This is just as indifference curves shape when two goods are perfect substitutes or complements.

2.2.5. The Isocost Line

An isocost line shows *all the combinations of labor and capital that can be purchased at a given total cost* (Browning & Zupan, 2015, p. 191). The total cost (TC) of producing any particular output is given by the sum of the firm's labor cost $P_L.L$ and its capital cost $P_K.K$, (it is similar to the budget line that the consumer faces): $\boldsymbol{TC = P_L.L + P_K.K}$ or, $\boldsymbol{TC = wL + rK}$

Where $\mathbf{P_L}$ is the price of L (or the wage rate **w**); $\mathbf{P_K}$ is the price of K (or the rental rate of capital **r**)

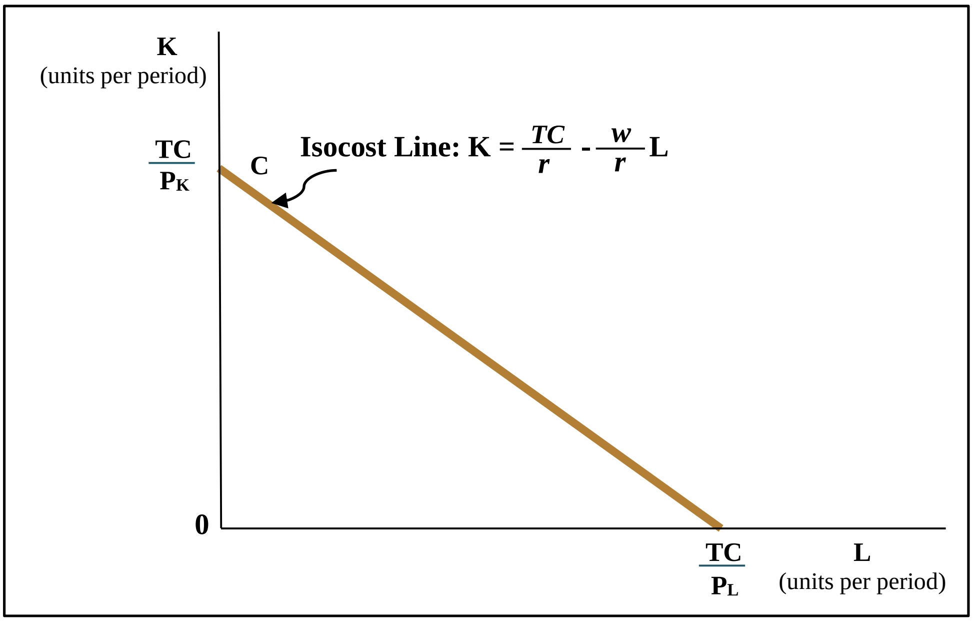
If we rewrite the TC equation as an equation for a straight line, we get:

$$K = \frac{TC}{P_K} - \frac{P_L}{P_K}L \qquad \text{or,} \qquad K = \frac{TC}{r} - \frac{w}{r}L$$

The slope of the isocost line is the derivative of K with respect to L: $\frac{dK}{dL} = -\frac{P_L}{P_K} = -\frac{w}{r}$

This slope indicates *how many units of capital a firm must give up to get one more unit of labor with spending the same total cost.*

Figure 7.5. The Isocost Line

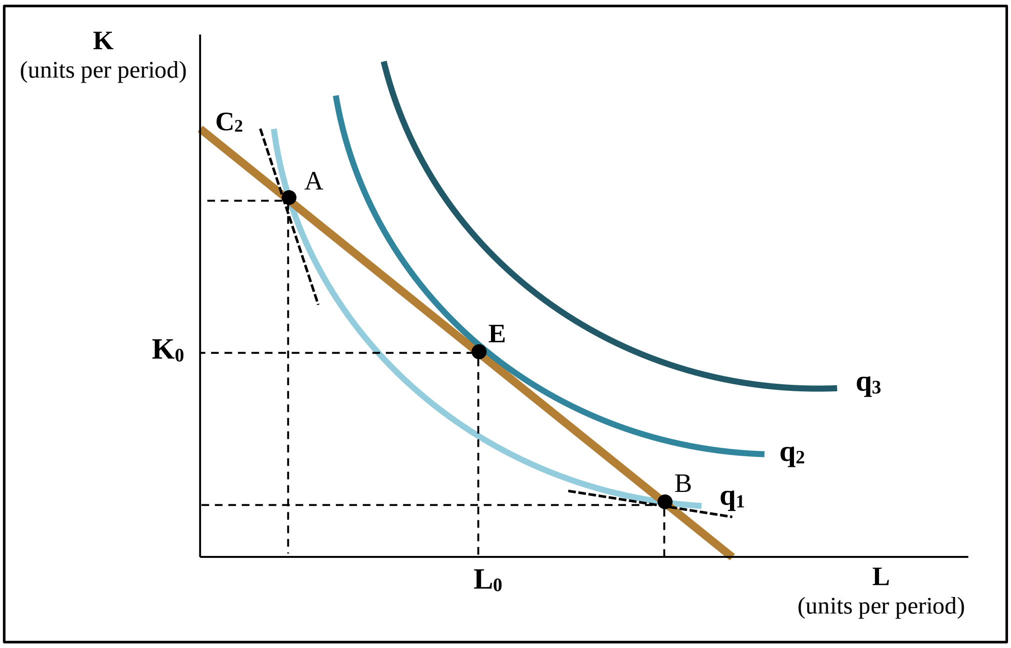


Note that a change in the firm's TC (with P_L and P_K unchanged) causes the isocost line to shift parallel to the original line, with no change in its slope. While, A change in the price of one input (with the price of the other input and the firm's TC unchanged) causes the isocost line to rotate about one intercept, but with a change in its slope, just like we have seen with the consumer's budget line.

2.2.6. The Optimal Choice of Inputs (The Producer's Equilibrium)

As in consumer theory, the optimal choice of inputs L and K can be analyzed as the problem of choosing the highest possible production isoquant *tangent to* a given isocost line. That means the slopes of the two curves must be equal. We can graphically illustrate the solution to the firm's choice problem and determine the point at which the firm maximizes its output.

Figure 7.6. Maximizing Firm's output



The equilibrium point of the firm is at point **E (L_0, K_0)** where isocost line **C_2** is tangent to the highest isoquant attainable **q_2** and the slopes of the two curves are equal ($MRTS_{L,K}= \frac{P_L}{P_K}$).

Note that, as in the consumer theory, there are two disequilibrium situations which are shown in Figure 7.6 by the two points **A** and **B** (Browning & Zupan, 2015, p. 193):

- At point **A**; $MRTS_{L,K} > \frac{P_L}{P_K}$: a firm can increase output without increasing production cost by hiring more labor and less capital. This causes a movement to the right along the isocost line C_2, which should continue until the equilibrium is established at the point E.
- At point **B**; $MRTS_{L,K} < \frac{P_L}{P_K}$: a firm can increase output without increasing production cost by hiring more capital and less labor. This causes a movement to the left along the isocost line C_2, which should continue until the equilibrium is established at the point E.

The condition of the producer's optimal choice (given the isocost line), can be expressed, mathematically, by the following equality:

$$\begin{cases} \frac{MP_L}{MP_K} = \frac{P_L}{P_K} \Rightarrow \frac{MP_L}{P_L} = \frac{MP_K}{P_K} \\ \text{Subject to the cost constraint: } \mathbf{TC_0 = P_L L + P_K L} \end{cases}$$

This is equivalent to the tangency condition and indicates that the firm should employ inputs in such a way that the marginal product per dinar spent is equal across all inputs.

We can also use the Lagrangian multiplier technique, by forming the Lagrangian expression:

$$L = f(L,K) + \lambda(TC_0 - P_L L - P_K K)$$

To solve this constrained optimization problem, we follow the same steps, as we did in dealing with the consumer's choice problem.

Also, as in consumer theory, the firm's input decision has a ***dual*** nature (Pindyck & Rubinfeld, 2013, p. 275). The optimum choice of K and L can be analyzed not only as the problem of choosing the highest production isoquant tangent to a given isocost line (output maximization problem), but also as the problem of choosing the lowest isocost line tangent to the production isoquant (cost minimization problem).

The cost-minimization problem can be written as:

$$\begin{cases} \text{Minimize } TC = wL + rK \\ \text{Subject to the constraint that: } f(K, L) = q_0 \end{cases}$$

The Lagrangian expression for this problem is: $L = P_L L + P_K K + \lambda[q_0 - f(L,K)]$

We solve for the optimal inputs **L_0** and **K_0** that minimize the firm's TC at a given output **q_0**, with the same way as we did before.

We can also derive Input demand functions from the firm's TC function through partial differentiation. These functions will depend on the quantity of output that the firm chooses to produce; therefore, they are called "*contingent*" demand functions (Nicholson & Snyder, 2012, p. 345).

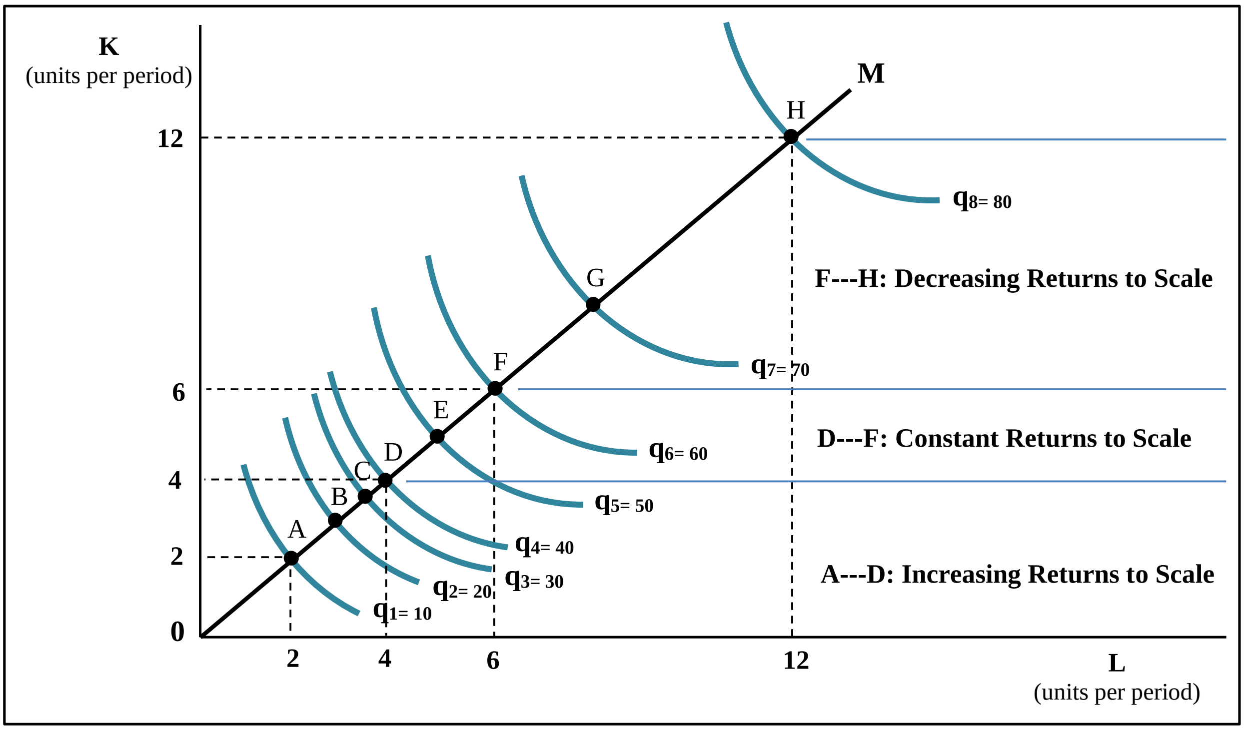
2.2.7. Returns to Scale

Returns to scale measure how output changes to a proportionate change in all inputs together. For example, if all inputs are doubled, returns to scale determine whether output will double, less than double, or more than double. Thus, there are three different cases: increasing, constant, and decreasing returns to scale (Browning & Zupan, 2015, p. 171).

- **Increasing Returns to Scale (IRS):** a situation in which output increases in greater proportion when all inputs increase in a given proportion.
- **Constant Returns to Scale (CRS):** a situation in which a proportional increase in all inputs increases output in the same proportion.
- **Decreasing Returns to Scale (DRS):** a situation in which output increases in lesser proportion when all inputs increase in a given proportion.

Returns to scale are usually assumed to be the same everywhere on the production surface. But they may vary over different ranges of output. Generally, returns to scale will be increasing at first (when the scale of operation is small), then pass through a stage of constant returns, and finally decreasing (when the scale of operation becomes large). That is, a production function can exhibit increasing, constant, and decreasing returns to scale at different levels of output, as shown in Figure 7.7.

Figure 7.7. Returns to Scale



The output varies along the ray OM. At first, in a move from point A to D, IRS occur: when both L and K are doubled, output more than doubles (it quadruples; from 10 to 40). Then, in a move from point D to F, a range of CRS prevail: increasing both L and K by a half, increases output by exactly a half (from 40 to 60). And then, in a move from point F to H, DRS result: when L and K are doubled, output less than doubles (it increases by only a third; from 60 to 80).

We can see that the spacing between successive multiple-isoquants provides a graphical method of ascertaining returns to scale. With IRS, from A to D, isoquants become closer and closer to one another as inputs are increased proportionately. With CRS, from D to F, the spacing is equidistant. And with DRS, from F to H, the spacing grows farther apart between isoquants.

2.2.8. Homogeneity of the Production Function and Returns to Scale

A homogeneous function is *one that if all inputs are multiplied by the same positive constant t (where t > 1), then t can be completely factored out of the function* (Koutsoyiannis, 1979, p. 77). That is, it may be taken out of the brackets as a common factor, and then the new level of output q' can be expressed as a function of t (to any power n): $q' = f(tL,tK) = t^n f(L,K) = t^n q$

The power ***n*** is the *degree of homogeneity* of the function and is a measure of the returns to scale:

- If ***n*** **< 1**; we have *DRS*.
- If ***n*** **= 1**; we have *CRS*. (This production function is sometimes called *linear homogeneous* production function).
- If ***n*** **> 1**; we have *IRS*.

Homogeneous production functions have two main properties (Nicholson & Snyder, 2012, p. 56):

- If a production function is homogeneous of degree *n* and can be differentiated, the partial derivatives of the function will be homogeneous of degree *n-1*.
- If a production function is homogeneous of degree *n*, then: $nq = Lf'_L + Kf'_K = L.MP_L + K.MP_K$

This equation is called '***Euler's theorem***' (named after the mathematician **Leonhard Euler**).

2.2.9. The Cobb-Douglas Production Functions

The **Cobb-Douglas** production function is *a specific form of production functions in the long run* (Pindyck & Rubinfeld, 2013, p. 276). It is the most widely used production function in empirical work. (it was named after American economists **Paul Douglas** and **Charles Cobb**). This function takes the following mathematical form: $q = f(L,K) = AL^{\alpha}K^{\beta}$

Where:

q *total output,*

L *labor input,*

K *capital input,*

A, α, and ***β*** *are positive constants.* (A is the efficiency of production, technical progress. The more efficient firm will have a larger **A** than the less efficient one. The higher the ratio **α/β** the more labor intensive the technique. Similarly, the lower the ratio **α/β** the more capital intensive the technique.)

α and β are the elasticities of output with respect to the factor inputs:

$\alpha = \frac{dq}{dL}\frac{L}{q}$, it measures the percentage increase in q resulting from a 1% increase in L while holding K constant.

$\beta = \frac{dq}{dK}\frac{K}{q}$, it measures the percentage increase in q resulting from a 1% increase in K while holding L constant.

The Cobb-Douglas function can exhibit any degree of returns to scale, depending on the sum of the powers (α + β) associated with L and K. Suppose all inputs were increased by a factor of *t*, then: $q' = f(tL,tK) = A(tL)^{\alpha}(tK)^{\beta} = t^{\alpha+\beta}AL^{\alpha}K^{\beta} = t^{\alpha+\beta}q$

- If **α + β > 1**, the function exhibits IRS.
- If **α + β = 1**, the function exhibits CRS (a *linear homogeneous* Cobb-Douglas production function. It takes the form: $q = A L^{\alpha}K^{1-\alpha}$).
- If **α + β < 1**, the function exhibits DRS.

The Cobb-Douglas production function has some properties. First, the function is a homogeneous of degree, n = α + β because, as we found above: $A(tL)^{\alpha}(tK)^{\beta} = At^{\alpha+\beta}q$. If β = 1-α, then: α + β = α + 1- α = 1, the function exhibits CRS and is said to be homogeneous of degree one, or *linearly homogeneous*. Second, since the function is homogeneous of degree n = α + β, then it satisfies Euler's theorem:

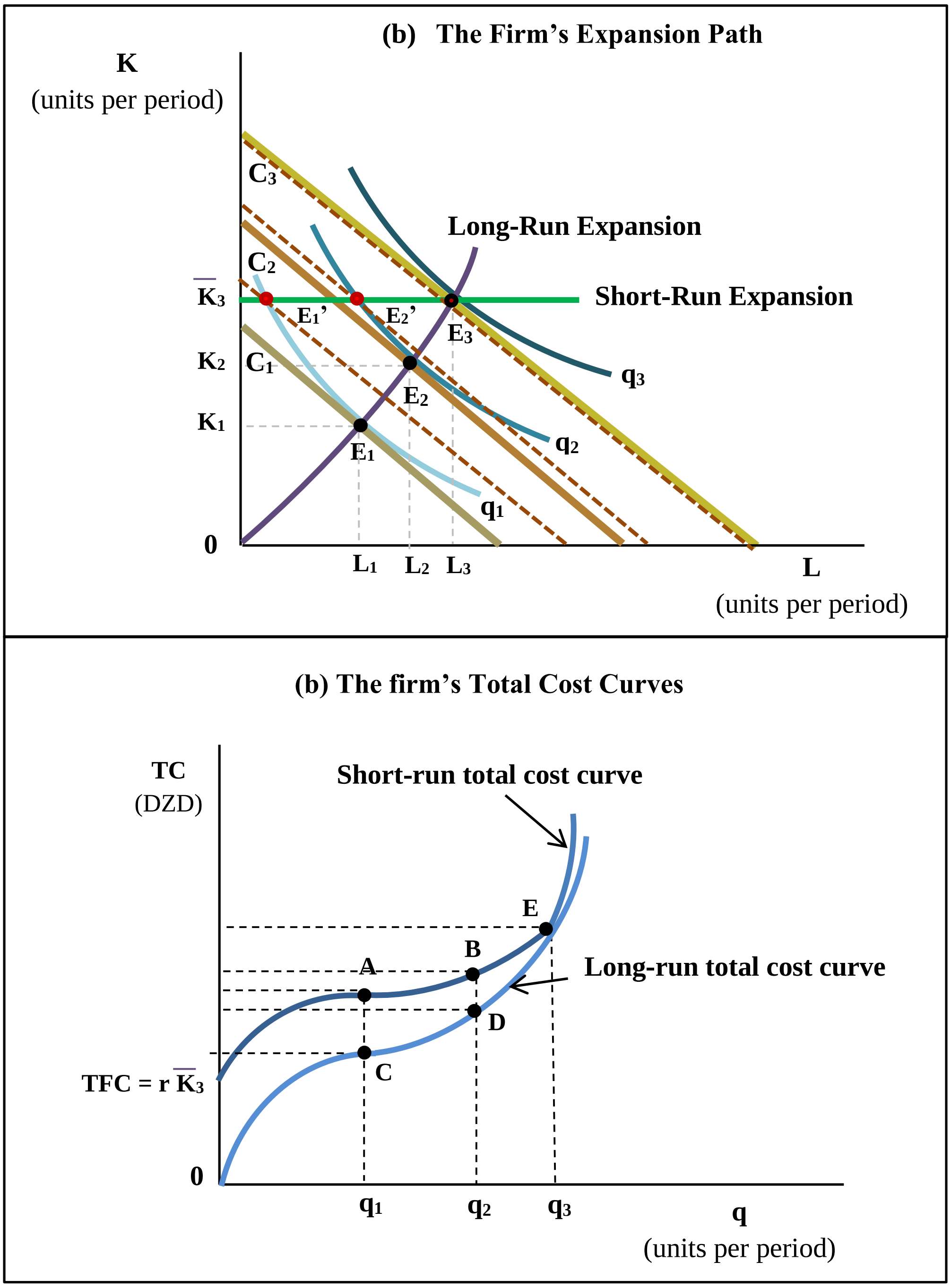
$(\alpha + \beta)q = Lf'_L + Kf'_K = \alpha AL^{\alpha-1}K^{\beta}L + \beta AL^{\alpha}K^{\beta-1}K = \alpha AL^{\alpha}K^{\beta} + \beta AL^{\alpha}K^{\beta} = (\alpha + \beta)AL^{\alpha}K^{\beta}$.

If: α + β = 1, the function is homogeneous of the degree one, thus: $q = MP_L L + MP_K K$.

2.2.10. The Firm's Expansion Path and Total Cost Curves

The firm's expansion path is *the locus of cost-minimizing tangencies between the firm's isocost lines and its isoquants resulting from the changes in total outlay (expenditure) while keeping the prices of inputs (L and K) constant* (Nicholson & Snyder, 2012, p. 338). This is analogous to the income-consumption pat) discussed in Chapter 3.

Figure 7.8. The Firm's Expansion Path and Total Cost Curves



Panel (a) shows the firm's expansion paths in the long-run and in the short-run. In the long run, the firm increases its output by using more of both inputs, so its long-run expansion path is upward sloping. In the short run, the firm cannot vary its capital, so its short-run expansion path is horizontal at the fixed level of output. That is, it increases its output by increasing the amount of labor it uses.

Panel (b) shows the firm's corresponding long-run and short-run total cost curves *LTC*(*q*) and *STC*(*q*). We see that *STC*(*q*) always lies above *LTC*(*q*) (i.e., short-run total cost is greater than long-run total cost) except at point **E**, where the two costs are equal (we will discuss the firm's costs in more details in the next chapter 8).

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Problems

1. The following table gives a hypothetical short-run production values for 'wheat'. Land is measured in acres, labor in number of workers per year, and total product (TP) in bushels per year.

Land (L_n)	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
Labor (L_b)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Total Product (TP)	**0**	**2**	**5**	**9**	**12**	**14**	**15**	**15**	**14**	**12**

a. Find AP_{Lb} and MP_{Lb}, and then plot TP_{Lb}, AP_{Lb} and MP_{Lb} curves?

b. In terms of "labor" and "land," what does the law of diminishing marginal returns state? And where this law starts operating?

c. Assuming: (1) constant returns to scale, (2) labor constant at one unit per year, and (3) alternative quantities of land used, ranging from $\frac{1}{9}$ to **1** acre per year.

- Find from the table: TP_{Ln}, AP_{Ln} and MP_{Ln}, and Plot them on the same set of axes with the curves of labor plotted in part (a)?
- Define the three stages of production for labor and land?
- Why does the producer operate in stage II?
- Explain where will the producer operate within stage II if: $P_{Lb} = 0$; $P_{Ln} = 0$; and $P_{Lb} = P_{Ln}$?

2. Suppose that the production function for 'flyswatters' during a particular period of time can be represented by the following equation: $q = f(L,K) = 600(LK)^2 - (LK)^3$

Where **q** is the total number of 'flyswatters' produced, **L** the amount of labor (workers), and **K** is capital used in production. Suppose that capital is constant at $\overline{K} = 10$.

a. Find AP_L and MP_L?

b. What is the amount of labor **L** at which the total output **q** reaches its maximum value?

c. What is the amount of labor **L** at the point when AP_L and MP_L are equal? And what is the importance of this point in analyzing the producer's behavior in the short-run.

3. Suppose that a firm has the following Cobb-Douglas production function: $q = f(L,K) = L^{3/4}K^{1/4}$

In the short-run, the firm can vary only its labor, *L*, but cannot vary its capital, $K = 16$.

a. Prove that the marginal product of labor MP_L is declining and positive?

b. What is the stage of production in which this firm operates?

In the long-run, the firm can vary both labor and capital.

c. Find the average and marginal products for both labor and capital?

d. If the price of labor, $P_L = \$15$, and the price of capital $P_K = \$5$, and given the total outlay of **TC = $500**. Calculate the amount of labor and capital that the firm should use in order to maximize its total output?

e. Calculate the Marginal Rate of Technical Substitution ($MRTS_{L,K}$) at the equilibrium point? Explain it?

f. What kind of returns to scale does the firm's production functions exhibit?

4. A Japanese synthetic rubber manufacturer's production function is given by: $q = f(L,K) = 10L^{0.5}K^{0.5}$

Suppose that its wage, **w**, is **$1** per hour, and the rental cost of capital, **r**, is **$4**.

- **a.** Is the combination (**L = 25, K = 16**) the optimal combination for this manufacturer or not, for a given output of **200** units? Justify your answer?
- **b.** Consider that the combination given above is the optimal one and **TC = $200**. What are the prices of labor and capital (**w**, and **r**) paid by this manufacturer?
- **c.** Given the same prices of labor and capital, **w** = **$1** per hour and **r** = **$4** per hour. What is the minimum total outlay to produce **300** units of output?

5. Suppose that we have the general Cobb–Douglas production function: $q = f(L,K) = AL^{\alpha}K^{\beta}$

Where **A** is the technical progress, **α** and **β** are positive constants ($0 < \alpha < 1$ and $0 < \beta < 1$).

- **a.** Show that **α** is the elasticity of output with respect to labor input $e_{q,L}$, and **β** is the elasticity of output with respect to capital input $e_{q,K}$?
- **b.** If both inputs **L** and **K** are doubled and $\alpha + \beta = 1$, by how much will the total output **q** double?
- **c.** Calculate the constants **α** and **β**, knowing that $e_{q,L} = 0.3$ and that the function is homogeneous of degree one?
- **d.** Show that this function satisfies Euler's theorem and implies that: $q = MP_L L + MP_K K$

6. Suppose that the production function for a good is given by: $q = f(L,K) = L + K + 2\sqrt{LK}$

- **a.** Find the marginal product for both labor and capital?
- **b.** Calculate the Marginal Rate of Technical Substitution ($MRTS_{L,K}$), knowing that: $K/L = 1/4$?
- **c.** Show that this function exhibits constant returns to scale?

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Chapter 8: Producer Behavior Analysis (Costs of Production)

Learning Objectives

By the end of this chapter, students will be able to:

- Understand the meaning of cost, and distinguish between explicit and implicit costs, economic and accounting costs, and explain the meaning of opportunity cost and sunk cost.
- Differentiate between a firm's production costs in the short-run and in long-run.
- Analyze the costs of production in the short-run, including analyzing different costs measures (total fixed and variable costs, and average and marginal costs), identifying the shapes of the short-run cost curves, and understanding the relationship between product and cost curves.
- Analyze the costs of production in the long-run, including identifying the shapes of the long-run cost curves, clarifying the relationship between the short-run and the long-run cost curves, and explaining the meaning of economies and diseconomies of scale.

1. Meaning of Cost and its Measurement

1.1. Meaning of Cost

A firm's cost *obviously includes all payments for inputs, such as labor, capital, energy, and materials used in production process within a given period of time* (Pindyck and Rubinfeld 2013, 229). A firm's manager (or accountant) can determine the cost of these inputs by multiplying the price of each input by the number of units used.

1.2. Explicit Costs versus Implicit Costs

The explicit costs are *the firm's direct, out-of-pocket payments for inputs used in production*, such as the costs labor, capital, energy, and materials used in production, which can be calculated by the manager of a firm (or accountant). While **the implicit costs** *reflect only a forgone opportunity rather than an explicit expenditure* (Perloff 2020, 210).

1.3. Economic Costs versus Accounting Costs

Economists think of cost differently from accountants, who are usually concerned with measuring **accounting costs,** *which include actual explicit expenses plus depreciation expenses for capital equipment*. But economists include all costs relevant to production. They are therefore concerned with **economic cost,** *which is the cost of utilizing scarce resources in production* (Pindyck and Rubinfeld 2013, 230). To run a firm profitably, a manager must think like an economist and must consider all relevant costs.

1.4. Opportunity Cost

An opportunity cost is *the cost associated with opportunities that are forgone by not putting the firm's resources to their best alternative use* (Pindyck and Rubinfeld 2013, 230). It takes the same meaning of **the economic costs**. Consider a firm that owns a building and therefore pays no rent for office space. But the firm could have earned rent on the office space by leasing it to another firm which means putting this resource to an alternative use that would have provided the firm with rental income. This forgone rent is the opportunity cost of utilizing the office space. And because the office space is a resource that the firm is utilizing, this opportunity cost is also an economic cost of doing business. The concept of the opportunity cost forces us to recognize that costs are not just money payments but also sacrificed alternatives. Although an opportunity cost is often hidden, it should be taken into account when making economic decisions.

1.5. Sunk Costs

A sunk cost is *a past expenditure that cannot be recovered* (Perloff 2020, 213). For example, consider a firm buys of specialized equipment for a plant. Suppose the equipment can be used to do only what it was originally designed for, and cannot be converted for alternative use. The expenditure on this equipment is a *sunk cost* and its *opportunity cost is zero*, because it has no alternative use. Therefore, it should not be included in the firm's current cost calculations. But if the equipment could be put to other use or could be sold or rented to another firm? In that case, its use would involve an opportunity cost of using it rather than selling or renting it to another firm.

The sunk cost is usually visible, but after it has been incurred it should always be ignored when making future economic decisions.

2. Costs of Production and Time Frames

It is important to distinguish our analysis of the costs of production between the short-run and the long-run, just as we did with production analysis.

2.1. Costs of Production in the Short-Run

2.1.1. Short-Run Cost Measures: Total Fixed Cost, Total Variable Costs, and Total Costs

In the short-run the firm can vary some inputs, such as labor and raw materials, but it cannot vary other inputs, such as capital. There are, however, costs associated with the use of both fixed and variable inputs.

- **Total Fixed Cost (TFC):** is the cost incurred by a firm that does not depend on how much output it produces. Thus, even if the firm produces nothing, it still incurs its TFC. This cost includes expenditures on inputs the firm cannot vary in the short run, which is capital, ($\mathbf{TFC = r\overline{K}}$).
- **Total Variable Cost (TVC):** is the cost incurred by a firm that depends on how much output it produces. This cost includes expenditures on inputs the firm can vary in the short-run, which is labor, ($\mathbf{TVC = wL}$).
- **Total Cost (STC):** is the sum of a firm's TFC and TVC in the short-run:

$$STC = TFC + TVC \quad STC = r\overline{K} + WL$$

Since TVC changes with the level of output, so does the STC.

2.1.2. Four Other Short-Run Cost Measures

To decide how much to produce, a firm uses other measures of costs. We can derive four such measures using the TFC, the TVC, and the STC.

- **Average Fixed Cost (AFC):** is the TFC divided by the amount of output: $AFC = \frac{TFC}{q}$
- **Average Variable Cost (AVC):** is the TVC divided by the amount of output:

$$AVC = \frac{TVC}{q}$$

$$AVC = \frac{wL}{q}$$

$$AP_L = \frac{q}{L} \Rightarrow \frac{1}{AP_L} = \frac{L}{q}$$

As a result, $\mathbf{AVC = \frac{w}{AP_L}}$

- **Average Total Cost (SAC):** is the STC divided by the amount of output. We can also define it as the sum of AFC and AVC: $\boldsymbol{SAC = \frac{STC}{q} = AFC + AVC = AFC + \frac{w}{AP_L}}$
- **Marginal Cost (SMC):** is the change in STC resulting from producing one extra unit of output. It can also be defined as the change in TVC resulting from producing one extra unit of output, because TFC does not change as output changes. $\boldsymbol{SMC = \frac{\Delta STC}{\Delta q} = \frac{\Delta TVC}{\Delta q}}$

Or, we can take the derivative from the total cost function, or from the total variable cost function.

$$\boldsymbol{SMC = \frac{dSTC}{dq} = \frac{dTVC}{dq}}$$

We also have: $TVC = wL \Rightarrow \Delta TVC = w\Delta L$

Thus, $SMC = \frac{w\Delta L}{\Delta q}$

$$MP_L = \frac{\Delta q}{\Delta L} \Rightarrow \frac{1}{MP_L} = \frac{\Delta L}{\Delta q}$$

As a result, **SMC =** $\boldsymbol{\frac{w}{MP_L}}$

2.1.3. Short-Run Cost Curves

We illustrate the relationship between output and the various cost measures using the following example.

Example (1): The table below shows the TFC, TVC, and STC at each level of output for a hypothetical firm.

Output (q)	0	1	2	3	4	5	6
TFC	60	60	60	60	60	60	60
TVC	0	30	40	45	55	75	120
STC	60	90	100	105	115	135	180

a. Calculate AFC, AVC, SAC, and SMC schedules?

b. Draw, on the same set of axes, TFC, TVC, and STC curves, and then draw, on another set of axes directly below the first set, the corresponding SMC, AFC, AVC, and SAC curves? Explain how these curves vary with the rate of output?

c. What is the relationship among the SMC, AVC, and SAC curves?

Answer:

a. Calculating AFC, AVC, SAC, and SMC schedules:

$$\mathbf{AFC} = \frac{TFC}{q} \;;\quad AFC_1 = \frac{TFC_1}{q_1} = \frac{60}{0} = / \;;\; AFC_2 = \frac{TFC_2}{q_2} = \frac{60}{1} = \mathbf{60} \ldots$$

$$\mathbf{AVC} = \frac{TVC}{q} \;;\quad AVC_1 = \frac{TVC_1}{q_1} = \frac{0}{0} = / \;;\; AVC_2 = \frac{TVC_2}{q_2} = \frac{30}{1} = \mathbf{30} \ldots$$

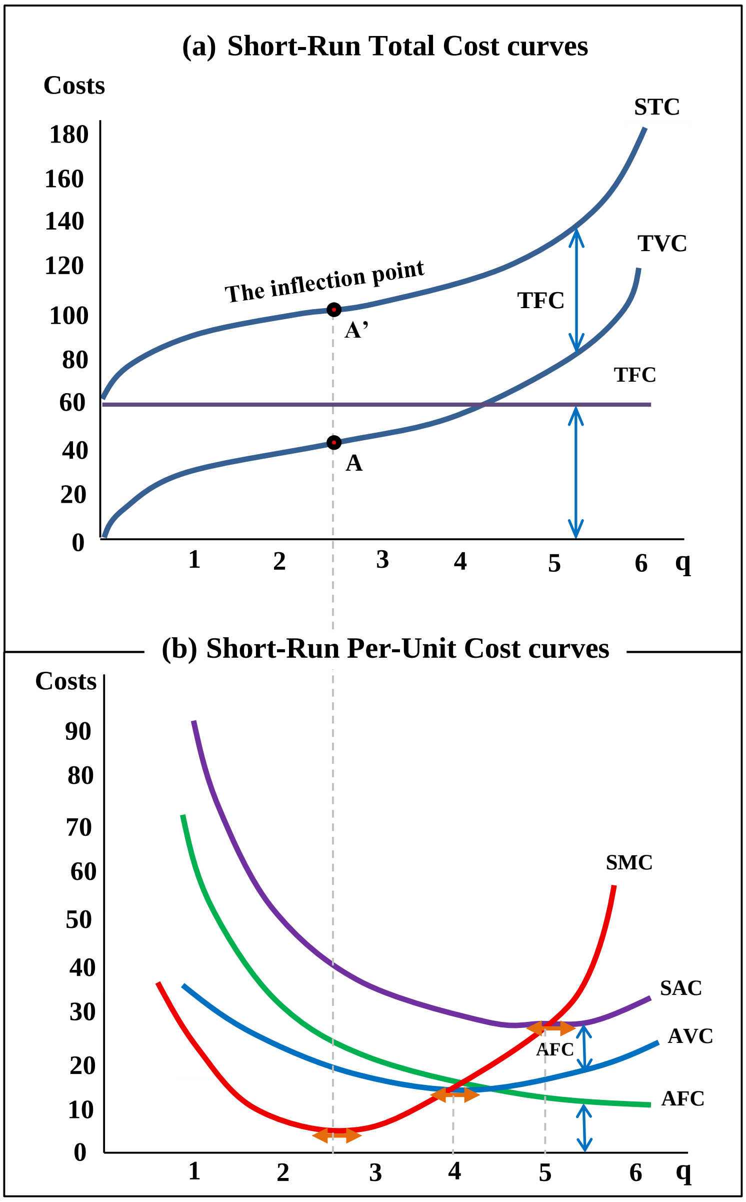
$$\mathbf{SAC} = \frac{STC}{q} \;;\quad SAC_1 = \frac{STC_1}{q_1} = \frac{60}{0} = / \;;\; SAC_2 = \frac{STC_2}{q_2} = \frac{90}{1} = \mathbf{90} \ldots$$

$$\mathbf{SMC} = \frac{\Delta STC}{\Delta q} \;;\quad SMC_1 = \frac{\Delta STC}{\Delta q} = \frac{90-60}{1-0} = \mathbf{30} \;;\; SMC_2 = \frac{\Delta STC}{\Delta q} = \frac{100-90}{2-1} = \mathbf{10} \ldots$$

AFC	—	60	30	20	15	12	10
AVC	—	30	20	15	13.75	15	20
SAC	—	90	50	35	28.75	27	30
SMC	—	30	10	5	10	20	45

b. Drawing cost curves and explaining how these curves vary with the rate of output:

Figure 8.1. Short-Run Cost Curves



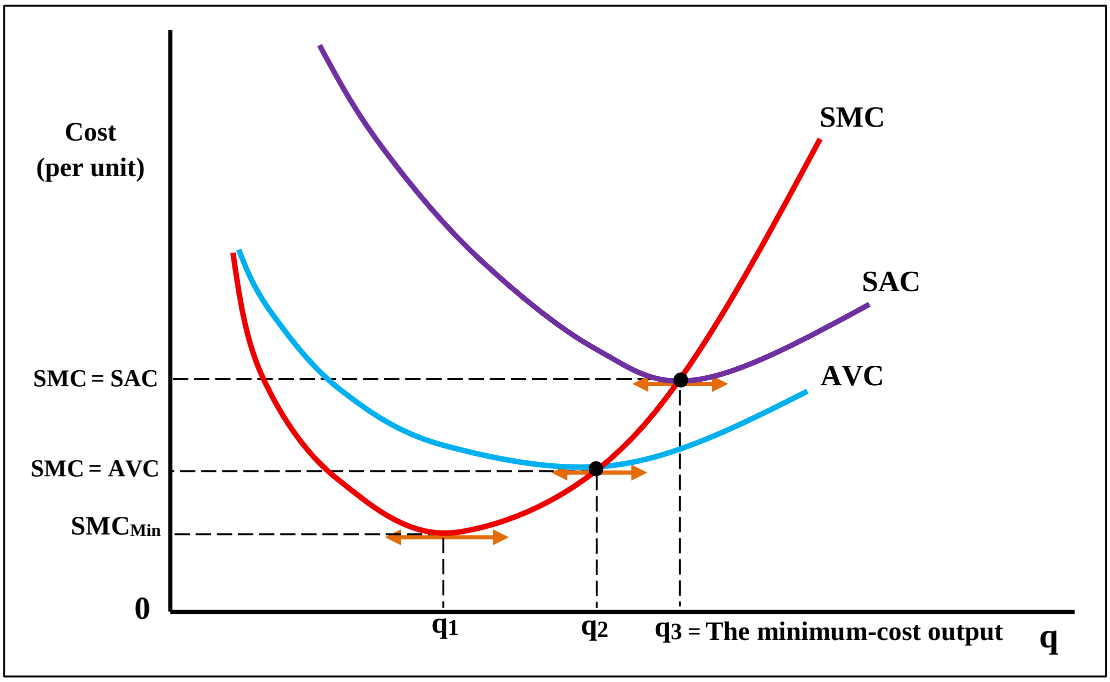
In panel **(a),** we see that the TFC curve is *a horizontal line* which is parallel to the *q* axis and DZD 60 above it. The TVC curve starts from the origin, increases at *a decreasing rate* (it is concave downward) until the point **A** (the inflection point), and up to this point, the firm is using so few of the variable inputs together with its fixed inputs that the law of DMR have not yet set in. Then the curve continues to increases at *an increasing rate* (it becomes concave upward), and starting from point A, the law of DMR will set in. Because, STC = TFC + TVC, at every output level, thus, STC curve has the same shape as the TVC curve (inverse ***S***-shaped curve) but is everywhere DZD 60 above it. The vertical distance between STC curve and TVC curve equals TFC, as illustrated by the two arrows.

In panel **(b),** the graph shows that the AFC curve falls continuously as output increases (because the same constant TFC is spread over a larger output). While the AVC, SAC, and SMC curves fall initially; reach their lowest points, and then rise (they are *U*-shaped curves). The SMC curve reaches its lowest point before both AVC and SAC curves. Also, the rising portion of the SMC curve intersects the AVC and SAC curves at their lowest points. The vertical distance between the SAC curve and the AVC curve, which becomes smaller as more output is produced, is equal to AFC, as illustrated by the two arrows. In addition, the area under the SMC curve gives the TVC.

c. The relationship among the SMC, AVC, and SAC curves

In the short-run all marginal curves are related to their average (total and variable) curves in the same way, as shown in Figure 8.2.

Figure 8.2. Short-Run Cost Curves



- **The relationship between AVC and SAC**

Both AVC and SAC initially decline, reaching their minimum points, and then each of them rises. However, the AVC reaches its minimum point before SAC does. This is due to the fact that SAC includes AFC, and the latter falls continuously with increases in output. After the AVC has reached its lowest point and starts rising, its rise is over a certain range offset by the fall in the AFC, so that the SAC continues to fall over that range. However, the rise in AVC eventually becomes greater than the fall in the AFC, so that the SAC starts increasing. The AVC approaches the SAC asymptotically as output increases.

- **The relationship between SMC, AVC and SAC**

The SMC and AVC are the same at the first unit of output. When SMC is *below* AVC and SAC, both AVC and SAC will *decline* (SMC pulls both average costs down), and when SMC is *above* AVC or SAC, both AVC and SAC will *rise* (SMC pulls both average costs up). When SMC curve rises, it intersects both AVC and SAC curves at their *minimum* points. Note that the quantity that corresponds to the minimum SAC is called **the minimum-cost output**.

2.1.4. The Geometry of Per-Unit Cost Curves

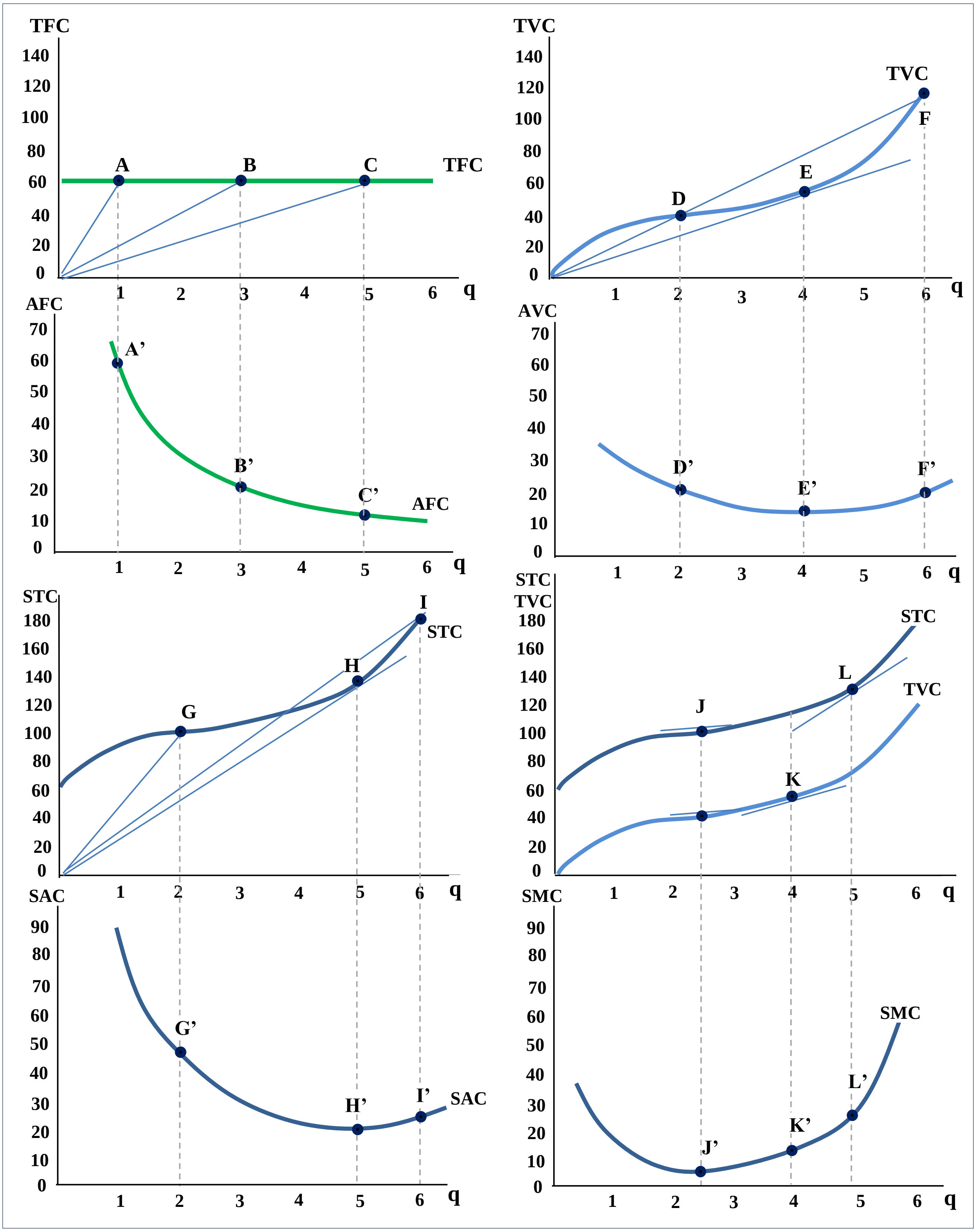
Short-run per-unit cost curves can be derived geometrically from the corresponding short-run TC curves in exactly the same way as the AP_L, and the MP_L curves were derived from the TP_L curve, as explained in Chapter 7.

<u>Example (2)</u>: Using the same graphs of the Figure 8.1, show geometrically how AFC, SAC, AVC, and SMC curves can be derived from TFC, STC, and TVC curves respectively?

<u>Answer:</u>

- **AFC** at any level of output is given by the slope of the ray drawn from the origin to each point on the TFC curve.
- **AVC** at any level of output is given by the slope of the ray drawn from the origin to each point on the TVC curve.
- **SAC** at any level of output is given by the slope of the ray drawn from the origin to each point on the STC curve.
- **SMC** at any level of output is given by the slope of the tangent at any point on either the STC curve or the TVC curve.

Figure 8.3. Graphical Derivation of Short-Run Cost Curves



2.1.5. From Total Product to Total Variable Cost

2.1.5.1. From the Short-Run Production Function to the Variable Cost Function

In the short-run, the firm's TVC is its cost of labor (***TVC = wL***). If input prices are constant, the production function determines the shape of the TVC curve. We can write the short-run production function as: $q = f(L, \overline{K}) = g(L) \Rightarrow L = g^{-1}(q)$

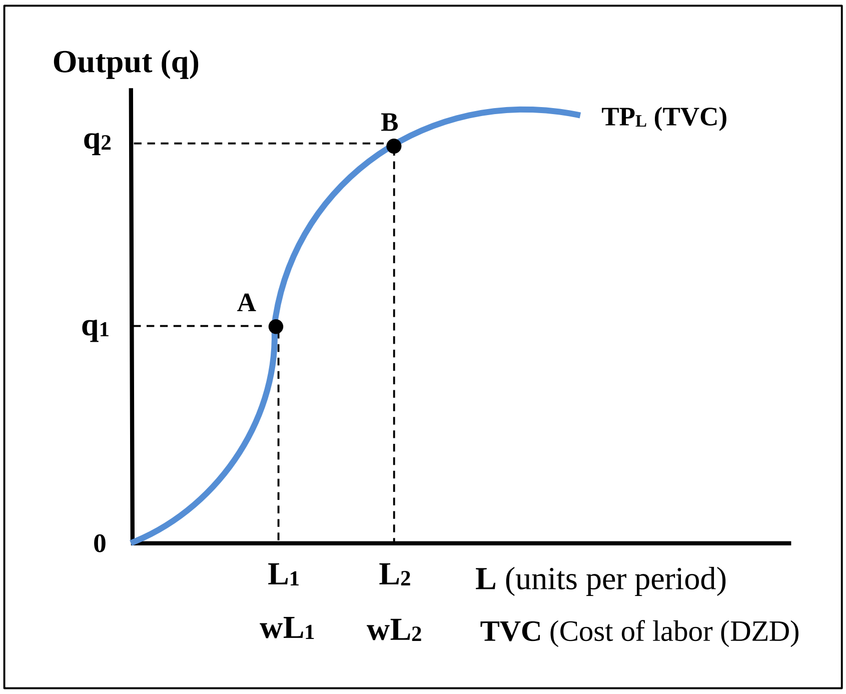
The TVC function is: $TVC(q) = wL = wg^{-1}(q)$

The TVC function is, therefore, the inverse of the short-run production function.

Similarly, the firm's STC function is: $STC(q) = TVC(q) + TFC \Rightarrow STC(q) = wg^{-1}(q) + TFC$

In the short-run, the only way the firm can increase its output is to use more labor. If the firm increases its labor enough, it reaches the point of DMR to labor. The law of DMR and the assumption of a fixed price per unit of labor determine the way TVC varies with output.

Figure 8.4. From Total Product to Total Variable Cost

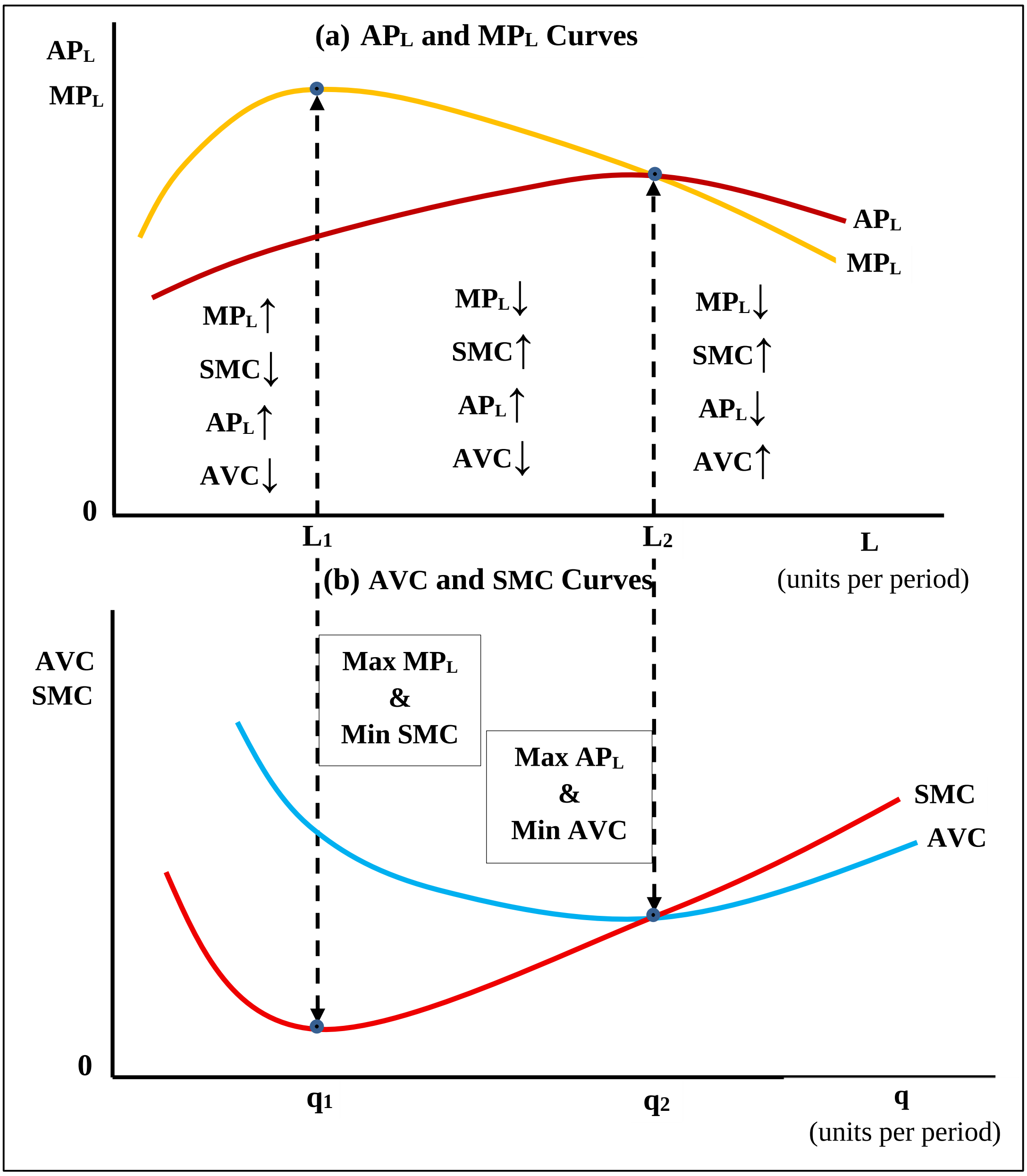


The quantity of labor is related to output through the TP curve, which can be transferred into TVC curve by multiplying each quantity of labor by its per-unit price. By convention, we draw TVC curve with TVC on the vertical axis and q on the horizontal axis, the reverse of the situation in Figure 8.4.

2.1.5.2.The Relationship Between Product Curves and Cost Curves

Figure 8.5 shows the relationship between the firm's product curves and its cost curves in the short-run. The AVC and the SMC curves in panel (b) are *the monetized mirror image* or *reciprocal* of the AP_L and MP_L curves in panel (a) (Salvatore 2006, 171).

Figure 8.5. The Relationship Between Product Curves and Cost Curves



As labor increases up to L_1 units (upper graph), output increases to q_1 units (lower graph). MP_L and AP_L rise and SMC and AVC fall (because $SMC = \frac{w}{MP_L}$ and AVC $= \frac{w}{AP_L}$). At the point of maximum MP_L, SMC is at a minimum.

As labor increases from L_1 to L_2 units, output increases from q_1 to q_2 unit. MP_L falls and SMC rises, but AP_L continues to rise and AVC continues to fall. At the point of maximum AP_L, AVC is at a minimum. As labor increases further, output increases. AP_L declines and AVC increases.

2.1.6. The Shapes of the Per-Unit Cost Curves

We examine the reasons behind the shape of each of per-unit cost curves, because firms rely more on these curves than on total cost curves to make decisions about labor and capital.

- **The shape of AFC curve:** this curve is *asymptotic* to both axes. That is, it approaches but never quite touches the axes. Also, AFC times output always gives the constant TFC. Thus, the

AFC curve is a *rectangular hyperbola*. The reason behind this is: *spreading TFC over ever-larger rates of output* (Salvatore 2006, 157).

The shapes of SMC, AVC, and SAC curves: these curves are ***U****-shaped*. The main reason behind this is: *the law of DMR*. This law, as we have seen in the previous chapter, leads to an inverse U-shaped AP_L and MP_L curves; that is, they rise initially, reach a maximum, and then fall. As a result, the AVC and the SMC curves must be U-shaped because they are a mirror reflection of product curves. The SAC curve must also be U-shaped. The reasons behind this are the influence of two opposing forces: *spreading total fixed cost over ever- larger rates of output*, and *eventually DMR*. This is, as we explained before, due to the fact that SAC includes AFC, which falls continuously with increases in output. With AFC decreasing more quickly than the increase in AVC, the S*AC* curve continues to fall. Eventually, AVC starts to increase more quickly than the fall in AFC, so that the SAC starts increasing (Browning and Zupan 2015, 187-188).

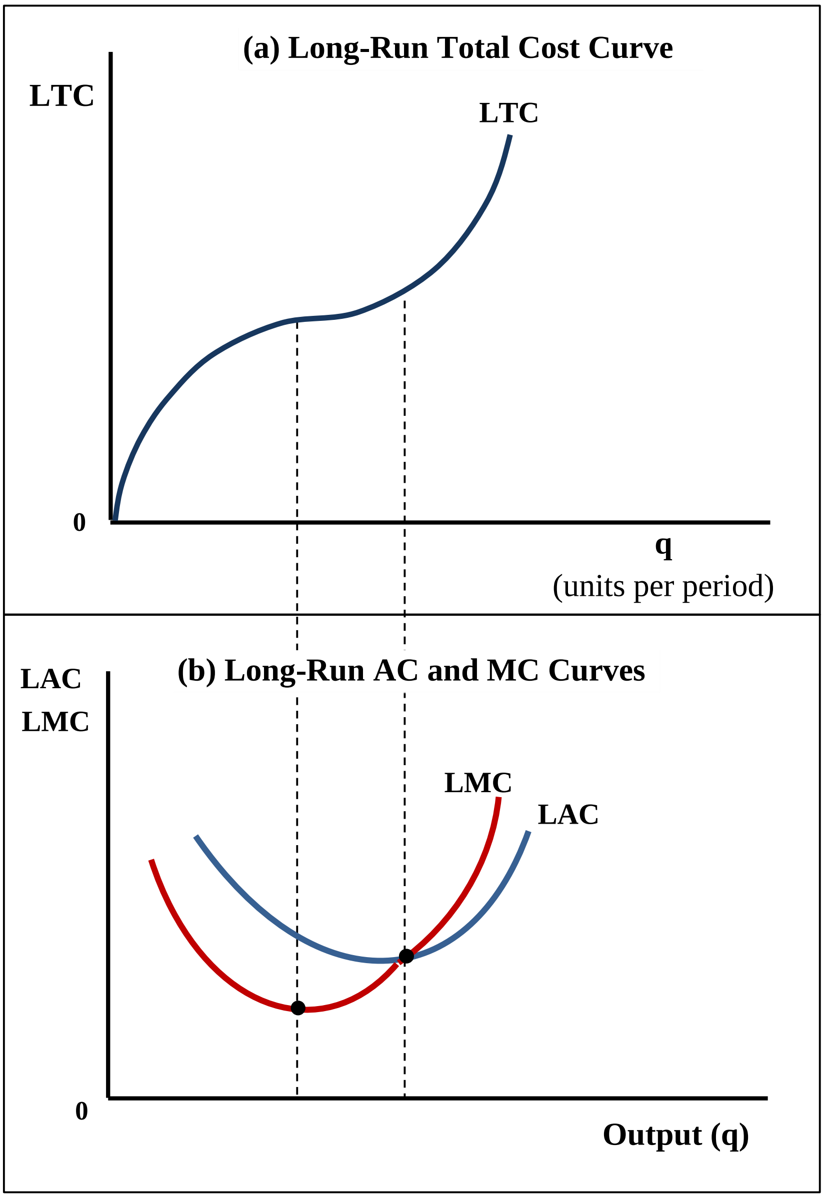
2.2. Costs of Production in the Long-Run

In the long-run, a firm can adjust all its inputs (both labor and capital). Therefore, all its costs are variable.

2.2.1. Long-Run Cost Curves

Given that all the firm's inputs are variable in the long-run, there is only one long-run total cost (LTC) curve, only one long-run average cost (LAC) curve, and a long-run marginal cost curve (LMC). Figure 8.6 shows LTC curve (in panel a), and the associated LAC and LMC curves (in panel b).

Figure 8.6. Long-Run Cost Curves



The graphical derivation of the AC and MC curves from the TC curve is the same for the long-run as for the short-run. The LTC curve shows the minimum total costs of producing each level of output when any desired scale of plant can be built.

The LAC curve shows the minimum per-unit cost of producing each level of output. **LAC** = $\frac{LTC}{q}$

The LMC measures the change in LTC per unit change in output. **LMC** = $\frac{\Delta LTC}{\Delta q}$

The relationship between average and marginal curves in the long-run is similar to that in the short-run. The LAC and LMC curves fall initially; reach their minimum points, and then rise (they are U-shaped curves). The LMC curve reaches its minimum point before The LAC curve. Also, the rising portion of the LMC curve intersects the LAC curve at its lowest points.

The LAC curve is also **U**-shaped, and *returns to scale* are the factors that determine this shape. IRS imply that LAC is falling, and DRS imply that LAC is rising (Browning and Zupan 2015, 200). Note that while a **U**-shape is typical, the LAC curve for many firms could have *a flat bottom* (see section **2.2.3** of this chapter). Because when LAC reaches a minimum, it stays at that level over a wide range of output (showing CRS) before it starts to rise, indicating that at that level of production there are *no economies of scale* (Salvatore 2006, 151).

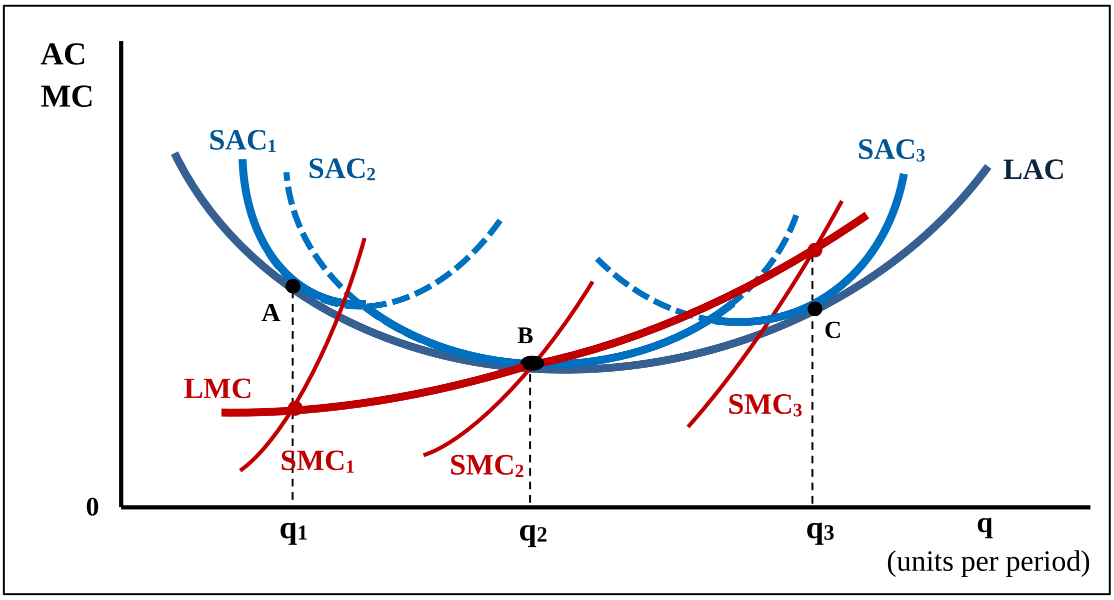
2.2.2. The Relationship between Short-Run and Long-Run Cost Curves

To clarify the relationship between the short-run and the long-run cost curves, we suppose that the firm has only *three* possible plant sizes.

- **The Relationship between Short-Run and Long-Run Average Cost Curves**

Considering that the firm has only three plant sizes, the associated SAC curves to each plant size are: SAC_1, SAC_2, and SAC_3.

Figure 8.7. LAC as the Envelope of SAC Curves and the Relationship with MC curves



The LAC curve is given by the *solid, scalloped* portion of the three SAC curves because they show the minimum cost of production for any output level (Perloff 2020, 236). By drawing a tangent to all these SAC curves we get the LAC curve, which is *the envelope* of all the SAC curves. If the firm can pick any possible plant size, LAC curve is *smooth* and ***U**-shaped*. The *dashed* portions of the SAC curves are irrelevant since they represent higher-than-necessary AC for the firm in the long-run.

Notice that, for outputs smaller than q_2 units, the LAC curve would be tangent to the SAC curves to the left of their minimum points (such as point A on SAC_1). For outputs larger than q_2 units, the LAC curve would be tangent to the SAC curves to the right of their minimum points (such as point C on SAC_3). At an output level of q_2 units, the LAC curve is tangent to SAC_2 at its minimum (at the point B), which is also the minimum point on the LAC curve. The scale of plant whose SAC curve forms the minimum point of the LAC curve (**SAC_2**) is called *the optimum scale of plant* (or *the minimum efficient scale*), while the minimum point on this curve determines *the optimum rate of output* for that plant (Salvatore 2006, 162).

- **The Relationship between Short-Run and Long-Run Marginal and Average Cost Curves**

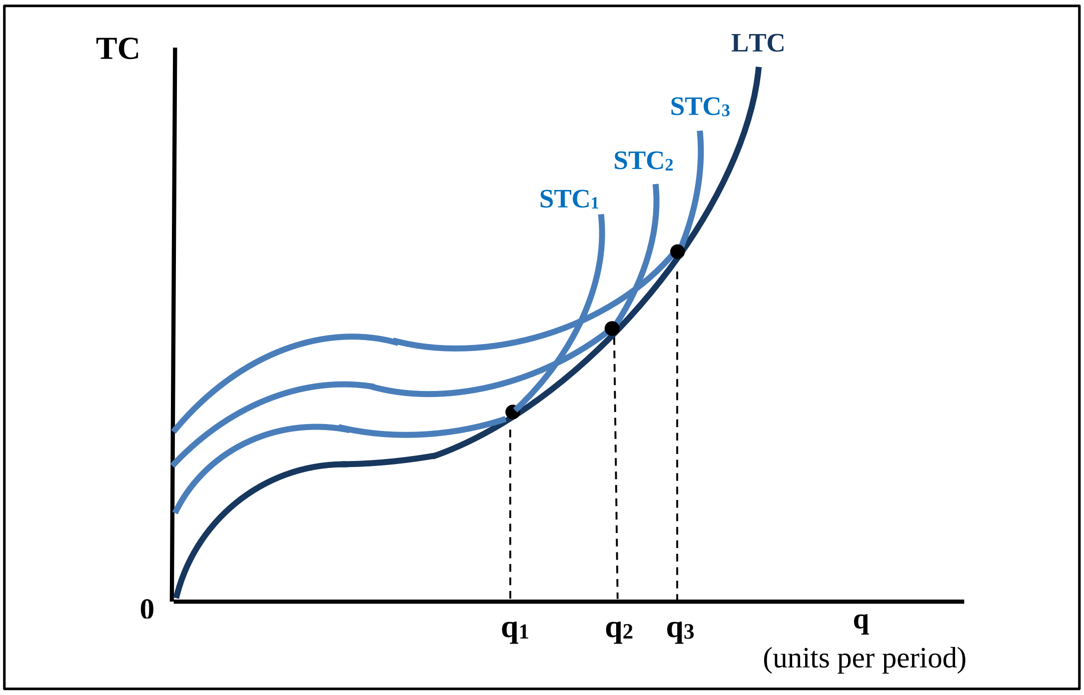
From figure 8.7 we can see that the three SMC curves intersect the corresponding SAC curves at their minimum points. The LMC curve applies to all possible plant sizes. Each point on the LMC curve is the SMC associated with the most cost-efficient plant at which each SAC curve is *tangent to* the LAC curve (at points A, B, and C). Thus, all SMC curves will intersect LMC curve at the output levels q_1, q_2, and q_3. the LMC curve *is not the envelope* of the SMC curves (Pindyck and Rubinfeld 2013, 258). At the point B, which represents the optimum scale of plant that generates the optimum rate of output: $\mathbf{SAC_2 = SMC_2 = LAC = LMC}$.

Note that whether dealing with the short-run or the long-run, the MC curve is below the corresponding AC curve when the AC curve is declining; MC equals AC when AC reaches its minimum; and the MC curve is above the AC curve when the AC curve is increasing.

- **The Relationship between Short-Run and Long-Run Total Cost Curves**

Figure 8.8 shows the relationship between the short-run and the long-run total costs.

Figure 8. 8. LTC Curve as the Envelope of STC Curves

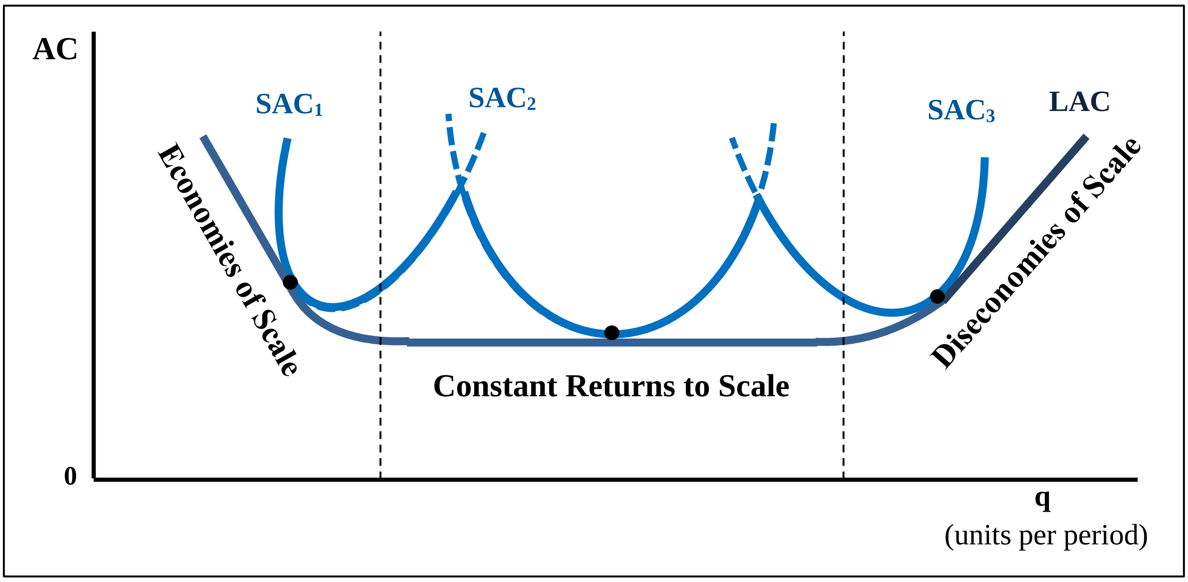


The LTC curve is given by a curve that is *tangent to* all the STC curves. It is also *the envelope* of all the STC curves, and no portion of the STC curves can ever be below the LTC curve derived from them. Hence the LTC curve gives the minimum LTC to produce any level of output (Salvatore 2006, 152). Note that, like the STC curves, the LTC curve is inverse ***S**-shaped*; and it starts from the origin, because there are no fixed costs in the long-run. While each STC curve starts from a given TFC, since there are fixed costs in the short-run.

2.2.3. Economies and Diseconomies of Scale

As we have said before, the LAC curve for many firms could have a much flatter **U**-shape. This shape gives us important information about how costs of a firm vary with the scale of its operations. When LAC curve falls, there are said to be *economies of scale*. When LAC curve stays constant, there are said to be *constant returns to scale*. When LAC rises, there are said to be *diseconomies of scale* (Taylor and Weerapana 2009, 230).

Figure 7. 9. Economies and Diseconomies of Scale



Economies of scale refer to *a situation in which the LAC falls as output increases*. The main reason for this is the gains from *greater specialization* among workers. As output increases and workers are added, each worker becomes more productive, therefore output will increase by a greater proportion than costs.

Diseconomies of scale refer to *a situation in which the LAC rises as output increases*. One reason for this is because of *management challenges.* As output continues to increase, most firms will reach a point where bigness starts to cause many problems that contribute to raise costs more than in proportion to output.

Constant returns to scale refer to *a situation in which the LAC remains unchanged as output increases*. A firm would experience constant returns to scale because as output increases, the impact of specialization will diminish, while the problems of bigness become more serious. At some level of production, these forces may just cancel out, so that an increase in output does not change *LAC* at all.

Note that since returns to scale imply economies or diseconomies of scale, we can say that the pattern explained above (economies of scale- constant returns to scale diseconomies of scale) is also the reason behind the **U**-shape of the LAC curve.

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Problems

1. The following table gives the costs of production for a 'bakery' business.

q (batches)		**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Costs (DZD)	**TFC**	50	50	50	50	50	50	50	50	50	50	50
	TVC	0	40	70	90	100	120	150	190	240	300	370
	STC	**50**	90	120	140	150	170	200	240	290	350	420

a. Plot, on the same set of axes, the **TFC**, **TVC**, and **STC** and explain them?

b. What is the relationship between the quantity of fixed inputs used and the short-run level of output?

c. Find, from the table: **AFC**, **AVC**, **SAC**, and **SMC** and Plot them on the same set of axes? How can the curves of these costs be derived geometrically from the corresponding total cost curves?

d. Explain the relationship among the **SMC**, **AVC**, and **SAC** curves?

2. A firm has the following total cost function: $TC = 20q^2 + 60q + 500$

a. Does the firm operate in the short-run or in the long-run? Justify your answer?

b. Determine all possible total, average, and marginal costs?

c. Find the amount of output, **q**, when the marginal cost intersects the average total cost?

d. What is the minimum-cost output?

3. A company specialized in 'Solar Panels Installation' has the following production function: $q = 4LK$

In the short-run, the company's amount of capital is fixed at $\overline{K} = 10$. It faces a wage rate of **w = $10** per hour and a rental rate of **r = $12** per hour.

a. Derive the company's short-run total cost (**STC**) function? What is the amount of this cost of producing **100** units of output?

b. Find the **AFC**, **AVC**, **SAC**, and **SMC** functions?

In the long-run, the company can adjust its labor and capital.

c. Derive the company's long-run total cost (**LTC**) function? What is the amount of this cost of producing **100** units of output?

d. Find the **LAC** and **LMC** functions?

e. Does this company experience economies or diseconomies of scale? Explain?

4. A high technology firm has the following Cobb-Douglas production function: $q = 10L^{0.3}K^{0.7}$

a. Calculate the MP_L and the MP_K?

b. Find the firm's expansion path equation?

c. Derive the contingent demand functions for labor and capital, considering that the firm seeks to minimize the total costs of producing a given quantity of output q_0?

d. If the firm faces factor prices of **w = $10** and **r = $20**. Calculate the amounts of **L** and **K** that the firm should utilize to produce an output, $q_0 = 200$ units?

5. Suppose that the production function for a producer is given by: $q = 2.66L^{1/2}K^{1/4}$

The cost of a unit of labor is, **w = $30** and the cost of a unit of capital is **r**.

a. Calculate the rental rate **r**, knowing that the total amount of capital investment is, **I = 50**, the interest rate on it is, ***i* = 5%**, and it has a useful life of **10** years and depreciated linearly?

b. Find the iso-cost equation for this producer?

c. Determine the producer's equilibrium in the following two cases, and what can you conclude?

- Maximizing total output, **q**, with total costs, **TC = $4500**.
- Minimizing total costs, **TC**, to produce a total output, **q = 100** units.

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Answers to Problems

Answers to Problems

Chapter 1

1. a. utility **b.** economics **c.** marginal analysis **d.** opportunity cost
e. economic problem **f.** scarcity **g.** rationality.

2. a. The difference between microeconomics and macroeconomics:

Aspect	Microeconomics	Macroeconomics
Definition	Studies the behavior of individual economic units such as consumers, firms, and industries.	Studies the behavior of the aggregates of the economy as a whole, including national income, inflation, and unemployment.
Main focus	Focuses on specific markets and individual decision-making.	Focuses on the overall economic performance and large-scale economic factors.
Key variables	Prices, output, demand and supply, costs, and profits.	GDP, national income, inflation rate, unemployment rate, fiscal and monetary policy.
Main objective	Aims to analyze how individuals and firms make decisions to allocate limited resources efficiently.	Aims to achieve overall economic stability and growth.
Examples	Determining the price of a good, analyzing consumer behavior, studying a firm's production costs..	Analyzing the causes of inflation, measuring economic growth, studying unemployment trends…

b. Indicating whether each of the following statements applies to microeconomics or macroeconomics:

- The unemployment rate in Algeria was 12.7 percent in 2024. (**Macroeconomics**)
- An unexpected heavy hailstorm damaged the grape crop and caused its price to rise. (**Microeconomics**)
- Air Algérie lowered ticket prices on domestic flights to attract more passengers. (**Microeconomics**)
- The European central bank raised interest rates to curb inflation in the Euro zone. (**Macroeconomics**)
- A clothing manufacturer hired more workers to increase production and meet the growing customer demand. (**Microeconomics**)
- Algeria's non-hydrocarbon exports increased by 6 percent in 2025. (**Macroeconomics**)

c. The difference between positive economics and normative economics:

Aspect	Positive economics	Normative economics
Definition	Deals with facts and objective analysis of how the economy actually works.	Deals with opinions, values, and what the economy should be like.
Nature	Descriptive and fact-based.	Prescriptive and value-based.
Focus	Explains economic phenomena without judgments.	Suggests polices or actions based on values or ethics.

Verifiability	Can be tested or proven by data.	Cannot be tested or proven; depends on beliefs or opinions.
Examples	An increase in the minimum wage leads to higher labor costs for firms.	The government should increase the minimum wage to improve living standards.

d. Indicating whether each of the following statements applies to positive or normative economics:
- The government should restrict imports. (**normative economics**)
- A fall in input prices leads to a rise in quantity supplied of output. (**positive economics**)
- All firms that produce cigarettes should be taxed by the government. (**normative economics**)
- Inflation in Algeria has increased during the recent years. (**positive economics**)
- Algeria should support national production. (**normative economics**)
- Granting subsidies for local farmers increases domestic food production. (**positive economics**)

3. a. The opportunity cost of studying is the enjoyment, fun, and exercise that the student gives up by not playing football.

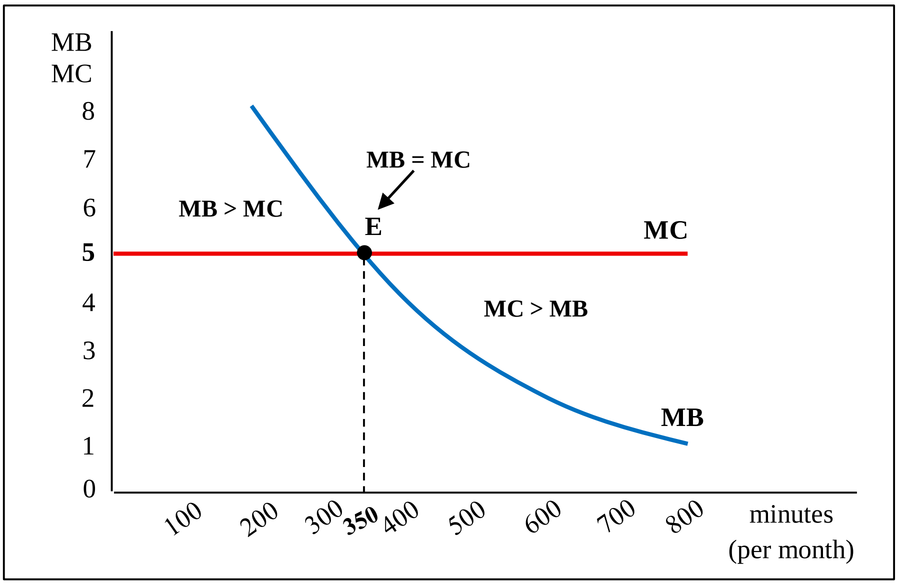
b. The opportunity cost of buying the book is the enjoyment, satisfaction, and experience of eating that the student forgoes at the restaurant.

4. a. If the firm cannot resell the specialized piece of equipment that is bought for $25,000, then its opportunity cost is zero (its original expenditure $25,000 is a sunk cost).

b. If the firm could resell this equipment for the same price, then its opportunity cost is $25,000.

c. If the firm could resell this equipment for $10,000, then its opportunity cost is $10,000 ($15,000 of the original expenditure is a sunk cost).

5. a. Drawing the curves MC and MB in the same set of axes:



b. The optimal amount of conversation per month is the quantity for which the marginal benefit of conversation is just equal to its marginal cost (when MC and MB curves cross), at point **E = 350** minutes per month (efficiency point).

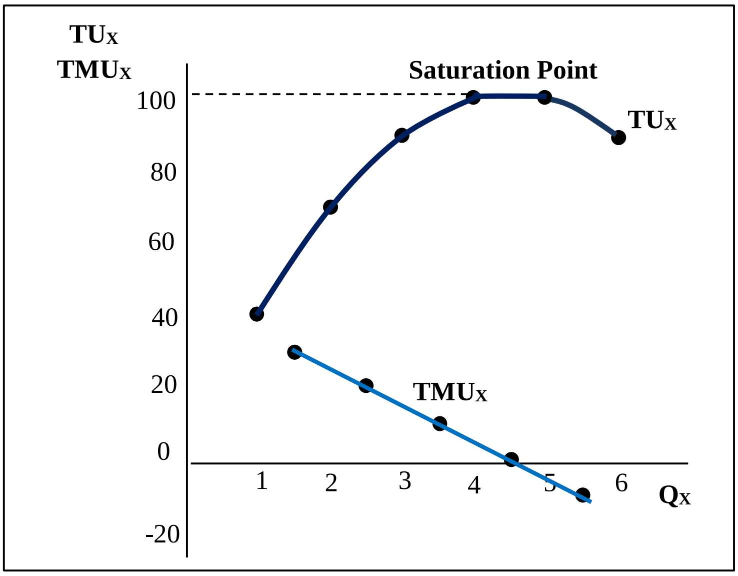
c. If Sami speaks with his mother for less than the optimal amount (350 minutes per month), the MB would exceed the MC, so he should talk longer (there is a net benefit to speaking). But if Sami speaks for more than 350 minutes per month, the MC would exceed the MB, so he should speak less.

Chapter 2

1. a. The marginal utility (MU_X) schedule: $\mathbf{MU_X = \frac{\Delta TU_X}{\Delta Q_X}}$ **(ex.** $MU_{X1} = \frac{(70-40)}{(2-1)}$ **= 30 …)**

Q_X	1	2	3	4	5	6
TU_X	40	70	90	100	100	90
MU_X	**-**	**30**	**20**	**10**	**0**	**-10**

b. TU_X and MU_X diagram, and the saturation point:



TUx curve starts from the origin (0), increases at a decreasing rate, reaches a maximum "saturation point" (100), and then starts falling. At the same time, MUx curve decreases until becomes zero, and then becomes negative.

c. The relationship between TU_X and MU_X curves:

- When *TUx* curve is increasing, *MUx* curve is decreasing and it is positive;
- When *TUx* curve reaches its maximum, *MUx* curve is zero;
- When *TUx* curve is decreasing, *MUx* curve becomes negative.

2. a. The number of hours should this student devote to studying each subject:

We calculate the marginal utility (MU) of the marks divided by the hours (H) of study for each course.

Microeconomics			Accounting			Statistics		
Hours of study	mark	$\frac{MU_M}{H_M}$	Hours of study	mark	$\frac{MU_A}{H_A}$	Hours of study	mark	$\frac{MU_S}{H_S}$
0	0	-	**0**	14	-	0	9	-
1	7	7	**1**	**15.5**	**1.5**	1	13	4
2	11	4	2	16.9	1.4	**2**	**14.5**	**1.5**
3	13.6	2.6	3	18.2	1.3	3	15.9	1.4
4	15.4	1.8	4	19.3	1.1	4	17.2	1.3
5	17	1.6	5	19.6	0.3	5	18.4	1.2
6	**18.5**	**1.5**	6	19.8	0.2	6	18.8	0.4
7	18.5	0	7	19.8	0	7	19	0.2

$$\frac{MU_M}{H_M} = \frac{MU_A}{H_A} = \frac{MU_S}{H_S} = 1.5$$

1 hour devoted to studying accounting, **2 hours** to studying statistics, and **6 hours** to studying microeconomics.

b. The possible high mark that can be got in each course:

microeconomics **18.5**, accounting **15.5**, statistics **14.5**

2. a. The budget constraint of Ahmed: I = \$70, P_G = \$10, P_T = \$5

$I = P_G.G + P_T.T \Rightarrow 70 = 10G + 5T \Rightarrow$ **T = 14 -2G**

b. the optimal quantities of golf and tennis, and the total amount of utility:

we calculate $\frac{MU_G}{P_G}$ and $\frac{MU_T}{P_T}$

Hours per month	Marginal utility from golf (MU_G)	Marginal utility from tennis (MU_T)	$\frac{MU_G}{P_G}$	$\frac{MU_T}{P_T}$
1	80	40	**8**	**8**
2	60	36	**6**	**7.2**
3	40	30	**4**	**6**
4	30	10	**3**	**2**
5	20	5	**2**	**1**
6	10	2	**1**	**0.4**
7	6	1	**0.6**	**0.2**

the equilibrium condition:

$$\begin{cases} \frac{MU_G}{P_G} = \frac{MU_T}{P_T} \Rightarrow \frac{20}{10} = 2 \\ \text{Subject to budget constraint: } I = P_G G + P_T T \Rightarrow 70 = 10(5) + 5(4) \end{cases}$$

G = **5 hours**

T = **4 hours**

$TU_{G,T} = \sum MU_{G,T}$ = (80 + 60 + 40 + 30 + 20) + (40 + 36 + 30 + 10) = **346 utils**

c. If “tennis” referred to savings rather than consumption, the equilibrium condition for Ahmed would remain completely unchanged. In order to maximize the total utility from his income, he should spend **\$50** of his income to purchase **5 hours** of tennis and saves the remaining money **\$20**.

3. a. Is there any basis for mutually advantageous exchange between individuals A and B:

The initial consumption:

Individual A: **6**X; **3**Y TU_A = (24 + 22 + 20 + 18 + 16 + 14) + (36 + 33 +30) = **213 utils**

Individual B: **1**X; **5**Y TU_B = (27) + (33 + 28 + 24 + 20 + 17) = **149 utils**

There is a basis for mutually advantageous exchange between individuals A and B if:

$$\left(\frac{MU_X}{MU_Y}\right)^A \neq \left(\frac{MU_X}{MU_Y}\right)^B \Rightarrow \left(\frac{14}{30}\right)^A \neq \left(\frac{27}{17}\right)^B$$

Since $\left(\frac{14}{30}\right)^A \neq \left(\frac{27}{17}\right)^B$ there is a basis for a mutually advantageous exchange between the two individuals.

- The direction of exchange:

Given that the initial consumption of individual A for the good X is the last one, and the initial consumption of individual B for the good Y is the first one, so individual A must give up units of X and get more units of Y, and the opposite is true for individual B.

b. The extent to which the exchange continues between the two parties, if the rate of exchange is one unit of good (**X**) against one unit of good (**Y**):

	Individual A		Individual B	
Q	**MUx**	**MUy**	**MUx**	**MUy**
1	24	36	27	33
2	22	33	22	28
3	20	30	18	24
4	18	27	16	20
5	16	24	14	17
6	14	21	12	15

The individual A would give up the **6th** unit of (X) in order to obtain one additional unit of good (Y). While, the individual B would give up the **5th** unit of (Y) in order to get one additional unit of good (X). We find that: $(\frac{16}{27})^A \neq (\frac{22}{20})^B$

Since: $(\frac{16}{27})^A \neq (\frac{22}{20})^B$, the process of exchange continues between the two individuals.
Once again, the individual A would give up the **5th** unit of (X) in order to obtain one additional unit of good (Y). While, the individual B would give up the **4th** unit of (Y) in order to get one additional unit of good (X). We find that: $(\frac{18}{24})^A = (\frac{18}{24})^B$

Since: $(\frac{18}{24})^A = (\frac{18}{24})^B$, the process of exchange would stop at this level.

Individual A: **4**X; **5**Y **TU_A'** = (24 + 22 + 20 + 18) + (36 +33 +30 + 27 + 24) = **234 utils**

Individual B: **3**X; **3**Y **TU_B'** = (27 + 22 +18) +(33 +28 +24) = **152 utils**

Both of the two individuals gained more utility. The total utility of individual A has increased by **21 utils**; (from 213 utils before the exchange to 234 utils after the exchange), and the total utility of individual B has also increased by **3 utils**; (from 149 utils before the exchange to 152 utils after the exchange).

4. a. The market basket of cell phones and sunglasses Maria can consume to maximize her utility:
I = \$200, P_C =\$100, P_S = \$50.

We calculate $\frac{MU_G}{P_G}$ and $\frac{MU_T}{P_T}$ at each quantity level.

Quantity of cell phones	TU_C (utils)	MU_C	$\frac{MU_C}{P_C}$	$\frac{MU_C}{P_C'}$	Quantity of sunglasses	TU_S (utils)	MU_S	$\frac{MU_S}{P_S}$
0	0	-	-	-	0	0	-	-
1	500	**500**	**5**	**10**	**1**	**300**	**300**	**6**
2	980	**480**	**4.8**	**9.6**	**2**	550	**250**	**5**
3	**1280**	**300**	**3**	**6**	3	700	**150**	**3**
4	1530	**250**	**2.5**	**5**	4	800	**100**	**2**

The equilibrium condition:

$$\begin{cases} \frac{MU_C}{P_C} = \frac{MU_S}{P_S} \Rightarrow \frac{500}{100} = \frac{250}{50} = 5 \\ \text{s. t.: } \mathbf{I = P_C C + P_S S \Rightarrow 200 = 100(1) + 50(2)} \end{cases}$$

$TU_{C,S}$ = 500 + 550 = **1050 utils**

Maria can buy **one cellphone** and **two pairs of sunglasses**, and she receives a total utility of **1050 utils** from consuming these two goods.

b. The new market basket of cell phones and sunglasses:

I = \$200, **P_C' =\$50**, P_S = \$50.

The equilibrium condition:

$$\begin{cases} \frac{MU_C}{P_C'} = \frac{MU_S}{P_S} \Rightarrow \frac{300}{50} = \frac{300}{50} = 6 \\ \text{s. t.: } I = P_c'C + P_SS \Rightarrow 200 = 50(3) + 50(1) \end{cases}$$

$TU_{C,S}'$ = 1280 + 300 = **1580 utils**

As the price of cellphones goes down to $50 each, Maria can buy **three cellphones** and **one pair of sunglasses**, and she receives a total utility of **1580 utils**.

c. Describing the substitution effect and the income effect of this fall in the price of cell phones. (Cell phones are a normal good.)

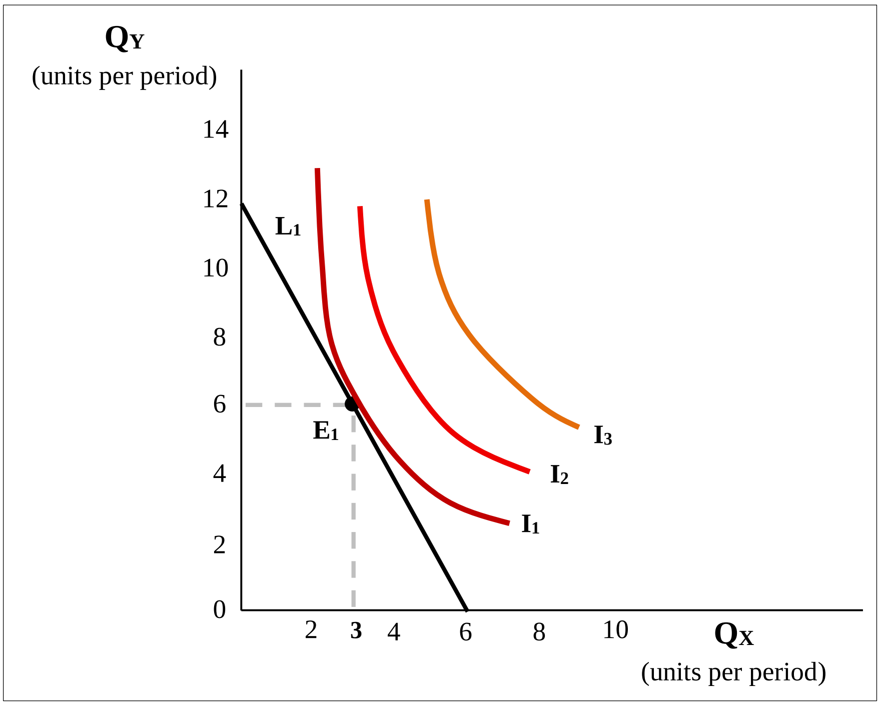
When the price of cellphones ***falls*** from $100 to $50, holding the price of sunglasses and income unchanged. The substitution effect ***increases*** quantity demanded of cellphones and **decreases** the quantity demanded of sunglasses, and Maria keeps the same level of utility (1050 utils). The fall in price will also increase Maria's purchasing power (her real income), which allowing her to ***increase*** the quantity demanded of cellphones even further (because they are a normal good), as she ends up purchasing three cellphones and only one pair of sunglasses (instead of one cellphone and two pairs of sunglasses before the fall in price). Thus, Maria's total utility increased to1580 utils.

The substitution and income effects work together in the same direction (they reinforce each other), causing quantity demanded to increase and move in the opposite direction of the price. Therefore, cellphones as a normal good obey the law of demand (its demand curve is downward-sloping.)

Chapter 3

1. **a.** Drawing the three indifference curves and the budget constraint line on the same set of axes:
   $I = \$12$, $P_X = \$2$, $P_Y = \$1$.

The budget constraint line: $I = P_XX + P_YY \Rightarrow 12 = 2X + Y \Rightarrow$ **$Y = 12 - 2X$**



b. The point at which this individual is in equilibrium:

The optimal combination is: 12 = 2X + Y => 12 = 2(**3**) + (**6**) => **X = 3 units ; Y = 6 units**

Adem is in equilibrium at point **E_1 (3, 6)**, where the budget line is tangent to the indifference curve I_1, which is the highest indifference curve that Adem can reach given his budget line. Because they are tangent, the absolute slope of indifference curve I_1 ($MRS_{X,Y} = \frac{Y_0}{X_0}$) and the absolute slope of the budget line ($MRT_{X,Y} = \frac{P_X}{P_Y}$) are equal at point E_1. That is, at this point: $\frac{Y_0}{X_0} = \frac{P_X}{P_Y} \Rightarrow \frac{6}{3} = \frac{2}{1} = 2$.

c. Deriving the income-consumption curve and the Engel curve for this individual.

The increase in Adem's money income from \$12 to \$16, and then to \$20 per time period, (with prices unchanged), causes the budget line to shift outward parallel to the original one, from L_1 to L_2 then to L_3.

The budget lines and optimal combinations:

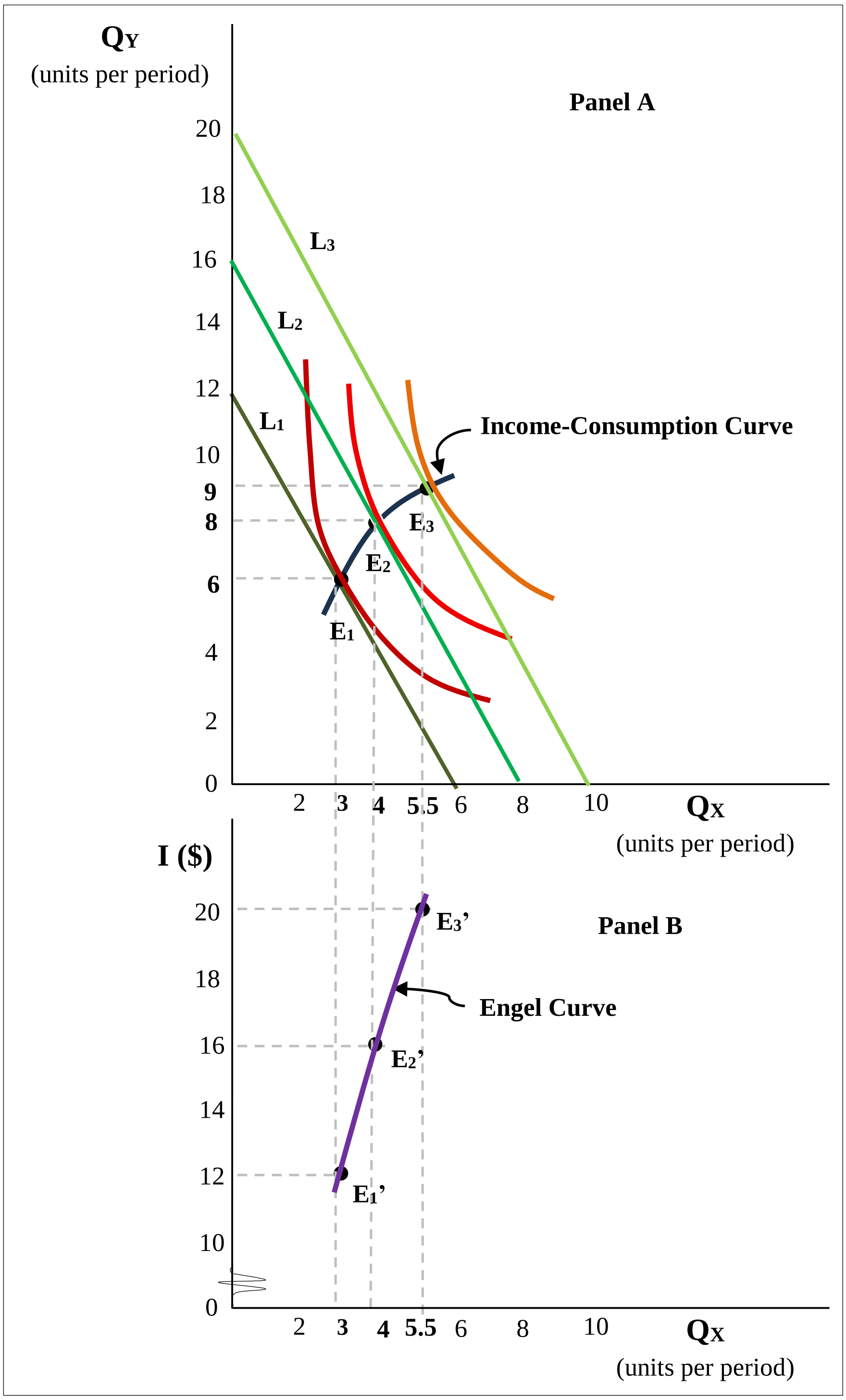
I_1 = \$12 => L_1: 12 = 2X + Y => **Y = 12 - 2X** ⟶ **E_1 (3, 6)**

I_2 = \$16 => L_2: 16 = 2X + Y => **Y = 16 - 2X** ⟶ 16 = 2(**4**) + (**8**) => **E_2 (4, 8)**

I_3 = \$20 => L_3: 20 = 2X + Y => **Y = 20 - 2X** ⟶ 20 = 2(**5.5**) + (**9**) => **E_3 (5.5, 9)**

In panel A of Figure below, when Adem's initial income is **\$12** per time period, he reaches equilibrium at point **E_1** on indifference curve **I_1** by purchasing **3X** and **6Y**. When his income rises to **\$16**, Adam attains equilibrium at point **E_2** on indifference curve **I_2** by buying **4X** and **8Y**. Then, when income rises to **\$20** per time period, he reaches equilibrium at point **E_3** on indifference curve **I_3** by purchasing **5.5X** and **9Y**. Connecting the optimal consumption points **E_1, E_2, E_3** associated with different income levels yields **the income-consumption curve** for Adam.

In panel B, line that joins points **E_1', E_2', E_2'** is Adam's **Engel curve** for good (X). It shows the relationship between the quantity demanded of good (X) and income, holding prices constant. At an income level of **\$12** per time period, Adam purchases **3** units of (X); at an income level of **\$16**, he purchases **4** units of (X); and at an income level of **\$20**, he purchases **5.5** units of (X).



d. The nature of good (X): When Adam's money income rises, the quantity demanded of good (X) rises. That is, there is a positive relationship between income and quantity demanded of the good. This relationship is represented graphically in Engel curve, which is upward sloping (positively sloped). Thus, the good (X) is **a normal good**.

2. a. The optimal quantities of food and clothing that maximize the satisfaction of Sarah, and the value of $TU_{F,C}$: $\mathbf{TU_{(F,C)} = FC}$, I = **1200** , P_F = **2** , P_C = **10**

Method 1: **The condition of tangency**

$$\begin{cases} \mathbf{MRS_{F,C} = MRT_{F,C} \Rightarrow \dfrac{MU_F}{MU_C} = \dfrac{P_F}{P_C}} \ \mathbf{\ldots(1)} \\ \mathbf{s.\ t.\ \ I = P_F F + P_C C \ldots(2)} \end{cases}$$

$$\mathbf{(1)} \Leftrightarrow \frac{\mathbf{dTU}}{\mathbf{dC}} = \frac{\mathbf{P_F}}{\mathbf{P_C}} \Leftrightarrow \frac{\mathbf{C}}{\mathbf{F}} = \frac{\mathbf{2}}{\mathbf{10}} \Leftrightarrow \frac{\mathbf{C}}{\mathbf{F}} = \frac{\mathbf{1}}{\mathbf{5}} \Leftrightarrow \mathbf{F = 5C \ldots(3)}$$

Substituting (**3**) into (**2**), we find:

$I = P_F F + P_C C \Rightarrow 1200 = 2F + 10C \Rightarrow 1200 = 2(\mathbf{5C}) + 10C \Rightarrow 1200 = 20\mathbf{C} \Rightarrow \mathbf{C_0 = 60\ units}$

Substituting C_0 into (**3**), we find: $F = 5(\mathbf{60}) \Rightarrow \mathbf{F_0 = 300\ units}$

Method 2: **The Lagrange multiplier**

Lagrange Expression: $\mathbf{L = TU_{F,C} + \lambda(I_0 - P_F F - P_F C)} \Rightarrow \mathbf{L = FC + \lambda(1200 - 2F - 10C)}$

We take the partial derivatives with respect to X, Y, and λ, and then equating them to zero.

$$\frac{dL}{dF} = 0 \Rightarrow C - 2\lambda = 0 \Rightarrow \mathbf{C = 2\lambda \ldots..(1)}$$

$$\frac{dL}{dC} = 0 \Rightarrow F - 10\lambda = 0 \Rightarrow \mathbf{F = 10\lambda \ldots..(2)}$$

$$\frac{dL}{d\lambda} = 0 \Rightarrow 1200 - 2F - 10C = 0 \Rightarrow \mathbf{1200 = 2F - 10C \ldots.(3)}$$

Dividing (**1**) over (**2**), we find:

$$\frac{C}{F} = \frac{2\lambda}{10\lambda} \Rightarrow \frac{C}{F} = \frac{1}{5} \Rightarrow \mathbf{F = 5C \ldots(4)}$$

Substituting (**4**) into (**3**), we find:

$1200 = 2F + 10C \Rightarrow 1200 = 2(\mathbf{5C}) + 10C \Rightarrow 1200 = 20\mathbf{C} \Rightarrow \mathbf{C_0 = 60\ units}$

Substituting C_0 into (**4**), we find: $F = 5(\mathbf{60}) \Rightarrow \mathbf{F_0 = 300\ units}$

- The value of total utility is: $TU_{F,C} = FC \Rightarrow TU_{F,C} = 300 \times 60 \Rightarrow \mathbf{TU_{F,C} = 18000\ utils}$

b. Sarah's Marginal Rate of Substitution ($MRS_{F,C}$) when utility is maximized:

$$\boldsymbol{MRS_{F,C}} = \frac{C_0}{F_0} = \frac{P_F}{P_C} = \frac{60}{300} = \frac{2}{10} = \mathbf{1/5}$$

Explanation: Sarah is willing to give up **1/5 unit** of *clothing* to obtain **one additional unit** of *food* while maintaining the same level of utility (remaining on the same indifference curve).

c. The demand equations for food and clothing as a function of the two prices and income:

$$\begin{cases} \mathbf{\dfrac{MU_F}{MU_C} = \dfrac{P_F}{P_C}} \ \mathbf{\ldots(1)} \\ \mathbf{s.\ t.\ \ I = P_F F + P_C C \ldots(2)} \end{cases}$$

$$\mathbf{(1)} \Leftrightarrow \frac{C}{F} = \frac{P_F}{P_C} \Leftrightarrow P_F F = P_C C \Rightarrow \mathbf{F = \frac{P_C C}{P_F}} \ \mathbf{\ldots(3)}$$

Substituting (**3**) into (**2**), we find:

$I = P_F F + P_C C \Rightarrow I = P_F\left(\mathbf{\dfrac{P_C C}{P_F}}\right) + P_C C \Rightarrow I = 2P_C C \Rightarrow \mathbf{C = \dfrac{I}{2P_C}}$ Sarah's demand function for clothing.

Substituting C into (3), we find:

$\mathbf{(3)} \Leftrightarrow F = \dfrac{P_C\left(\mathbf{\dfrac{I}{2P_C}}\right)}{P_F} \Leftrightarrow F = \mathbf{\dfrac{I}{2P_F}}$ Sarah's demand function for food.

d. Sarah's income rises to **I' = \$1500**, and the prices remain unchanged. The combination of food and clothing should she buy to maximize her utility:

we can use demand functions to find the new quantities of food and clothing:

$$\mathbf{F_0' = \frac{I'}{2P_F}} \Rightarrow \mathbf{F_0' = \frac{1500}{2(2)}} \Rightarrow \mathbf{F_0' = 375\ units}$$

$$\mathbf{C_0' = \frac{I'}{2P_C}} \Rightarrow \mathbf{C_0' = \frac{1500}{2(10)}} \Rightarrow \mathbf{C_0' = 75\ units}$$

- The type of each good: When Sarah's income rises, the quantity demand for both food and clothing increases (there is a positive relationship between quantity demanded of the two goods and income). Thus, both food and clothing are **normal goods**.

3. a. The amount of minimum income spent on goods (X) and (Y) to receive a total utility of **3000:**

$TU_{(X,Y)} = 3X^2Y$, P_X = DZD 10, P_Y = DZD 5, TU_0 = 3000 utils

We use the Lagrangian multiplier:

$$L = P_X X + P_Y Y + \lambda(TU_0 - 3X^2Y) \Rightarrow L = 10X + 5Y + \lambda(3000 - 3X^2Y)$$

$$\frac{dL}{dX} = 0 \Rightarrow 10 - 6\lambda XY = 0 \Rightarrow \mathbf{10 = 6\lambda XY \;.....(1)}$$

$$\frac{dL}{dY} = 0 \Rightarrow 5 - 3\lambda X^2 = 0 \Rightarrow \mathbf{5 = 3\lambda X^2 \;.....(2)}$$

$$\frac{dL}{d\lambda} = 0 \Rightarrow 3000 - 3X^2Y = 0 \Rightarrow \mathbf{3000 = 3X^2Y \;....(3)}$$

Dividing (**1**) over (**2**), we find:

$$\frac{10}{5} = \frac{6\lambda XY}{3\lambda X^2} \Rightarrow \frac{2Y}{X} = 2 \Rightarrow \mathbf{X = Y \;...(4)}$$

Substituting (**4**) into (**3**), we find:

$$3000 = 3X^2(X) \Leftrightarrow \mathbf{X = Y = 10\ units}$$

The minimum income: I = 10X + 5Y = 10(10) + 5(10) => **I = DZD 150**

b. Deriving the demand equations for the two goods as a function of prices and income:

$$\begin{cases} \dfrac{MU_X}{MU_Y} = \dfrac{P_X}{P_Y} \;...(1) \\ \text{s. t. } I = P_X X + P_Y Y \;...(2) \end{cases}$$

$$\mathbf{(1)} \Leftrightarrow \frac{6XY}{3X^2} = \frac{P_X}{P_Y} \Leftrightarrow \frac{2Y}{X} = \frac{P_X}{P_Y} \Rightarrow 2P_Y Y = P_X X \Rightarrow \mathbf{X = \frac{2P_Y Y}{P_X} \;...(3)}$$

Substituting (**3**) into (**2**), we find:

$I = P_X X + P_Y Y \Rightarrow I = P_X(\frac{2P_Y Y}{P_X}) + P_Y Y \Rightarrow I = 3P_Y Y \Rightarrow \mathbf{Y = \frac{I}{3P_Y}}$ the consumer's demand function for good (Y).

Substituting **Y** into (**3**), we find:

$\mathbf{(3)} \Leftrightarrow X = \dfrac{2P_Y(\frac{I}{3P_Y})}{P_X} \Leftrightarrow \mathbf{X = \frac{2I}{3P_X}}$ the consumer's demand function for good (X).

- The relationship between the two goods:

The two goods are independent from each other, because each function is written as only its own price of the good and income. In addition, there is a positive relationship between quantities demanded and income, and a negative relationship between quantities demanded and prices.

4. a. The marginal utility functions (MU_X) and (MU_Y):

$TU_{(X,Y)} = \sqrt{XY} = X^{½}Y^{½}$, $P_X = \$60$, $P_Y = \$40$.

$$MU_X = \frac{dL}{dX} = ½\, X^{-½}Y^{½} = \frac{Y^{½}}{2X^{½}}$$

$$MU_Y = \frac{dL}{dX} = ½\, X^{½}Y^{-½} = \frac{X^{½}}{2Y^{½}}$$

b. Is the market basket (**X = 5, Y = 10**) the consumer's optimal basket or not?

We check the two conditions:

$$\begin{cases} \dfrac{MU_X}{MU_Y} = \dfrac{P_X}{P_Y} \quad ...(1) \\ \text{s. t. } I = P_X X + P_Y Y ...(2) \end{cases}$$

Let's start with the first one: $\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$

$$\frac{MU_X}{MU_Y} = \frac{½\, X^{-½}Y^{½}}{½\, X^{½}Y^{-½}} = \frac{Y}{X} = \frac{10}{5} = \mathbf{2}$$

$$\frac{P_X}{P_Y} = \frac{60}{40} = \mathbf{1.5}$$

$\frac{MU_X}{MU_Y} \neq \frac{P_X}{P_Y}$ so, the market basket (**X = 5, Y = 10**) is not the woman's optimal basket.

c. Calculating the optimal basket that maximizes her total utility:

$\sqrt{XY} = X^{½}Y^{½}$, $P_X = \$60$, $P_Y = \$40$, $I = \$1200$

$$\begin{cases} \dfrac{MU_X}{MU_Y} = \dfrac{P_X}{P_Y} \quad ...(1) \\ \text{s. t. } I = P_X X + P_Y Y ...(2) \end{cases}$$

(1) $\Leftrightarrow \frac{Y}{X} = \frac{P_X}{P_Y} \Leftrightarrow \frac{Y}{X} = \frac{60}{40} \Rightarrow \frac{Y}{X} = \frac{3}{2} \Rightarrow \mathbf{Y = \frac{3X}{2}}$ **…(3)**

Substituting (**3**) into (**2**), we find:

$I = P_X X + P_Y Y \Rightarrow 1200 = 60X + 40Y \Rightarrow 1200 = 60X + 40(\frac{3X}{2}) \Rightarrow 1200 = 60X + 60X \Rightarrow 1200 = 120X$

$\Rightarrow$ **X_0 = 10 units**

Substituting X_0 into (**3**), we find: $Y = \frac{3(10)}{2} \Rightarrow$ **Y_0 = 15 units**

- The total utility : $TU_{(X,Y)} = (10)^{½}(15)^{½} \Rightarrow$ **$TU_{(X,Y)}$ = 12.25 utils**

d. Calculating (MU_X) and (MU_Y) at the point of equilibrium:

$$MU_X = \frac{Y^{½}}{2X^{½}} = \frac{(15)^{½}}{2(10)^{½}} = \mathbf{0.61\ unit}$$

$$MU_Y = \frac{X^{½}}{2Y^{½}} = \frac{(10)^{½}}{2(15)^{½}} = \mathbf{0.41\ unit}$$

5. a. Deriving the demand functions for good (X) and good (Y):

$TU_{(X,Y)} = X^2Y + 4$

$$\begin{cases} \dfrac{MU_X}{MU_Y} = \dfrac{P_X}{P_Y} \quad ...(1) \\ \text{s. t. } I = P_X X + P_Y Y ...(2) \end{cases}$$

(1) <=> $\frac{2XY}{X^2} = \frac{P_X}{P_Y}$ <=> $\frac{2Y}{X} = \frac{P_X}{P_Y}$ => $2P_YY = P_XX$ => $\mathbf{X = \frac{2P_YY}{P_X}}$ **…(3)**

Substituting (**3**) into (**2**), we find:

$I = P_XX + P_YY$ => $I = P_X(\frac{2P_YY}{P_X}) + P_YY$ => $I = 3P_YY$ => $\mathbf{Y = \frac{I}{3P_Y}}$ Ali's demand function for good (Y).

Substituting **Y** into (**3**), we find:

(3) <=> $X = \frac{2P_Y(\frac{I}{3P_Y})}{P_X}$ <=> $\mathbf{X = \frac{2I}{3P_X}}$ Ali's's demand function for good (X).

b. The optimal basket of the two goods:

$\mathbf{P_X}$ = £**2** , $\mathbf{P_Y}$ **= £4** , **I = £24**

$\mathbf{X_0} = \frac{2I}{3P_X}$ => $\mathbf{X_0} = \frac{2(24)}{3(2)}$ => $\mathbf{X_0 = 8}$ **units**

$\mathbf{Y_0} = \frac{I}{3P_Y}$ => $\mathbf{Y_0} = \frac{(24)}{3(4)}$ => $\mathbf{Y_0 = 2}$ **units**

c. The amount of total utility and the indifference curve equation at the equilibrium point:

- Total utility : $\mathbf{TU_0} = X^2Y + 4 = (\mathbf{8})^2(\mathbf{2}) + 4 =$ **132 utils**

- The indifference curve equation:

$\mathbf{TU_0} = X^2Y + 4$ => $\mathbf{132} = X^2Y + 4$ => $\mathbf{128} = X^2Y$ => $\mathbf{Y = \frac{128}{X^2}}$

d. The new optimal basket of the two goods:

$\mathbf{P_X'}$ = £**4** , P_Y = £4 , I = £24

$\mathbf{X_0'} = \frac{2I}{3P_X'}$ => $\mathbf{X_0'} = \frac{2(24)}{3(4)}$ => $\mathbf{X_0 = 4}$ **units**

$\mathbf{Y_0'} = \frac{I}{3P_Y}$ => $\mathbf{Y_0'} = \frac{(24)}{3(4)}$ => $\mathbf{Y_0 = 2}$ **units**

The curve that traces the utility maximizing combinations of the two goods as the price of good (X) increases, while the price of good (Y), and income remain constant is called: **the price-consumption curve**.

e. Separating the substitution effect and income effect resulting from the rise in the price of good (X):

- **The initial situation of Ali**:

P_X = £2 , P_Y = £4 , I = £24

The optimal market basket: $\mathbf{E_1}$ **(8, 2)**

The indifference curve equation ($\mathbf{I_1}$): $\mathbf{Y = \frac{128}{X^2}}$

The budget line equation ($\mathbf{L_1}$): 24 = 2X + 4Y => **Y = - ½X + 6**

- **The substitution effect** (a movement along the original indifference curve $\mathbf{I_1}$ from $\mathbf{E_1}$ to $\mathbf{E_1'}$)**:**

$\mathbf{P_X'}$ **= £4** , P_Y = £4 , I = £24

The indifference curve equation ($\mathbf{I_1}$): $\mathbf{Y = \frac{128}{X^2}}$

The budget line equation ($\mathbf{L_1'}$): 24 = 4X + 4Y => **Y = -X + 6**

- Finding the new optimal market basket $\mathbf{E_1'}$:

The slope of indifference curve ($\frac{dY}{dX}$) = the slope of the budget line (-1)

$\frac{dY}{dX} = -1 \Rightarrow (\frac{128}{X^2})' = -1 \Rightarrow \frac{-256}{X^3} = -1 \Rightarrow$ **X_0' = 6.35 units**

$Y = \frac{128}{X^2} \Rightarrow Y = \frac{128}{(6.35)^2} \Rightarrow$ **Y_0' = 3.17 units**

E_1' (6.35, 3.17)

Quantity of (X): decreases from 8 to 6.35 => **ΔX^S = -1.65 units**

Quantity of (Y): increases from 2 to 3.17 => **ΔY^S = +1.17 units**

- **<u>The income effect</u>** (a shift from **E_1'** on indifference curve **I_1** to **E_2** on indifference curve **I_2**):

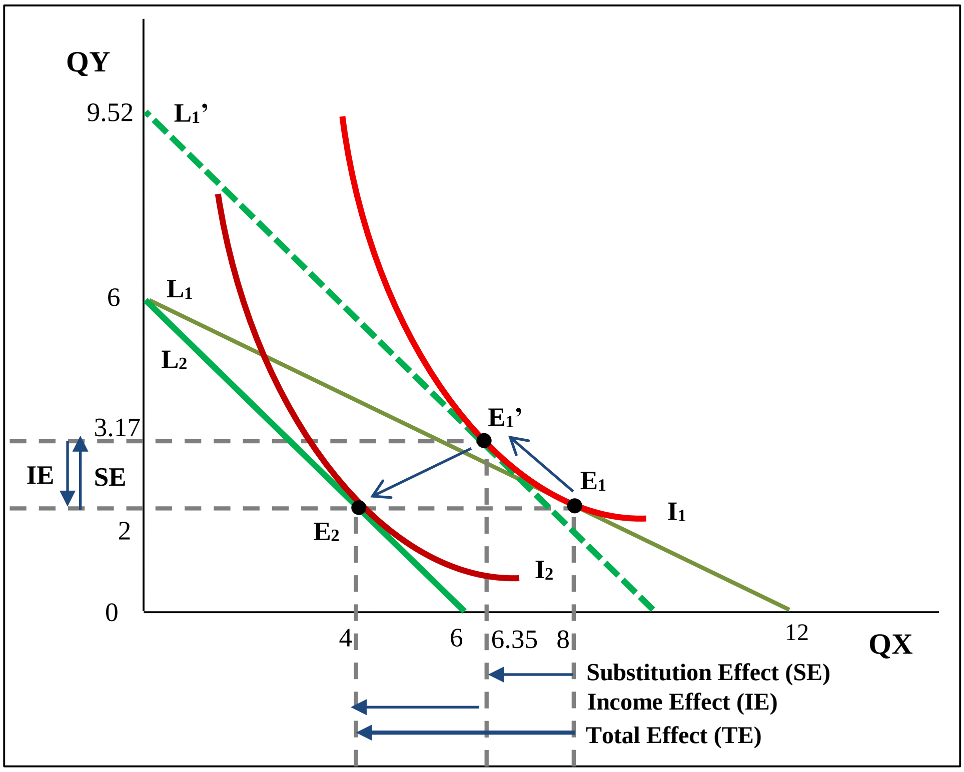
Quantity of (X): decreases from 6.35 to 4 => **ΔX^I = -2.35 units**

Quantity of (Y): decreases from 3.17 to 2 => **ΔY^I = -1.17 units**

- **<u>The total effect</u>** (a shift from **E_1** on indifference curve **I_1** to **E_2** on indifference curve **I_2**):

Quantity of (X): ΔX^T = -1.65 +(-2.35) => **ΔX^T = - 4 units**

Quantity of (Y): ΔY^T = +1.17 +(-1.17) => **ΔY^T = 0 units**



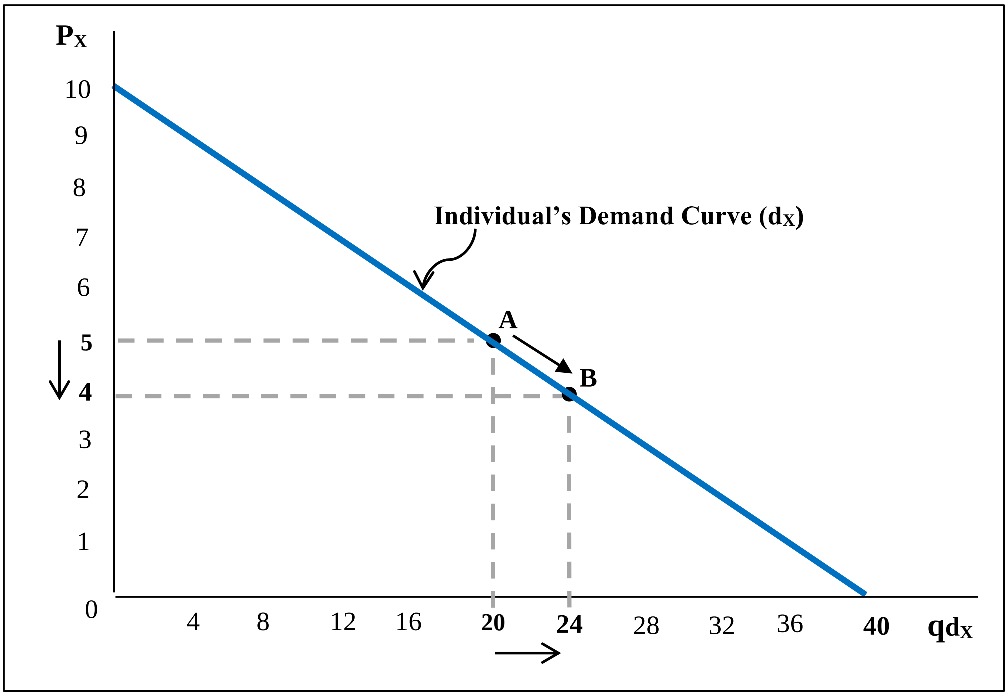
<u>Chapter 4</u>

1. a. Deriving the individual's demand schedule and the individual's demand curve: **q_{dx} = 40 - 4Px**

The individual's demand schedule

Px	0	1	2	3	4	5	6	7	8	9	10
q_{dx}	40	36	32	28	24	20	16	12	8	4	0

The individual's demand curve



b. The maximum quantity of this good that the individual will ever demand per unit of time is: **40 units**. This occurs at **zero** price. This is called *the saturation point* for the individual. Additional units of good (X) result in a storage and disposal problem for the individual. Thus the "relevant" points on a demand curve are all in the first quadrant.

c. If the price of good (X) falls from DZD **5** to DZD **4**, holding all other factors constant, the quantity demanded increases from **20** to **24 units** from point **A** to point B, expressed by *a downward movement along the same demand curve*. This is called *a change in quantity demanded* (an increase in quantity demanded, or extension of demand).

2. a. The world supply function for 'corn':

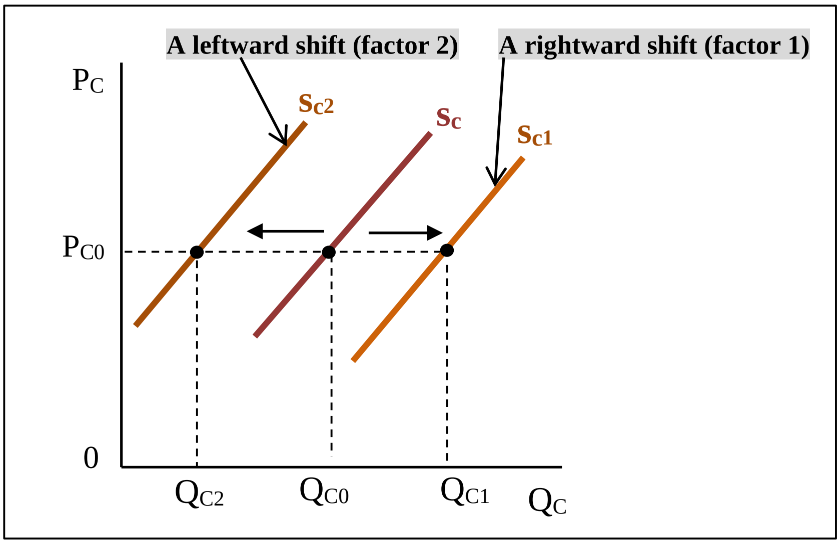
The U.S. supply function for 'corn' is: $\boldsymbol{Q_{C(US)} = 10 + 10P_C}$

The supply function of the rest of the world for 'corn' is: $\boldsymbol{Q_{C(RW)} = 5 + 20P_C}$

$\boldsymbol{Q_{C(W)} = Q_{C(US)} + Q_{C(RW)}}$ => $\boldsymbol{Q_{C(W)}} = (10 + 10P_C) + (5 + 20P_C)$ => $\boldsymbol{Q_{C(W)} = 15 + 30P_C}$

b. How each of the following factors will affect the supply of corn:

- **Factor 1:** If the US producers of corn expect that its price will fall in the next few months, then the producers will supply more quantities of corn to the market today, and there will be an increase in supply, *shifting the supply curve rightward.*
- **Factor 2:** If bad weather destroys the crops of corn in many parts of UAS, then productive capacity of those farms decreases, and there will be a decrease in supply, *shifting the supply curve leftward*.



3. a. Writing the quantity demanded as a function of only the price of good (X):

$Q_{DX} = 300 - 2P_X + 4I$, **I = 25**.

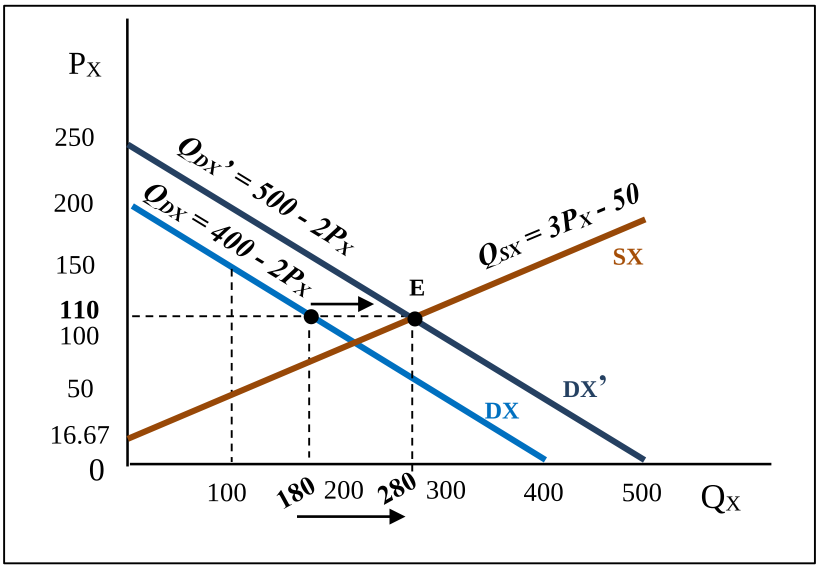
$Q_{DX} = 300 - 2P_X + 4I \Rightarrow Q_{DX} = 300 - 2P_X + 4(25) \Rightarrow Q_{DX} = 400 - 2P_X$

b. If the income is *doubled* (rises from **25** to **50**), and the price of the good remains constant at DZD **110**.

At $P_{X1} = 110$, $I_1 = 25$: $Q_{X1} = 300 - 2(110) + 4(25) \Rightarrow Q_{X1} = 180$ **units**

At $P_{X1} = 110$, $I_2 = 50$: $Q_{X2} = 300 - 2(110) + 4(50) \Rightarrow Q_{X2} = 280$ **units**

There will be an increase in demand, and the entire demand curve, D_X, shifts rightward to a new position D_X'. The quantity demanded increases by: $\Delta Q_X = 100$ **units** (from 180 units to 280 units) at the same price 110, as shown in the following diagram.



c. Finding the equilibrium market price and quantity for the good after the income's change:

The supply function for this good is given by: $Q_{SX} = 3P_X - 50$.

The market demand function after the income has doubled is:

$Q_{DX} = 300 - 2P_X + 4I \Rightarrow Q_{DX} = 300 - 2P_X + 4(50) \Rightarrow Q_{DX} = 500 - 2P_X$

The condition of equilibrium is: $Q_{DX} = Q_{SX} \Rightarrow 500 - 2P_X = 3P_X - 50 \Rightarrow P_0 =$ DZD **110**.

Substituting P_0 into the market demand function or the market supply function, we find:

$Q_0 = 500 - 2(110) = 3(110) - 50 \Rightarrow Q_0 = 280$ units.

The point of equilibrium is: **E (280, 110)**

4. a. Finding the market demand function and the market supply function for good (X):

$\mathbf{N_B = 10,000}$, $\mathbf{q_{dx} = 12 - 2Px}$, $\mathbf{N_S = 1,000}$, $\mathbf{q_{Sx} = 20Px}$

The market demand function for good (X):

$Q_{DX} = N_B.q_{dX} = 10,000(12 - 2Px)$ => $\mathbf{Q_{DX} = 120,000 – 20,000Px}$

The market supply function for good (X):

$Q_{SX} = N_S.q_{SX} = 1,000(20Px)$ => $\mathbf{Q_{SX} = 20,000Px}$

b. Calculating the market equilibrium price and quantity:

$Q_{DX} = Q_{SX}$ => $120,000 – 20,000P_X = 20,000P_X$ => P_0 = DZD **3**

$Q_0 = 120,000 – 20,000(\mathbf{3}) = 20,000(\mathbf{3})$ => $\mathbf{Q_0 = 60,000}$ units

c. Calculating the quantity bought by each individual, and the quantity sold by each seller:

- The quantity bought by each individual is: $q_{dx1} = 12 - 2Px$ => $q_{dx1} = 12 – 2(3)$ => $\mathbf{q_{dx1} = 6}$ **units**
- The quantity sold by each seller is: $q_{Sx1} = 20Px$ => $q_{Sx1} = 20(3)$ => $\mathbf{q_{Sx1} = 60}$ **units**

5. a. Finding the function of market demand and the function of market supply, (both are linear functions):

- The linear function of market demand is given by: $Q_{DX} = a – bP_X$

At the price, $\mathbf{P_1 = \$1.2}$, the quantity demanded, $\mathbf{Q_{D1} = 170}$ units

At the price $\mathbf{P_2 = \$1.3}$, the quantity demanded, $\mathbf{Q_{D2} = 155}$ units

We substitute the values of prices and quantities into the market demand function to find the values of the two constants ***a*** and ***b***:

$Q_{D1} = a – bP_1$ => $170 = a – b(1.2)$ => $a = 1.2b + 170$

$Q_{D2} = a – bP_2$ => $155 = a – b(1.3)$ => $a = 1.3b + 155$

$1.2b + 170 = 1.3b + 155$ => $0.1b =15$ => $b = 150$

$a =1.2(150) + 170 = 1.3(150) + 155$ => $a = 350$

The market demand function is: $Q_{DX} = 350 – 150P_X$

- The linear function of market supply is given by: $Q_{SX} = c + dP_X$

At the price, $\mathbf{P_1 = \$1.2}$, the quantity supplied, $\mathbf{Q_{S1} = 80}$

At the price $\mathbf{P_2 = \$1.3}$, the quantity supplied, $\mathbf{Q_{S2} = 110}$

We substitute the values of prices and quantities into the market supply function to find the values of the two constants ***c*** and ***d***:

$Q_{S1} = c + dP_1$ => $80 = c + d(1.2)$ => $c = 80 - 1.2d$

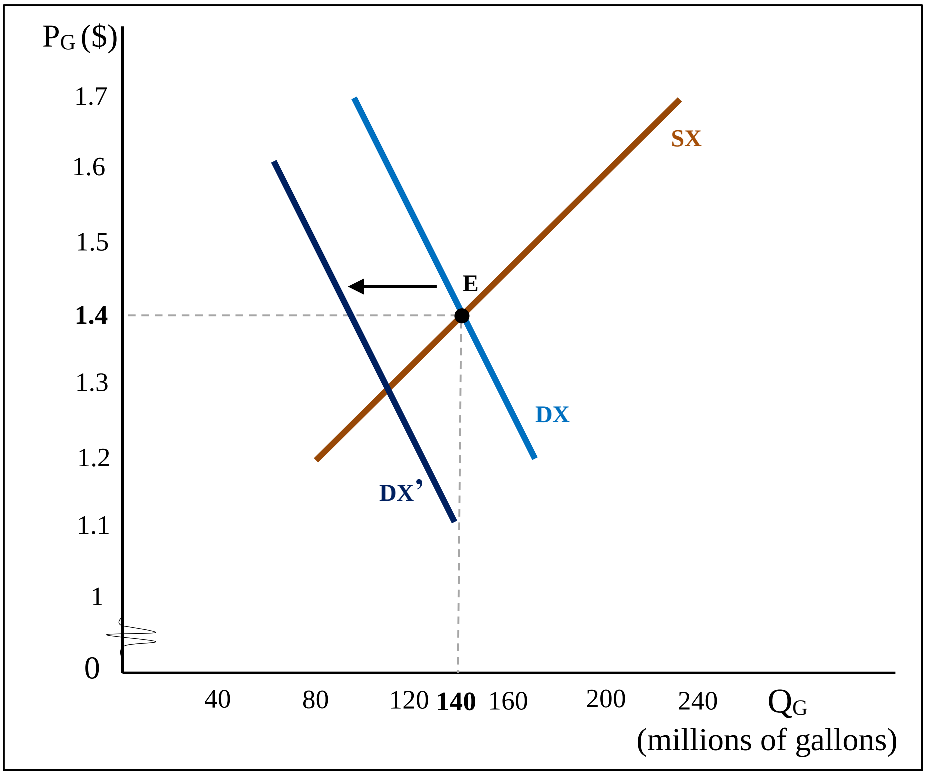
$Q_{S2} = a + dP_2$ => $110 = c + d(1.3)$ => $c = 110 - 1.3d$

$80 - 1.2d = 110 - 1.3d$ => $0.1d =30$ => $d = 300$

$c = 80 - 1.2(300) = 110 - 1.3(300)$ => $c = - 280$

The market supply function is: $Q_{SX} = - 280 +300P_X$

b. Drawing the market demand and supply curves, and illustrating how a rise in the price of 'automobiles' would affect the gasoline market:



If the price of automobiles rises, then the demand for gasoline would decrease, because automobiles and gasoline are complements. This causes a leftward shift of the entire market demand curve of gasoline.

6. a. The quantity demanded and the quantity supplied as functions of only the price of coffee:

$\boldsymbol{Q_{DC} = 8.56 - P_{Cf} - 0.3P_S + 0.1I}$, $\boldsymbol{Q_{SC} = 9.6 + 0.5P_{Cf} - 0.2P_{Cc}}$, $\boldsymbol{P_S = \$0.20}$, $\boldsymbol{I = \$35}$, $\boldsymbol{P_{Cc} = \$3}$

The market demand function:

$Q_{DC} = 8.56 - P_{Cf} - 0.3P_S + 0.1I \Rightarrow Q_{DC} = 8.56 - P_{Cf} - 0.3(\mathbf{0.2}) + 0.1(\mathbf{35})$

$\Rightarrow Q_{DC} = 8.56 - P_{Cf} - 0.06 + 3.5$

$\Rightarrow \boldsymbol{Q_{DC} = 12 - P_{Cf}}$

The market supply function:

$Q_{SC} = 9.6 + 0.5P_{Cf} - 0.2P_{Cc} \Rightarrow Q_{SC} = 9.6 + 0.5P_{Cf} - 0.2(\mathbf{3})$

$\Rightarrow Q_{SC} = 9.6 + 0.5P_{Cf} - 0.6$

$\Rightarrow \boldsymbol{Q_{SC} = 9 + 0.5\,P_{Cf}}$

b. The equilibrium price and quantity:

The condition of equilibrium: $\boldsymbol{Q_{DC} = Q_{SC}} \Rightarrow 12 - P_{Cf} = 9 + 0.5P_{Cf} \Rightarrow \boldsymbol{P_0 = \$2}$

Substituting $\mathbf{P_0}$ into the market demand function or the market supply function, we find:

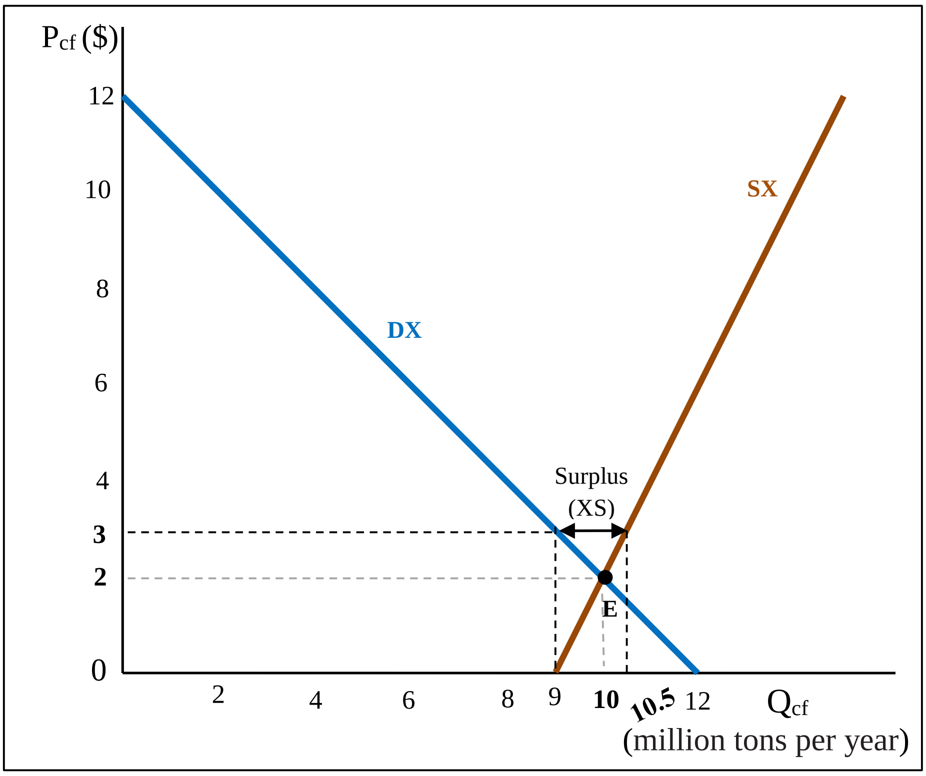
$\boldsymbol{Q_0} = 12 - (\mathbf{2}) = 9 + 0.5(\mathbf{2}) \Rightarrow \boldsymbol{Q_0} = \mathbf{10}$ million tons per year

c. If the equilibrium price *rises* to **$3.** There will be a surplus (excess supply) of:

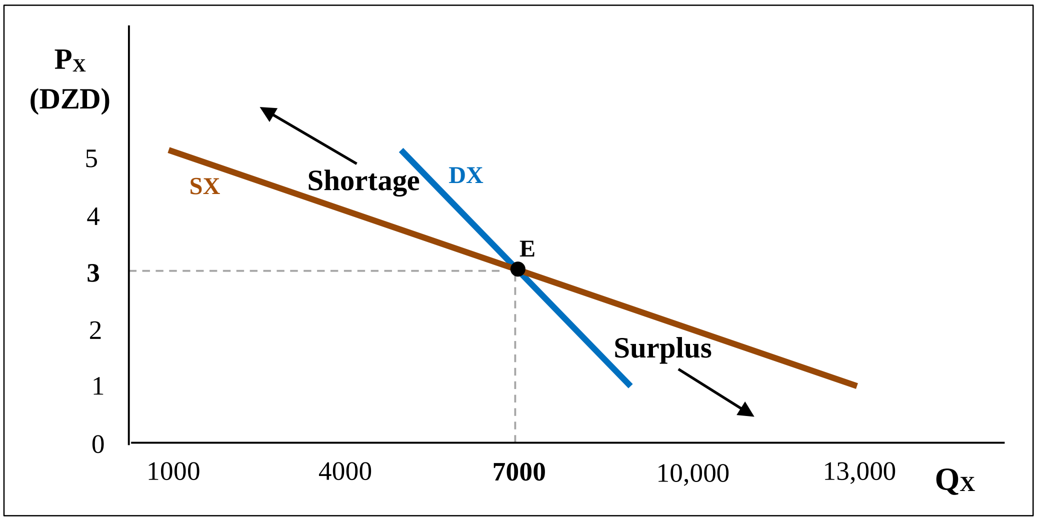
$\boldsymbol{Q_{DC}} = 12 - (\mathbf{3}) = \mathbf{9}$ million tons per year

$\boldsymbol{Q_{SC}} = 9 + 0.5(\mathbf{3}) = \mathbf{10.5}$ million tons per year

$\boldsymbol{\Delta Q_C} = Q_{SC} - Q_{SC} \Rightarrow \Delta Q_C = 10.5 - 9 \Rightarrow \Delta Q_C = \mathbf{1.5}$ million tons per year.



7. a. The demand and supply curves, and the equilibrium price and quantity:



b. The equilibrium price is DZD **3** and the equilibrium quantity is **7000** units. At prices below the equilibrium level, a surplus arises and the market forces tend to produce a lower price which moves us further away from the equilibrium level. At prices above the equilibrium price, a shortage arises and the market forces tend to produce a higher price which moves us further away from the equilibrium level. Thus, the equilibrium for good (X) is **unstable**.

Chapter 5

1. Calculating the percentage change in quantity demanded for good (X), if:

a. The price of the good (X) increases by 10%, with all other factors remaining constant:

$$\mathbf{Q_{DX} = P_X^{-0.3} P_Y^{0.1} I^{0.4}}$$

$$e_{DX} = \frac{dQ_{DX}}{dP_X}\frac{P_X}{Q_{DX}} = -0.3P_X^{-1.3}P_Y^{0.1}I^{0.4}\frac{P_X}{P_X^{-0.3}P_Y^{0.1}I^{0.4}} \Rightarrow e_{DX} = -0.3$$

$$e_{DX} = \frac{\%\Delta Q_{DX}}{\%\Delta P_X} \Rightarrow \%\Delta Q_{DX} = e_{DX}(\%\Delta P_X) \Rightarrow \%\Delta Q_{DX} = -0.3\,(10\%) \Rightarrow \%\Delta Q_{DX} = -3\%$$

If the price of good (X) *increases* by **10%**, with all other factors remaining constant, then the quantity demanded for good (X) *decreases* by **3%**.

Since (**0 <** $|e_{DX}|$ **= 0.3 < 1**), the demand for good (X) is *relatively inelastic*.

b. The price of the other good (Y) increases by 5%, with all other factors remaining constant:

$$e_{X,Y} = \frac{dQ_{DX}}{dP_Y}\frac{P_Y}{Q_{DX}} = 0.1P_X^{-0.3}P_Y^{-0.9}I^{0.4}\ \frac{P_Y}{P_X^{-0.3}P_Y^{0.1}I^{0.4}} \Rightarrow e_{X,Y} = 0.1$$

$$e_{X,Y} = \frac{\%\Delta Q_{DX}}{\%\Delta P_Y} \Rightarrow \%\Delta Q_{DX} = e_{X,Y}(\%\Delta P_Y) \Rightarrow \%\Delta Q_{DX} = 0.1\ (5\%) \Rightarrow \%\Delta Q_{DX} = 0.5\%$$

If the price of the other good (Y) *increases* by **5%**, with all other factors remaining constant, then the quantity demanded for good (X) *increases* by **0.5%**.

Since ($e_{X,Y}$ **= 0.1 > 0**), the two goods (X) and (Y) are *substitutes*.

c. The consumer's income increases by 10%, with all other factors remaining unchanged:

$$e_I = \frac{dQ_{DX}}{dI}\frac{I}{Q_{DX}} = 0.4P_X^{-0.3}P_Y^{0.1}I^{-0.6}\ \frac{I}{P_X^{-0.3}P_Y^{0.1}I^{0.4}} \Rightarrow e_I = 0.4$$

$$e_I = \frac{\%\Delta Q_{DX}}{\%\Delta I} \Rightarrow \%\Delta Q_{DX} = e_I(\%\Delta I) \Rightarrow \%\Delta Q_{DX} = 0.4\ (10\%) \Rightarrow \%\Delta Q_{DX} = 4\%$$

If the price of the consumer's income *increases* by **10%**, with all other factors remaining constant, then the quantity demanded for good (X) *increases* by **4%**.

- The type of the good (X): Since (0 < e_I **= 0.4 < 1**), the good (X) is a *necessity*.

2. a. Finding the income elasticity of demand between the various successive levels of income:

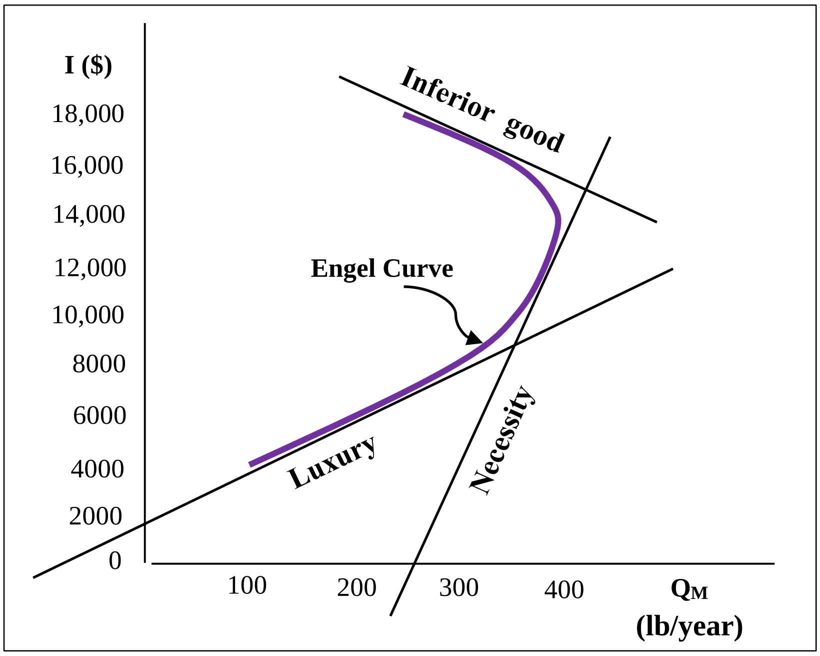
Income (I)	4,000	6,000	8,000	10,000	12,000	14,000	16,000	18,000
Quantity demanded (Q_{DM})	100	200	300	350	380	390	350	250
Income elasticity of demand (e_I)		**2**	**1.5**	**0.67**	**0.43**	**0.16**	**-0.72**	**-2.28**
Type of good	**Luxury**			**Necessity**			**Inferior**	

$$e_{I1} = \frac{\Delta Q_{DX}}{\Delta I}\frac{I}{Q_{DX}} = \frac{200-100}{6000-4000}\ \frac{4000}{100} = 2$$

$$e_{I2} = \frac{\Delta Q_{DX}}{\Delta I}\frac{I}{Q_{DX}} = \frac{300-200}{8000-6000}\ \frac{6000}{200} = 1.5$$

b. At very low levels of income ($8000 per year or less), this family presumably consumes mostly cheap cuts of meat, regular cuts representing a *luxury*. At intermediate levels of income (between $8000 and $14,000 per year), regular cuts of meat become a *necessity*. At high levels of income (above $14,000), this family begins to reduce its consumption of regular cuts of meat, which became *an inferior good*, and consumes more steaks and roast beef.

c. The curve that represents the income-quantity relationship is called *Engel Curve*. It is a curve that measures the change in quantity demanded of a good when the consumer's income changes, other things remaining the same.



We can determine the type of the good using Engel curve. At the points on the curve where *the tangent is downward sloping*, the good is *an inferior good*. At the points on the curve where the tangent is *upward sloping and crosses the horizontal axis (the quantity axis)*, the good is *a necessity* (income inelastic demand). At the points on the curve where the tangent is upward sloping and crosses the vertical axis (the income axis), the good is *a luxury* (income elastic demand).

3. a. Calculating the price elasticity of demand at each price level, and determining when the demand for hotel rooms is elastic, inelastic, or unitary elastic:

Price ($ per chip)	500	450	400	350	300	250	200
Quantity Demanded (millions of chips per year)	20	25	30	35	40	45	50
Total Expenditure	**10,000**	**11,250**	**12,000**	**12,250**	**12,000**	**11,250**	**10,000**
Price elasticity of demand (| e_{DC} |)	**2.5**	**1.8**	**1.33**	**1**	**0.75**	**0.55**	
Type of elasticity of demand	Elastic				Unitary elastic	Inelastic	

$$e_{DC1} = \frac{\Delta Q_{DC}}{\Delta P_C}\frac{P_C}{Q_{DC}} = \frac{25-20}{450-500}\;\frac{500}{20} = \mathbf{-2.5}$$

$$e_{DC2} = \frac{\Delta Q_{DC}}{\Delta P_C}\frac{P_C}{Q_{DC}} = \frac{30-25}{400-450}\;\frac{450}{25} = \mathbf{-1.8}$$

b. See the last row of the table above.

c. When the price of computer chips falls, total expenditure (TE) rises as long as demand is elastic (**$|e_{DC}| > 1$)**. This is because; the percentage increase in quantity demanded is greater than the percentage fall in price. TE reaches a maximum when demand becomes unitary elastic (**$|e_{DC}| = 1$)**. When the price continues to fall, TE decreases as long as demand is inelastic (**$|e_{DC}| < 1$)**. This is because; the percentage increase in quantity is smaller than the percentage fall in price. Thus, TE moves in the opposite direction of price when demand is elastic and in the same direction as price when demand is inelastic.

4. a. The type of demand and goods:

Goods	Price Elasticity of Demand	Income Elasticity of Demand	Cross Elasticity of Demand
Butter	0.35 **Inelastic**	+0.42 **Necessity**	Butter & margarine: +1.54 **Substitutes** Cheese (type 1) & butter: -0.61 **Complements** Cheese (type 1) & cheese (type 2): +0.82 **Substitutes**
Margarine	0.27 **Inelastic**	-0.20 **Inferior**	
Cheese (type 1)	0.62 **Inelastic**	+0.89 **Necessity**	
Cheese (type 2)	1.34 **Elastic**	+1.22 **Luxury**	

b. When the consumers' income rises by **10%**, while the prices of the goods remain the same, total revenue (TR) will change:

- For butter, TR increases by: 10 (+0.42)% = **4.2%**
- For margarine, TR decreases by: 10 (-0.20)% = **-2%**
- For cheese (type 1), TR increases by: 10 (+0.89)% = **8.9%**
- For cheese (type 2), TR increases by: 10 (+1.22)% = **12.2%**

c. We know that TR moves in the same direction as price when demand is inelastic and in the opposite direction of price when demand is elastic. Based on that, the best price policy for the firm, if it seeks to rise its TR is:

- *Increasing* the prices of *butter*, *margarine*, and *cheese (type 1)*, because demand for them is *inelastic*.
- *Decreasing* the price of *cheese (type 2)*, because demand for it is *elastic*.

d. The price elasticity of demand can help the government in formulating its taxation policy through imposing higher rates of taxes on goods with inelastic demand which are, in this case: butter, margarine, and cheese (type 1). While, it can impose lower rate of taxes on goods with elastic demand, cheese (type 2), in this case. Such taxation policy allows the government to rise its revenue from taxes.

5. a. Calculating the price elasticity of supply for 'jeans' at the price **$120** a pair:

The supply function of 'Jeans' is given by: $\mathbf{Q_{SJ} = -72 + \frac{4}{5}P_J}$.

$Q_{SJ} = -72 + \frac{4}{5}(120) \Rightarrow Q_{SJ} = 24$ millions of pairs per year

$$e_{SJ} = \frac{dQ_{SJ}}{dP_J}\frac{P_J}{Q_{SJ}} = \frac{4}{5}\,\frac{120}{24} \Rightarrow e_{SJ} = 4$$

The supply of 'jeans' is elastic.

b. Calculating the price elasticity of supply for 'long-distance phone calls' at the quantity supplied **600**:

The supply function of 'long-distance phone calls' is given by: $\mathbf{Q_{SC} = 20P_C}$.

$(600) = 20P_C \Rightarrow P_C = 30$ cents per minute

$$e_{SC} = \frac{dQ_{SC}}{dP_C}\frac{P_C}{Q_{SC}} = 20\,\frac{30}{600} \Rightarrow e_{SJ} = 1$$

The supply of 'long-distance phone calls' is unitary elastic.

c. If a big surge in demand for the two goods occurred on a given day. The momentary supply elasticity for 'jeans' will be *inelastic or perhaps perfectly inelastic—a vertical supply curve*. Because, on a given day, firms cannot make more adjustments to production (hiring or training additional workers, or buying additional tools and other equipment etc.) to provide more quantity to satisfy the increased demand in the market. By contrast, the momentary supply elasticity for 'long-distance phone calls' will be *perfectly elastic—a horizontal supply curve*. Because, on a given day, firms can easily make adjustments, using the same inputs (telephone cables, computer switching, and satellite time) and they can ensure that the quantity supplied equals the quantity demanded without changing the price.

6. a. Finding the market demand function and the market supply function of the good (X):

- The market demand function:

Group (1): $\mathbf{q_{dx1} = 15 - 3P_X}$; Group (2): $\mathbf{q_{dx2} = 7 - P_X}$ (**5** consumers in each group)

$\mathbf{Q_{DX}} = 5q_{dX1} + 5q_{dX2}$ => $\mathbf{Q_{DX}} = 5(15 - 3P_X) + 5(7 - P_X)$ => $\mathbf{Q_{DX} = 110 - 20P_X}$

- The market supply function:

Firm (A): $\mathbf{q_{SXA} = 15 + 7P_X}$; Firm (B): $\mathbf{q_{SXB} = 5 + 3P_X}$

$\mathbf{Q_{SX}} = q_{SXA} + q_{SXB}$ => $\mathbf{Q_{SX}} = (15 + 7P_X) + (5 + 3P_X)$ => $\mathbf{Q_{SX} = 20 + 10P_X}$

b. The market equilibrium price and quantity:

$\mathbf{Q_{DX} = Q_{SX}}$ => $110 - 20P_X = 20 + 10P_X$ => $30P_X = 90$ => $\mathbf{P_0}$ = DZD **3**

Substituting P_0 into Q_{DX} or Q_{SX}, we find: $\mathbf{Q_0} = 110 - 20(\mathbf{3}) = 20 + 10(\mathbf{3})$ => $\mathbf{Q_0 = 50}$ **units**

c. Calculating the price elasticity of demand for each individual consumer, and for the market:

- For each individual consumer:

The quantity demanded by each consumer in group (1) is: $\mathbf{q_{dX1}} = 15 - 3(\mathbf{3})$ => $\mathbf{q_{01} = 6}$ **units**

The quantity demanded by each consumer in group (2) is: $\mathbf{q_{dX2}} = 7 - (\mathbf{3})$ => $\mathbf{q_{02} = 4}$ **units**

$$e_{dX1} = \frac{dq_{dX1}}{dP_X}\frac{P_0}{q_{01}} = -3\,\frac{3}{6} = \mathbf{-1.5}$$ The individual demand by consumers of group (1) is elastic.

$$e_{dX2} = \frac{dq_{dX2}}{dP_X}\frac{P_0}{q_{02}} = -1\,\frac{3}{4} = \mathbf{-0.75}$$ The individual demand by consumers of group (2) is inelastic.

- For the market:

$$e_{DX} = \frac{dQ_{DX}}{dP_X}\frac{P_0}{Q_0} = -20\,\frac{3}{50} = \mathbf{-1.2}$$ The market demand for good (X) is elastic.

d. Calculating the price elasticity of supply for each firm, and for the market:

- For each firm:

The quantity supplied by firm (A) is: $\mathbf{q_{SXA}} = 15 + 7(\mathbf{3})$ => $\mathbf{q_{0A} = 36}$ **units**

The quantity supplied by firm (B) is: $\mathbf{q_{SXB}} = 5 + 3(\mathbf{3})$ => $\mathbf{q_{0B} = 14}$ **units**

$$e_{SXA} = \frac{dq_{SXA}}{dP_X}\frac{P_0}{q_{0A}} = 7\,\frac{3}{36} = \mathbf{0.58}$$ The individual supply by firm (A) is inelastic.

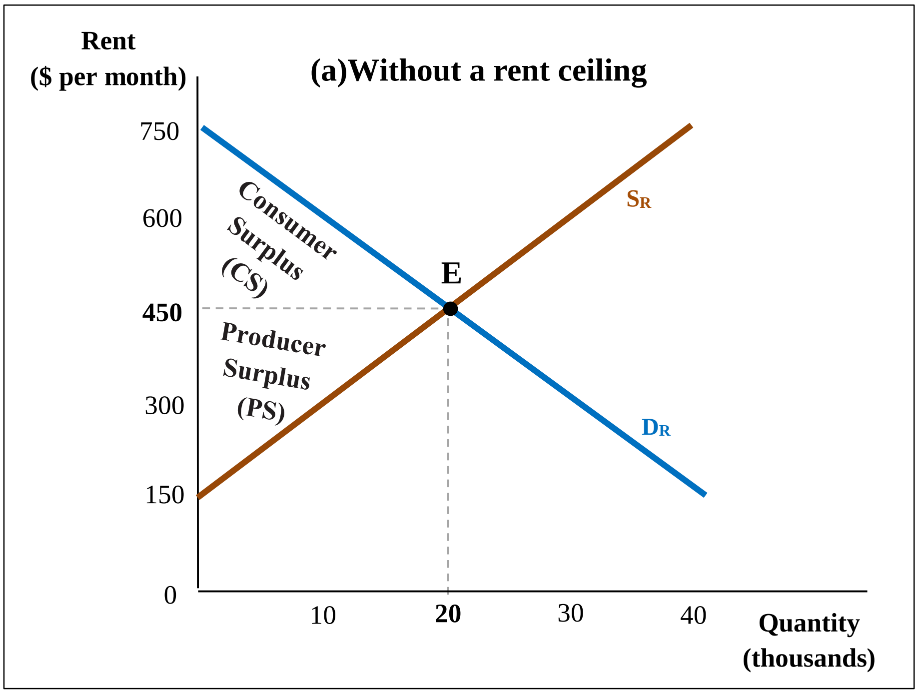
$$e_{SXB} = \frac{dq_{SXB}}{dP_X}\frac{P_0}{q_{0B}} = 3\,\frac{3}{14} = \mathbf{0.64}$$ The individual supply by firm (B) is inelastic.

- For the market:

$$e_{SX} = \frac{dQ_{SX}}{dP_X}\frac{P_0}{Q_0} = 10\,\frac{3}{50} = \mathbf{0.6}$$ The market supply for good (X) is inelastic.

Chapter 6

1. a. The market demand and supply curves, and the equilibrium point:



Without a rent ceiling, the market produces an efficient **20,000** units of housing at a rent of **$450** a month.

b. Calculating the consumer surplus (CS) and the producer surplus (PS) geometrically.

CS = the area of triangle below the demand curve and above the price

$$= \frac{\text{(the base x the height)}}{2}$$

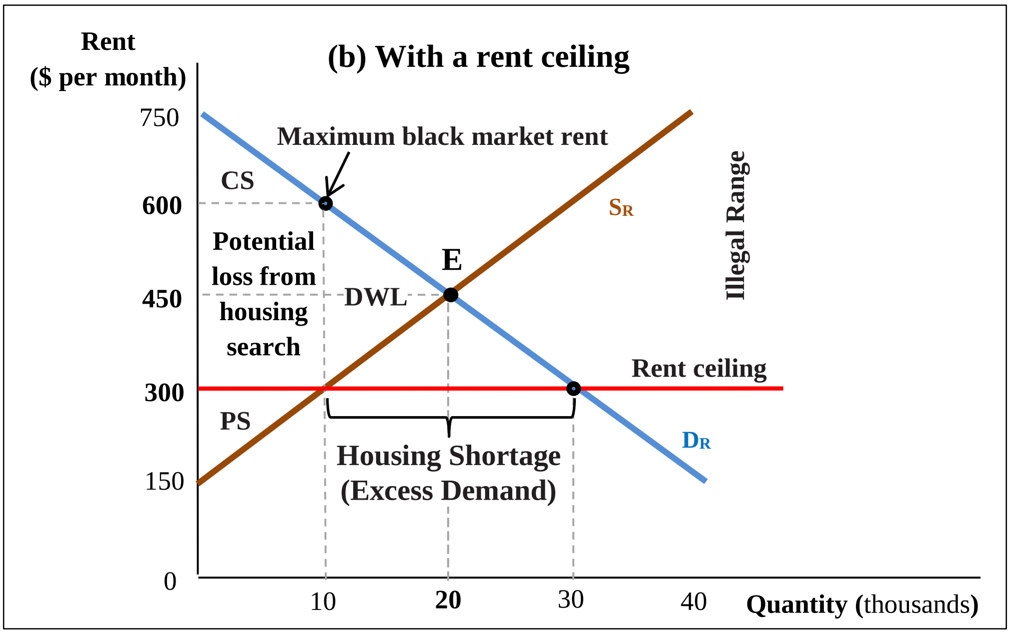
$$= \frac{[20{,}000 \times (750-450)]}{2} = \$3 \textbf{ million}$$

PS = the area of the triangle above the supply curve and below the price

$$= \frac{\text{(the base x the height)}}{2}$$

$$= \frac{[20{,}000 \times (450-150)]}{2} = \$3 \textbf{ million}$$

c. If a rent ceiling is set at **$300** a month.



At a rent of \$300 a month, the quantity of housing supplied is 10,000 units and the quantity demanded is 30,000 units. So, there is a **shortage** (excess demand) of: ΔQ_{DX} = 30,000 – 10,000 = **20,000 units** of housing.

d. At a rent of \$300 a month, the quantity of housing available is 10,000 units. Frustrated renters spend time searching for housing and they make deals with landlords in a black market. Someone is willing to pay **\$600** a month for the **10,000th** unit. CS and PS shrink and a deadweight (DWL) loss arises.

e. The unintended negative effects that can result in the market from setting the rent ceiling are:

- **A persistent housing shortage:** There will be a shortage of housing (20,000 units).
- **Increased search activity:** Frustrated renters spend more time and other resources in searching for the restricted quantity available.
- **Wasted resources:** There will be an opportunity cost which is equal to the rent (a regulated price) plus the time and other resources spent searching for scarce housing.
- **Inefficiently low quality:** In many cases tenants would be willing to pay much more for improved conditions of housing, but landlords have no incentive to provide such improvements because they cannot raise rents to cover their repair costs.
- **Inefficient allocation of scarce housing:** Allocating scarce housing based on some mechanisms that do not produce a fair outcome including: *A lottery*; allocates housing to those who are lucky. *First-come, first-served*; allocates housing to those who have the greatest foresight and who get their names on a list first. *Discrimination*; allocates housing based on the views and self-interest of the apartment's owner.
- **The emergence of black market:** A market in which landlords rent apartments illegally at a rent higher than the legal rent ceiling (at rent of **\$600** a month) and the tenants who would be willing to pay much more than the maximum legal rent will accept it.

2. a. Which of the two following functions represents the market demand or supply function of 'peanuts':

$$P = 1200 - 4Q \quad ; \quad P = 200 + Q$$

$$P = 1200 - 4Q \Rightarrow Q = 300 - \tfrac{1}{4}P \quad ; \quad \frac{dQ}{dP} = -\tfrac{1}{4}$$

The slope of the function is *negative*, so there is *an inverse relationship between quantity and price,* thus this function represents: **the market demand function**.

$$P = 200 + Q \Rightarrow Q = P - 200 \quad ; \quad \frac{dQ}{dP} = +1$$

The slope of the function is *positive*, so there is *a direct relationship between quantity and price,* thus this function represents: **the market supply function**.

b. Finding the market equilibrium price and quantity:

The condition of equilibrium: $P_D = P_S \Rightarrow 1200 - 4Q = 200 + Q \Rightarrow Q_0 = 200$ tons per day

Substituting Q_0 into the market demand function or the market supply function, we find:

$P_0 = 1200 - 4(200) = 200 + (200) \Rightarrow P_0 = \$400$ per ton.

c. Calculating the consumer surplus and the producer surplus mathematically:

- The consumer surplus:

$$CS = \int_0^{Q_0} (f(Q_{Dx})dQ) - P_0Q_0 \Rightarrow CS = \int_0^{200} [(1200 - 4Q_{DX})dQ] - P_0Q_0$$

$$CS = {}_0^{200}[(1200Q_{DX} - 2Q_{DX}^2)] - (400)(200)$$

$\boldsymbol{CS} = [(1200\,(200) - 2(200)^2)] - [(1200\,(0) - 2(0)^2)] - (400)(200)$

CS = $80,000

- The producer surplus:

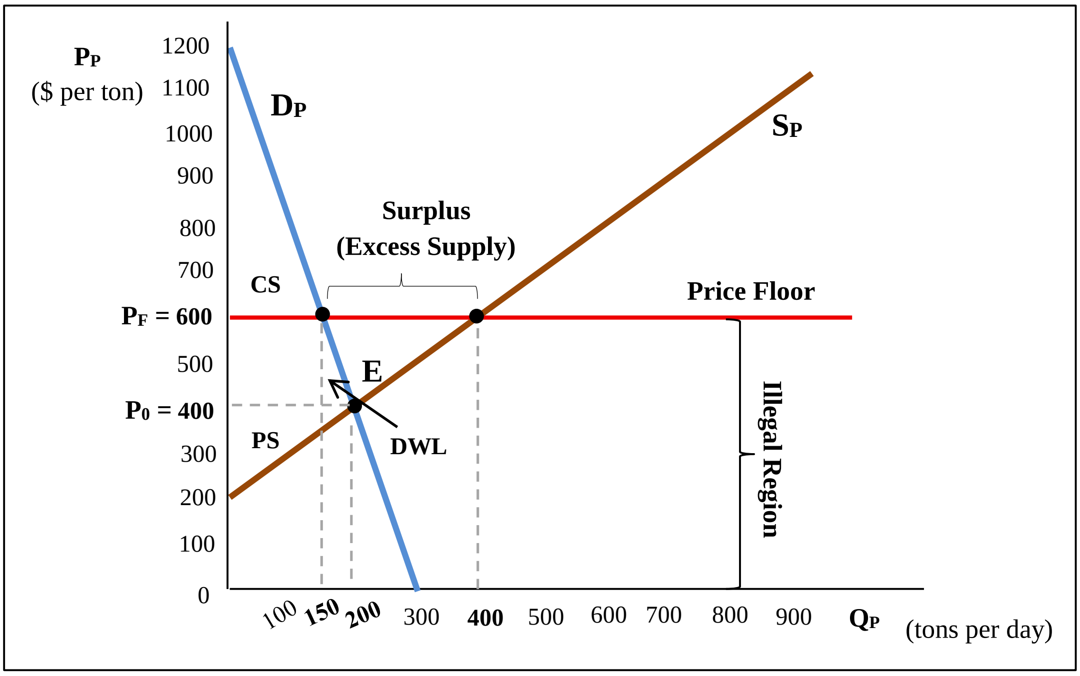
$PS = P_0Q_0 - \int_0^{Q_0}(f(Q_{SX})dQ) \Rightarrow \boldsymbol{PS} = P_0Q_0 - \int_0^{200}(200 + Q_{SX})dQ$

$PS = P_0Q_0 - [(200Q + \frac{1}{2}Q_{SX}^2)]_0^{200}$

$PS = (400)(200) - [(200\,(200) + \frac{1}{2}\,(200)^2)] - [(200\,(0) - \frac{1}{2}\,(0)^2)]$

PC = $20,000

d. Imposing a price floor of **$600** per ton by the government results in a **surplus (excess supply)** of: $\boldsymbol{\Delta Q_{SX}}$ = 400 - 150 = **250 tons per day**. In order to maintain the price floor and prevent the excess supply from driving down the market price. The government must *buy up* the unwanted surplus of peanuts at the floor price and it cannot resell it domestically. Instead, it can be stored, destroyed, given to the poor, or shipped abroad.



e. The unintended negative effects that can result in the market from imposing the price floor are:

- **A persistent surplus of the good:** There will be a surplus of peanuts of (250 tons per day) which is in an inefficient overproduction of the good.
- **Rationing mechanisms:** The sellers who appeal to the personal biases of the buyers, perhaps due to racial or familial ties, may be better able to sell their goods than those who do not.
- **Wasted Resources:** When the government buys up the unwanted surplus of peanuts. This surplus may be destroyed, which is a pure waste.
- **Inefficient allocation of sales among sellers:** Those who would be willing to sell the good at the lowest price are not always those who actually manage to sell it.
- **Inefficiently high quality**: Sellers may offer high-quality of peanuts at a high price, even though buyers would prefer a lower quality at a lower price.

3. a. The market equilibrium price and quantity of 'cigarettes' in New York City:

$Q_{DC} = 425 - 25P_C \quad ; \quad Q_{SC} = -70 + 140P_c$

$Q_{DC} = Q_{SC} \Rightarrow 425 - 25P_C = -70 + 140P_C \Rightarrow P_0 = \$3$ a pack

Substituting P_0 into Q_{DC} or Q_{SC} we find:

$Q_0 = 425 - 25(3) = -70 + 140(3) \Rightarrow Q_0 = 350$ millions of packs per year

b. The government imposes a specific tax of $t = \$1.50$ per pack on *sellers* of cigarettes.

Calculating the new equilibrium price and quantity, the buyers' price P_b, the sellers' price P_S, the burden of the tax that falls on buyers t_b, the burden of the tax that falls on sellers t_S, and the total revenue earned by the government:

The tax is imposed on sellers of cigarettes, so the supply curve S_C shifts leftward by $t = \$1.50$ to the new curve S_C'. The new supply function after imposing the tax becomes:

$Q_{SC}' = -70 + 140(P_C - t) \Rightarrow Q_{SC}' = -70 + 140(P_C - 1.50) \Rightarrow Q_{SC}' = -70 + 140P_C - 210 \Rightarrow Q_{SC}' = -280 + 140P_C$

- The new equilibrium price and quantity P_1 and Q_1:

  $Q_{DC} = Q_{SC}' \Rightarrow 425 - 25P_C = -280 + 140P_C \Rightarrow P_1 = \$4.27$ a pack

Substituting P_1 into either Q_{DC} or Q_{SC}' we find:

$Q_1 = 425 - 25(4.27) = -280 + 140(4.27) \Rightarrow Q_1 = 318.18$ million of packs per year

- The buyers' price P_b: $P_b = P_1 = \$4.27$

Or, we substitute Q_1 into the demand function: $Q_1 = 425 - 25P_b \Rightarrow 318.18 = 425 - 25P_b \Rightarrow P_b = \$4.27$

- The sellers' price P_S: $P_b - P_S = t \Rightarrow P_S = P_b - t \Rightarrow P_S = 4.27 - 1.50 \Rightarrow P_S = \$2.77$

Or, we substitute Q_1 into Q_{SC}: $Q_1 = -70 + 140P_s \Rightarrow 318.18 = -70 + 140P_s \Rightarrow P_S = \$2.77$

- The burden of the tax that falls on buyers t_b:

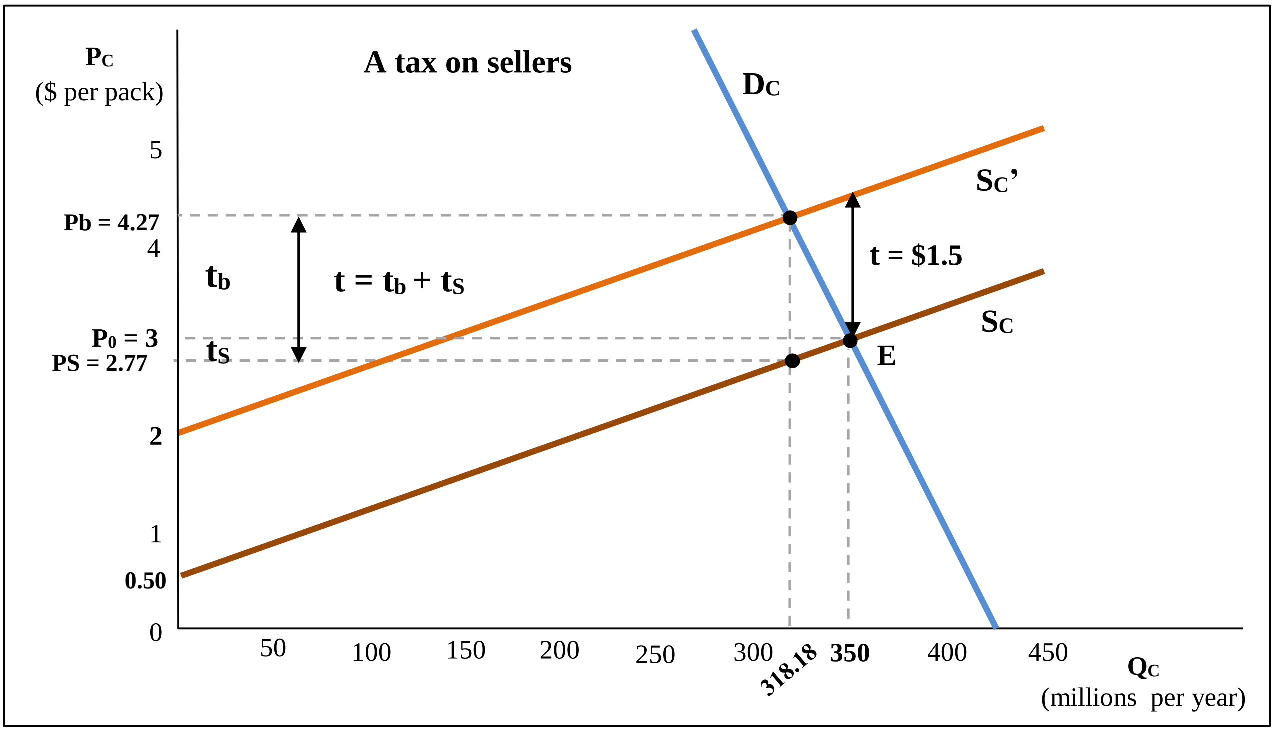
$t_b = P_b - P_0 \Rightarrow t_b = 4.27 - 3 \Rightarrow t_b = \$1.27$

- The burden of the tax that falls on sellers t_S:

$t_S = P_0 - P_S \Rightarrow t_S = 3 - 2.77 \Rightarrow t_S = \$0.23$

- The total revenue earned by the government TR:

$TR = Q_1 t \Rightarrow TR = 318.18 \times 1.50 \Rightarrow TR = \$477.27$ **millions**



c. The best amount of the tax that maximizes the government's total revenue:

When a tax ***t*** is imposed, the supply function becomes:

$Q_{SC}' = -70 + 140(P_C - t) \Rightarrow Q_{SC}' = -70 - 140t + 140P_C$

- The new equilibrium price and quantity P_1 and Q_1:

$Q_{DC} = Q_{SC}' \Rightarrow 425 - 25P_C = -70 - 140t + 140P_C \Rightarrow P_1 = 3 + 0.85t$

$Q_1 = 425 - 25(3 + 0.85t) \Rightarrow Q_1 = 350 - 21.25t$

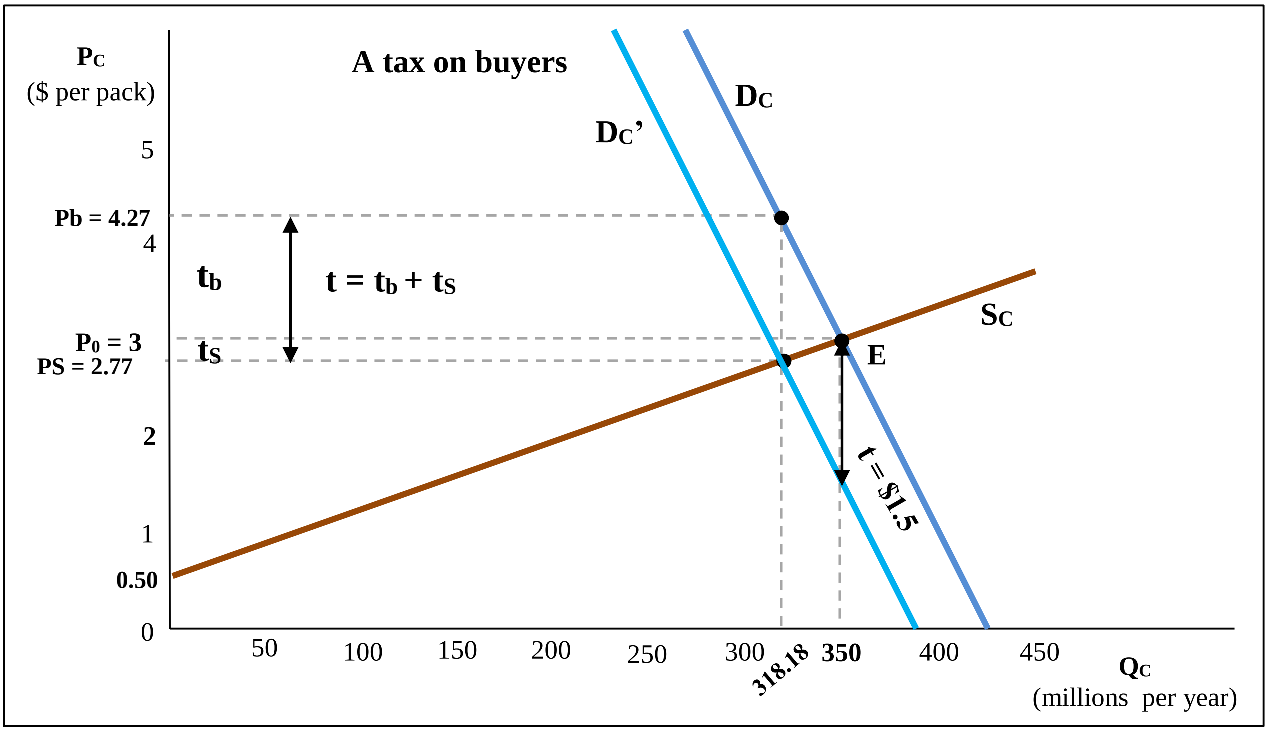
Total revenue : $TR = Q_1 t \Rightarrow TR = (350 - 21.25t) \times t \Rightarrow TR = 350t - 21.25t^2$

Total revenue is at a maximum when:

$$\frac{dTR}{dt} = 0 \Rightarrow 350 - 2(21.25)t = 0 \Rightarrow t = \$8.23$$

TR = $350t - 21.25t^2 \Rightarrow TR = 350(\mathbf{8.23}) - 21.25(\mathbf{8.23})^2 \Rightarrow$ **TR = $1441.17 million = $1.44117 billion**

d. A tax (with the same amount, ***t* = $1.50** a pack) on *buyers* has the same effects as the tax on sellers. In both cases, the equilibrium quantity decreases to 325 million packs a year, the price paid by buyers (P_b) rises to $4.27 a pack, and the price received by sellers (P_S) falls to $2.77 a pack. The burden of the tax that falls on buyers (t_b) is $1.27, and that falls on sellers (t_S) is $0.23. The only difference is that the market demand curve shifts leftward by the amount of the tax.



4. a. The market equilibrium price and quantity of 'wheat':

$Q_{DW} = 3100 - 125P_W \quad ; \quad Q_{SW} = 1540 + 115P_W$

$Q_{DW} = Q_{SW} \Rightarrow 3100 - 125P_W = 1540 + 115P_W \Rightarrow 240P_W = 1540 \Rightarrow P_0 = \$6.5$ per bushel

Substituting P_0 into Q_{DW} or Q_{SW} we find:

$Q_0 = 3100 - 125(6.5) = 1540 + 115(6.5) \Rightarrow Q_0 = 2287.5$ million bushels

b. Calculating the price elasticity of demand and the price elasticity of supply at the equilibrium point:

$$e_{DW} = \frac{dQ_{DW}}{dP_W}\,\frac{P_0}{Q_0} \Rightarrow e_{DW} = (-125)\,\frac{6.5}{2287.5} \Rightarrow e_{DW} = -0.35$$ **0 < |e_{DW}| < 1** thus demand is inelastic.

We can likewise calculate the price elasticity of supply: 35799.375

$e_{SW} = \frac{dQ_{SW}}{dp_W}\frac{P_0}{Q_0}$ => $e_{SW} = (115)\frac{6.5}{2287.5}$ => $e_{SW} = 0.33$ **0 < e$_{SW}$ < 1** thus supply is inelastic.

c. Calculating the consumer surplus and producer surplus:

- The consumer surplus: $Q_{DW} = 3100 - 125P_W$ => $P_W = 24.8 - \frac{1}{125}Q_{DW}$

$CS = \int_0^{Q_0}(f(Q_{DW})dQ) - P_0Q_0$ => $CS = \int_0^{2287.5}(24.8 - \frac{1}{125}Q_{DW})dQ - P_0Q_0$

$CS = {}_0^{2287.5}[(24.8Q_{DW} - \frac{1}{250}Q_{DW}^2)] - (6.5)(2287.5)$

$CS = [(24.8(2287.5) - \frac{1}{250}(2287.5)^2)] - [(24.8(0) - \frac{1}{250}(0)^2)] - (6.5)(2287.5)$

CS = \$20,930.625 million = **\$20.930 billion**

- The producer surplus: $Q_{SW} = 1540 + 115P_W$ => $P_W = Q_{SW} - 13.39$

$PS = P_0Q_0 - \int_0^{Q_0}(f(Q_{SW})dQ)$ => $PS = P_0Q_0 - \int_0^{2287.5}(\frac{1}{115}Q_{SW} - 13.39)dQ$

$PS = (6.5)(2287.5) - {}_0^{2287.5}[(\frac{1}{230}Q_{SW}^2 - 13.39\,Q_{SW})]$

$PS = (6.5)(2287.5) - [(\frac{1}{230}(2287.5)^2 - 13.39\,(2287.5)] - [(\frac{1}{230}(0)^2 - 13.39\,(0)]$

PS = 22,747.695 million = **\$22.75 billion**

d. The government grants wheat producers a subsidy ***s* = 20%** of the equilibrium price.

- Who will benefit more from the subsidy?

We found that: $e_{DW} = 0.35 > e_{SW} = 0.33$, thus sellers will benefit more than buyers from the subsidy.

- The new equilibrium price and quantity:

The amount of the subsidy is: **S** = 20%(6.5) => **S = \$1.30**

After granting the subsidy, the supply curve shifts rightward by ***s* = \$1.30** to the new curve labeled S_W'.

The new supply function becomes:

$Q_{SW}' = 1540 + 115(P_W + S)$ => $Q_{SW}' = 1540 + 115(P_W + 1.30)$ => $Q_{SW}' = 1540 + 115P_W + 149.5$

=> $Q_{SW}' = 1689.5 + 115P_W$

$Q_{DW} = Q_{SW}'$ => $3100 - 125P_W = 1689.5 + 115P_W$ => $P_1 = \$5.88$

Substituting P_1 into either Q_{DW} or Q_{SW}' we find:

$Q_1 = 3100 - 125(5.88) = 1689.5 + 115(5.88)$ => $Q_1 = 2365.3$ **million bushels**

- The buyers' price P_b and the sellers' price P_S:

  $P_b = P_1$ = \$5.88

Or, we substitute Q_1 into Q_{DW}: $Q_1 = 3100 - 125P_b$ => $2365.3 = 3100 - 125P_b$ => P_b = **\$5.88**

  $P_S - P_b = S$ => $P_S = P_b + S$ => $P_S = 5.88 + 1.30$ => P_S = **\$7.18**

Or, we substitute Q_1 into Q_{SW}: $Q_1 = 1540 + 115P_S$ => $2365.3 = 1540 + 115P_S$ => P_S = **\$7.18**

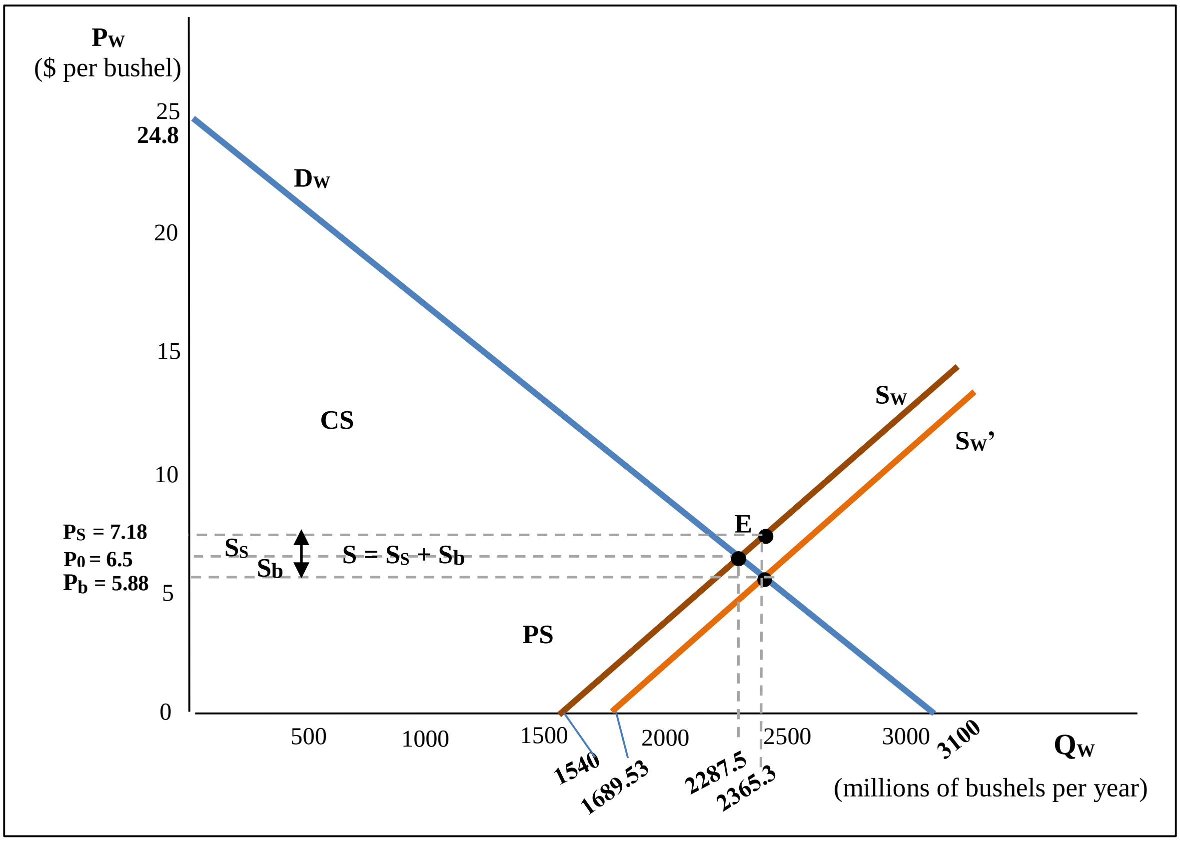
- the part of the subsidy that sellers benefit from S_S and the part of the subsidy that buyers benefit from S_b:

  $S_S = P_S - P_0$ => $S_S = 7.18 - 6.5$ => S_S = **\$0.68**

  $S_b = P_0 - P_b$ => $S_b = 6.5 - 5.88$ => S_b = **\$0.62**

- The government's total expenditure **TE**:

  TE = Q_1S => **TE** = 2365.3x1.30 => **TR = \$3074.89 million = \$3.075 billion**



5. a. The market equilibrium price and quantity of 'water':

Q_{DW} = 200 – 2P_W, ; **Q_{SW} = 100**

Q_{DW} = Q_{SW} => 200 – 2P_W = 100 => **P_0 = 50¢ = $0.5**

Q_0 = 100 = 200 – 2(**50**) => **Q_0 = 100,000** bottles per week

b. Calculating the consumer surplus and the producer surplus:

CS = the area of triangle below the demand curve and above the price

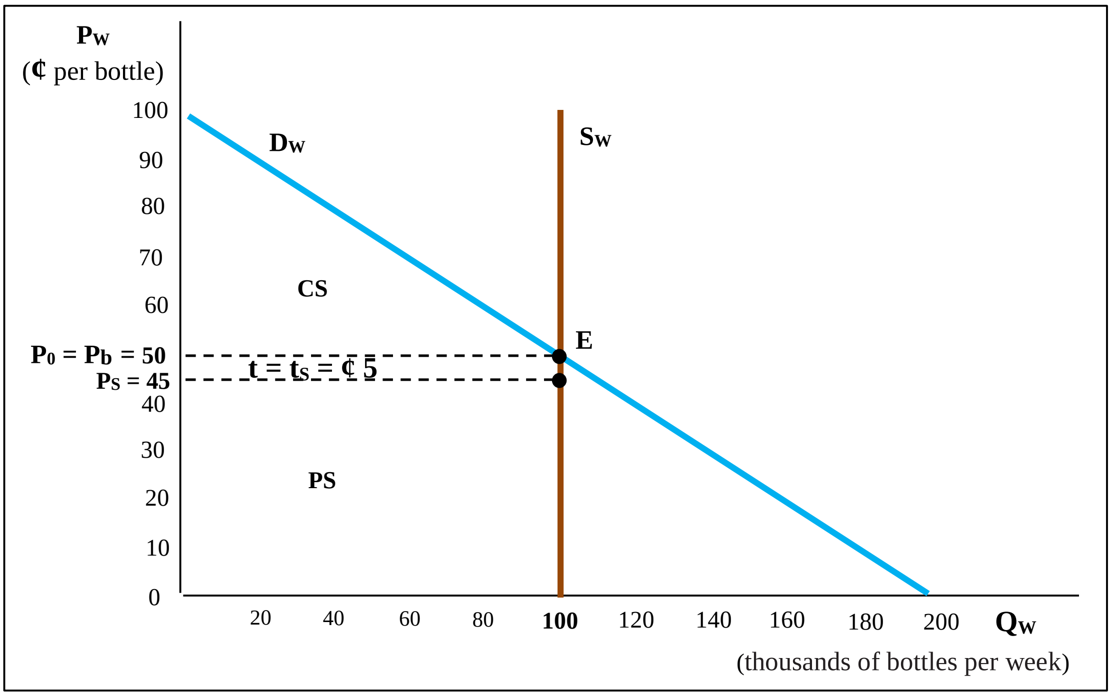
= $\frac{\text{(the base x the height)}}{2}$

= $\frac{[100 \text{ x } (100-50)]}{2}$ = **2500,000¢ = $25,000**

PS = the area of the rectangle above the supply curve and below the price

= (the length x the width)

= (100 x 50) = **5000,000¢ = $50,000**



c. Finding the new equilibrium price and quantity, the buyers' price P_b, and the sellers' price P_S after imposing a specific tax of **5¢** a bottle:

- The new equilibrium price and quantity:

The supply curve does not change because the spring owners still produce 100,000 bottles a week.

$Q_{Dw} = 200 - 2P_1$

$Q_{Sw} = 100$

$100 = 200 - 2P_1 \Rightarrow \mathbf{P_1 = 50¢ = P_0}$

$\mathbf{Q_1 = Q_0 = 100{,}000}$ bottles per week

The buyers are willing to buy the 100,000 bottles only if the price is 50¢ a bottle, so there is no change in equilibrium price and quantity.

- The buyers' price P_b: $\mathbf{P_b = P_1 = 50¢}$
- The sellers' price P_s: $\mathbf{P_b - P_S = t} \Rightarrow \mathbf{P_S} = P_b - t \Rightarrow \mathbf{P_S} = 50 - 5 \Rightarrow \mathbf{P_S = 45¢}$

d. Sellers bear the entire burden of the tax, **t** = **5¢** and buyers bear nothing, because *supply is perfectly inelastic* $\mathbf{e_{SW} = 0}$.

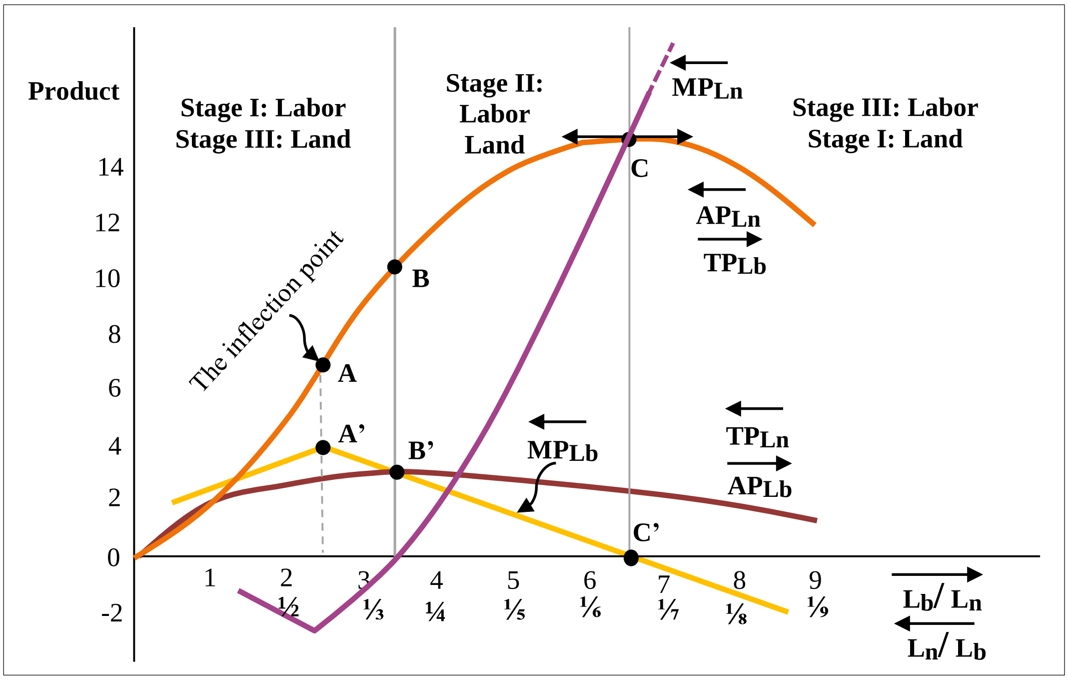
Chapter 7

1. a. Finding $\mathbf{AP_{Lb}}$ and $\mathbf{MP_{Lb}}$:

$$\mathbf{AP_{Lb} = \frac{TP_{Lb}}{L_b}}\ ; \quad AP_{Lb1} = \frac{TP_{L1}}{L_1} = \frac{0}{0} = /\ ;\ AP_{Lb2} = \frac{TP_{L2}}{L_2} = \frac{2}{1} = \mathbf{2}\ ;\ AP_{Lb3} = \frac{TP_{L3}}{L_3} = \frac{5}{2} = \mathbf{2.5} \dots$$

$$\mathbf{MP_{Lb} = \frac{\Delta TP_L}{\Delta L_b}}\ ; \quad MP_{Lb1} = \frac{\Delta TP_L}{\Delta L} = \frac{2-0}{1-0} = \mathbf{2}\ ;\ MP_{Lb2} = \frac{\Delta TP_L}{\Delta L} = \frac{5-2}{2-1} = \mathbf{3} \dots$$

Land (L_n)	1	1	1	1	1	1	1	1	1	1
Labor (L_b)	0	1	2	3	4	5	6	7	8	9
Total Product (TP_{Lb})	0	2	5	9	12	14	15	15	14	12
AP_{Lb}	—	**2**	**2.5**	**3**	**3**	**2.8**	**2.5**	**2.14**	**1.75**	**1.33**
MP_{Lb}	—	**2**	**3**	**4**	**3**	**2**	**1**	**0**	**-1**	**- 2**



b. The law of diminishing marginal returns (DMR) states that: "As more units of labor per unit of time are used to cultivate a fixed amount of land, after a point the MP_{Lb} will necessarily decline". Note that to observe the law of DMR, one input (either land or labor) must be kept fixed while the other input is varied. Technology is also assumed to remain constant. This means that this law is a short-run concept. The law of DMR begins to operate where the MP_{Lb} starts declining (at point A').

c. Finding $\mathbf{TP_{Ln}}$, $\mathbf{AP_{Ln}}$ and $\mathbf{MP_{Ln}}$, and Plotting them on the same set of axes:

We can obtain the values of $\mathbf{TP_{Ln}}$, through using the information of the table given above, by keeping labor fixed at **1** unit per time period and using alternative quantities of land, ranging from ⅑ to **1** acre.

Starting from the end of the table, we see that **9** units of labor on **1** unit of land produces **12** units of output; therefore, using ⅑ of the quantity of labor and land should result in ⅑ of **12** units of output, because of constant returns to scale. Thus, **1** unit of labor used on ⅑ unit of land produces ⅑ of **12** or $\frac{12}{9} = 1.33$ units of output. The other figures of $\mathbf{TP_{Ln}}$ are obtained by following the same procedure.

Labor (L_b)	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
Land (L_n)	**1**	**½**	**⅓**	**¼**	**⅕**	**⅙**	**⅐**	**⅛**	**⅑**
TP_{Ln}	**2**	**2.5**	**3**	**3**	**2.8**	**2.5**	**2.14**	**1.75**	**1.33**
AP_{Ln}	**2**	**5**	**9**	**12**	**14**	**15**	**15**	**14**	**12**
MP_{Ln}	**-**	**- 1**	**- 3**	**0**	**4**	**9**	**15**	**22**	**30**

We find the values of $\mathbf{AP_{Ln}}$ and $\mathbf{MP_{Ln}}$ by applying their formulas: $AP_{Ln} = \frac{TP_{Ln}}{L_n}$ / $MP_{Ln} = \frac{\Delta TP_{Ln}}{\Delta L_n}$

We can observe that the $\mathbf{TP_{Ln}}$ is identical with the $\mathbf{AP_{Lb}}$ and that the $\mathbf{AP_{Ln}}$ is identical with the $\mathbf{TP_{Lb}}$

- Defining the three stages of production for labor and land:

▪ **Stage I**: starts from the origin to the point where the AP is maximum (or AP intersects MP) at point B'.

▪ **Stage II**: starts from the end of the first stage to the point where the MP = 0 at point C' (or TP is maximum at point C).

▪ **Stage III**: covers the range over which the MP is negative (or when TP falls).

We see that the $\mathbf{TP_{Ln}}$ coincides precisely with the $\mathbf{AP_{Lb}}$ and the $\mathbf{AP_{Ln}}$ coincides with the $\mathbf{TP_{Lb}}$. Because of this, stage I for labor corresponds to stage III for land, stage II for labor covers the same range as stage II for land, and stage III for labor corresponds to stage I for land. Thus, there is *perfect symmetry* between the stages of production for labor (moving from left to right) and those for land (moving from right to left) under constant returns to scale.

- The producer will not operate in stage I for labor (= stage III for land), because the $\mathbf{MP_{Ln}}$ is negative. The producer will not operate in stage III for labor (= stage I for land), because the $\mathbf{AP_{Lb}}$ is negative. The producer will operates in stage II, which is the only optimal (efficient) stage of production, because the $\mathbf{MP_{Lb}}$ and $\mathbf{MP_{Ln}}$ are both positive and declining.
- If $\mathbf{P_{Lb} = 0}$, the producer will produce at the end of stage II for labor (where the $\mathbf{MP_{Lb}} = 0$ and the $\mathbf{AP_{Ln}}$ is maximum). If $\mathbf{P_{Ln} = 0}$, the producer will produce will produce at the beginning of stage II for labor (where the $\mathbf{AP_{Lb}}$ is maximum and the $\mathbf{MP_{Ln}} = 0$). If $\mathbf{P_{Lb} = P_{Ln}}$, the producer will produce at the point where $\mathbf{MP_{Lb}} = \mathbf{MP_{Ln}}$ (at the intersection point of the two curves). The higher the price of labor in relation to the price of land, the closer to the beginning of stage II for labor will the producer operate. The higher the price of land in relation to the price of labor, the closer to the beginning of stage II for land (which is the end of stage II for labor) will the producer operate.

2. a. Finding $\mathbf{AP_L}$ and $\mathbf{MP_L}$:

$\mathbf{q = f(L,K) = 600(LK)^2 - (LK)^3}$; $\mathbf{\overline{K} = 10}$

$\mathbf{\overline{K} = 10} \Rightarrow \mathbf{q} = 600(\mathbf{10}L)^2 - (\mathbf{10}L)^3 \Rightarrow \mathbf{q = 60{,}000L^2 - 1000L^3}$

$$\mathbf{AP_L} = \frac{q}{L} = \frac{60{,}000L^2 - 1000L^3}{L} = \mathbf{60{,}000L - 1000L^2}$$

$$\mathbf{MP_L} = \frac{dq}{dL} = \mathbf{120{,}000L - 3000L^2}$$

b. The amount of labor **L** at which the total output **q** reaches its maximum value:

$$\mathbf{q_{Max}} = \frac{dq}{dL} = \mathbf{0} \Rightarrow \mathbf{MP_L = 0} \Rightarrow 120{,}000L - 3000L^2 = 0 \Rightarrow \mathbf{L = 40} \text{ workers.}$$

c. The amount of **L** at the point when $\mathbf{AP_L}$ equals $\mathbf{MP_L}$, and the importance of this point in analyzing the producer's behavior in the short-run:

$\mathbf{AP_L} = \mathbf{MP_L} \Rightarrow 60{,}000L - 1000L^2 = 120{,}000L - 3000L^2 \Rightarrow 60{,}000L - 2000L^2 = 0 \Rightarrow \mathbf{L = 30}$ workers.

- The importance of this point is to determine the beginning of the stage II of production, which is the optimal stage for the producer in the short-run.

3. a. Proving that the $\mathbf{MP_L}$ is declining and positive:

$\boldsymbol{q = f(L,K) = L^{3/4}K^{1/4}}$; $\boldsymbol{\overline{K} = 16}$

$\boldsymbol{\overline{K} = 16} \Rightarrow q = L^{3/4}\boldsymbol{(16)^{1/4}} \Rightarrow \boldsymbol{q = 2L^{3/4}}$

$$\mathbf{MP_L} \text{ is declining and positive: } \begin{cases} \dfrac{dMP_L}{dL} < 0 \\ \dfrac{d^2MP_L}{dL^2} > 0 \end{cases}$$

$$MP_L = \frac{dq}{dL} \Rightarrow \mathbf{MP_L} = \frac{3(2)L^{-1/4}}{4} \Rightarrow \mathbf{MP_L} = \frac{3L^{-1/4}}{2}$$

$$\frac{dMP_L}{dL} = \frac{-1}{4}\frac{3L^{\frac{-5}{4}}}{2} = \frac{-3L^{\frac{-5}{4}}}{8} < 0 \qquad \textbf{MP}_\textbf{L} \text{ is declining and positive}$$

$$\frac{d^2MP_L}{dL^2} = \frac{-3}{8}\frac{-5L^{\frac{-9}{4}}}{4} = \frac{15L^{\frac{-9}{4}}}{32} > 0$$

b. The producer operates in **stage II**, because **MP_L** is *declining* and *positive*. This stage is the only efficient stage of production, where the law of DMR is operating.

c. Finding the average and marginal products for both labor and capital:

In the long-run, the firm can vary both L and K.

$$AP_L = \frac{q}{L} = \frac{L^{3/4}K^{1/4}}{L} = \frac{K^{1/4}}{L^{1/4}} = \left(\frac{K}{L}\right)^{1/4}$$

$$AP_K = \frac{q}{K} = \frac{L^{3/4}K^{1/4}}{K} = \frac{L^{3/4}}{K^{3/4}} = \left(\frac{L}{K}\right)^{3/4}$$

$$MP_L = \frac{dq}{dL} = \frac{3}{4}L^{-1/4}K^{1/4} = \frac{3}{4}\left(\frac{K}{L}\right)^{1/4}$$

$$MP_K = \frac{dq}{dK} = \frac{1}{4}L^{3/4}K^{-3/4} = \frac{1}{4}\left(\frac{L}{K}\right)^{3/4}$$

d. Calculating the amount of L and K that the firm should use in order to maximize its total output:

P_L = \$15, **P_K = \$5**, **TC = \$500**

Let's use the condition of tangency (we can also use the Lagrange multiplier method):

$$\begin{cases} \dfrac{MP_L}{MP_K} = \dfrac{P_L}{P_K} \quad \ldots(1) \\ \text{s. t. } TC = P_L L + P_K K \ \ldots(2) \end{cases}$$

$$(1) \Leftrightarrow \frac{\frac{3}{4}X^{-1/4}Y^{1/4}}{\frac{1}{4}X^{3/4}Y^{-3/4}} = \frac{P_L}{P_K} \Leftrightarrow \frac{3K}{L} = \frac{15}{5} \Leftrightarrow \frac{3K}{L} = 3 \Leftrightarrow \mathbf{L = K \ \ldots(3)}$$

Substituting (**3**) into (**2**), we find:

TC = P_LL + P_KK => **500** = **15**L + **5**K **=>** 500 = 15**K** + 5K => 500 = 20K **=> K_0** = **25** units.

L_0 = K_0 = 25 units.

e. Calculating the Marginal Rate of Technical Substitution (**$MRTS_{L,K}$**) at the equilibrium point:

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{3K}{L} = \frac{3(\mathbf{20})}{\mathbf{20}} = 3$$

Explanation: The firm can give up **3 units** of *capital* by using **one extra unit** of *labor* and still produce the same level of output (remaining on the same isoquant).

f. The kind of returns to scale that the firm's production functions exhibits:

The firm's production function is a Cobb-Douglas production function. Thus, it is homogeneous of degree $n = \alpha + \beta$.

$n = \alpha + \beta$ => **$n = \frac{3}{4} + \frac{1}{4} = 1$**, the production function is homogeneous of degree **one**. So, this function exhibits **constant returns to scale** (when inputs L and K increase by a given proportion, the output q increases with the same proportion.)

4. a. Is the combination (**L = 25, K = 16**) the manufacturer's optimal one for a given **q = 200** units:

$q = f(L,K) = 10L^{0.5}K^{0.5}$; $W = P_L = \$1$; $r = P_K = \$4$.

The two conditions of equilibrium must be satisfied:

$$\begin{cases} \dfrac{MP_L}{MP_K} = \dfrac{W}{r} \quad ...(1) \\ \text{s. t. } TC = WL + rK ...(2) \end{cases}$$

$$(1) \Leftrightarrow \frac{10(0.5)L^{-0.5}K^{0.5}}{10(0.5)L^{0.5}K^{-0.5}} = \frac{W}{r} \Leftrightarrow \frac{K}{L} = \frac{W}{r} \Leftrightarrow \frac{16}{25} \neq \frac{1}{4}$$

Since the first condition is not satisfied, thus (**L = 25, K = 16**) is not the the manufacturer's optimal combination.

b. The prices of labor and capital (**w** and **r**) paid by the manufacturer:

(**L = 25, K = 16**) is the optimal combination ,**TC = $200**.

$$\begin{cases} \dfrac{MP_L}{MP_K} = \dfrac{W}{r} \quad ...(1) \\ \text{s. t. } TC = WL + rK ...(2) \end{cases} \Leftrightarrow \begin{cases} \dfrac{K}{L} = \dfrac{W}{r} \quad ...(1) \\ TC = WL + rK ...(2) \end{cases} \Leftrightarrow \begin{cases} \dfrac{16}{25} = \dfrac{W}{r} \quad ....(1) \\ 200 = 25W + 16r ...(2) \end{cases}$$

$$\Leftrightarrow \begin{cases} W = 0.64r \\ 200 = 25(0.64r) + 16r \Rightarrow 200 = 32r \Rightarrow r = \$6.25 \end{cases}$$

$W = 0.64r \Rightarrow W = 0.64(6.25) \Rightarrow W = \$4$

c. The minimum total outlay to produce $q_0 = 300$ units of output:

$W = \$1$; $r = \$4$

Using Lagrange multiplier: $\mathcal{L} = WL + rK + \lambda(q_0 - f(L,K)) \Rightarrow \mathcal{L} = L + 4K + \lambda(300 - 10L^{0.5}K^{0.5})$

We take the partial derivatives with respect to L, K, and λ, and then equating them to zero.

$$\frac{d\mathcal{L}}{dL} = 0 \Rightarrow 1 - 10(0.5)\lambda L^{-0.5}K^{0.5} = 0 \Rightarrow 1 = 5\lambda L^{-0.5}K^{0.5} \quad .....(1)$$

$$\frac{d\mathcal{L}}{dK} = 0 \Rightarrow 4 - 10(0.5)\lambda L^{0.5}K^{-0.5} = 0 \Rightarrow 4 = 5\lambda L^{0.5}K^{-0.5} \quad .....(2)$$

$$\frac{d\mathcal{L}}{d\lambda} = 0 \Rightarrow 300 - 10L^{0.5}K^{0.5} = 0 \Rightarrow 300 = 10L^{0.5}K^{0.5} \quad ....(3)$$

Dividing (**1**) over (**2**), we find:

$$\frac{1}{4} = \frac{5\lambda L^{-0.5}K^{0.5}}{5\lambda L^{0.5}K^{-0.5}} \Rightarrow \frac{1}{4} = \frac{K}{L} \Rightarrow L = 4K \quad ...(4)$$

Substituting (**4**) into (**3**), we find:

$300 = 10L^{0.5}K^{0.5} \Rightarrow 300 = 10(4K)^{0.5}K^{0.5} \Rightarrow 300 = 20K \Rightarrow K_0 = 15$ **units**

Substituting K_0 into (**4**), we find: $L_0 = 4(15) \Rightarrow L_0 = 60$ **units**

- The minimum outlay to produce $q_o = 300$ units is: $TC = L + 4K \Rightarrow TC = 60 + 4(15) \Rightarrow TC = \$120$

5. a. Showing that **α** and **β** is the elasticities of output with respect to **L** and **K** respectively:

$q = f(L,K) = AL^{\alpha}K^{\beta}$

$e_{qL} = \frac{dq}{dL}\frac{L}{q} \Rightarrow e_{qL} = A\alpha L^{\alpha-1}K^{\beta}\frac{L}{AL^{\alpha}K^{\beta}} \Rightarrow e_{qL} = \alpha$

$e_{qK} = \frac{dq}{dK}\frac{K}{q} \Rightarrow e_{qK} = A\beta L^{\alpha}K^{\beta-1}\frac{K}{AL^{\alpha}K^{\beta}} \Rightarrow e_{qK} = \beta$

b. If **L** and **K** are doubled and **α + β = 1**, by how much will the total output **q** double?

$q' = f(2L,2K) = A(2L)^{\alpha}(2K)^{\beta} = A2^{\alpha}L^{\alpha}2^{\beta}K^{\beta} = 2^{\alpha+\beta}A\,L^{\alpha}K^{\beta} = 2^{\alpha+\beta}q = 2q$

If both **L** and **K** *double*, the total output **q** *doubles* as well.

c. Calculating **α** and **β**, knowing that $e_{q,L} = 0.3$ and that the function is homogeneous of degree one:

$e_{qL} = \alpha = 0.3$

The function is homogeneous of degree one: n= **α + β = 1**

α + β = 1 => **0.3** + β = 1 => **β = 0.7**

$q = f(L,K) = AL^{0.3}K^{0.7}$

d. Showing that the production function satisfies Euler's theorem and implies that: $q = MP_L L + MP_K K$

- The production function satisfies Euler's theorem if: $f_L'L + f_K'K = nq = (\alpha + \beta)q$

$f_L'L + f_K'K = (0.3AL^{-0.7}K^{0.7})L + (0.7AL^{0.3}K^{-0.3})K = (0.3AL^{0.3}K^{0.7} + 0.7AL^{0.3}K^{0.7} = 0.3q + 0.7q$

$= (0.3 + 0.7)q = (\alpha + \beta)q$

The function satisfies Euler theorem: $f_L'L + f_K'K = (0.3 + 0.7)q$

$f_L' = \frac{dq}{dL} = MP_L$; $f_K' = \frac{dq}{dK} = MP_K$

$(0.3+ 0.7)q = f_L'L + f_K'K \Rightarrow q = MP_L L + MP_K K$

6. a. Finding the marginal product for both labor and capital:

$q = f(L,K) = L + K + 2\sqrt{LK} = L + K + 2L^{½}K^{½}$

$MP_L = \frac{dq}{dL} = 1 + L^{-½}K^{½}$

$MP_K = \frac{dq}{dK} = 1 + L^{½}K^{-½}$

b. Calculating the Marginal Rate of Technical Substitution ($MRTS_{L,K}$):

$K/L = ¼$

$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{1 + L^{-½}K^{½}}{1 + L^{½}K^{-½}} = \frac{1 + (K/L)^{½}}{1 + (K/L)^{-½}} = \frac{1 + (1/4)^{½}}{1 + (1/4)^{-½}} = \frac{1 + (½)}{1 + (2)} = ½$

c. Showing that this function exhibits constant returns to scale:

First, the production function must be homogeneous of degree *n*.

$q' = f(tL,tK) = t^n f(L,K) = t^n q \Rightarrow q' = tL + tK + 2\sqrt{tLtK}$

$\Rightarrow q' = tL + tK + 2\sqrt{t^2LK}$

$\Rightarrow q' = tL + tK + 2t\sqrt{LK}$

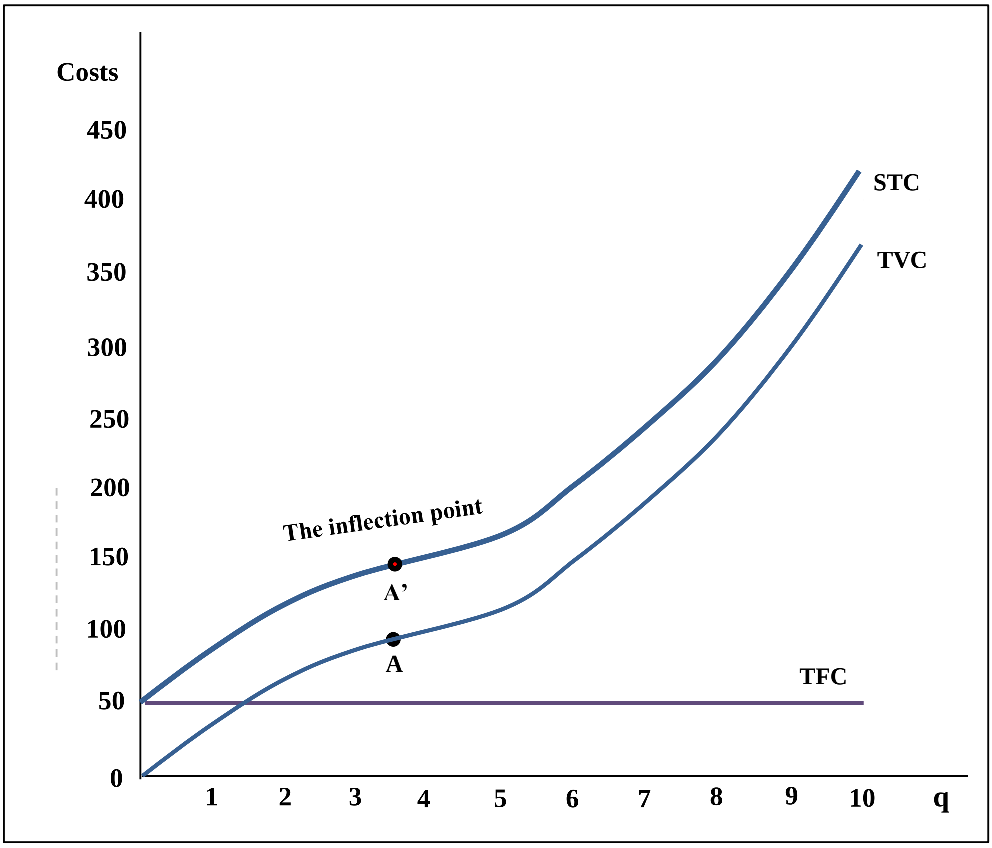
$\Rightarrow q' = t(L + K + 2\sqrt{LK})$

$\Rightarrow q' = tq$

The production function is homogeneous of degree one (**n = 1**). Thus, it exhibits constant returns to scale.

Chapter 8

1. a. Plotting the **TFC**, **TVC**, and **STC** curves:



b. The quantity of fixed inputs used determines the size or the scale of plant in which the firm operates in the short-run. Within the limits imposed by its scale of plant, the firm can vary its output in the short-run by varying the quantity of variable inputs used per unit of time.

c. Finding: **AFC**, **AVC**, **SAC**, and **SMC** and Plotting them on the same set of axes:

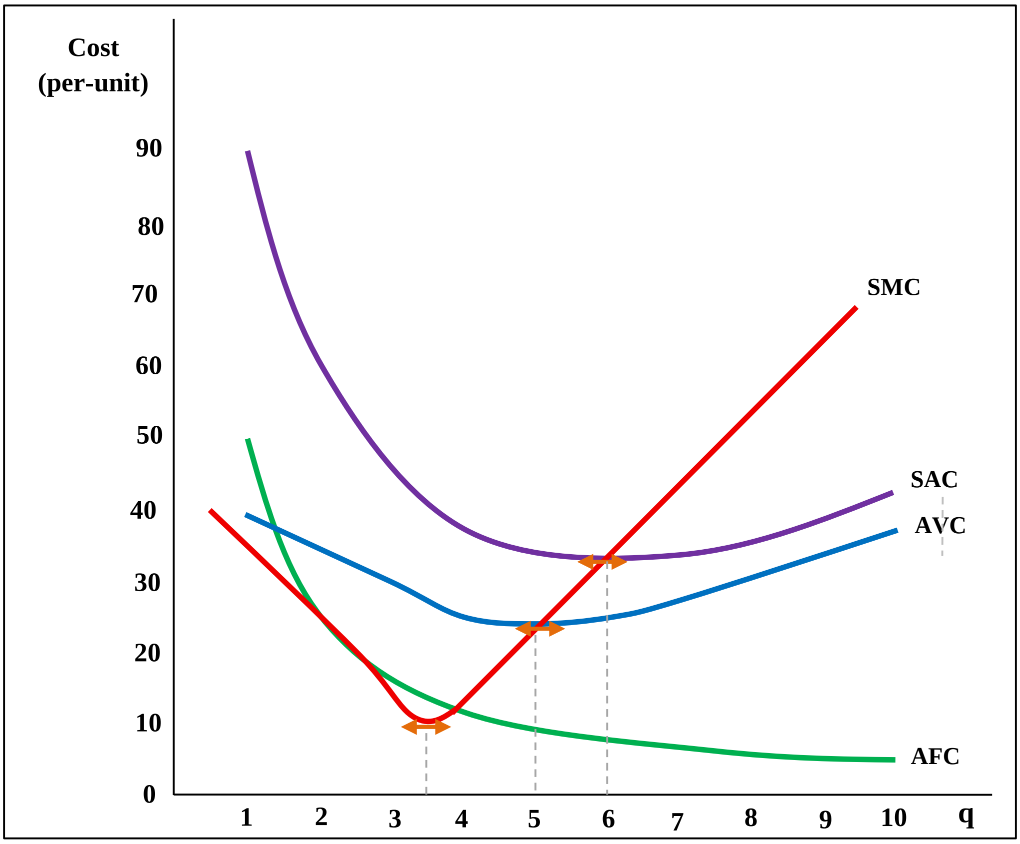
$\mathbf{AFC} = \frac{TFC}{q}$; $AFC_1 = \frac{TFC_1}{q_1} = \frac{50}{0} = /$; $AFC_2 = \frac{TFC_2}{q_2} = \frac{50}{1} = \mathbf{50}$ …

$\mathbf{AVC} = \frac{TVC}{q}$; $AVC_1 = \frac{TVC_1}{q_1} = \frac{0}{0} = /$; $AVC_2 = \frac{TVC_2}{q_2} = \frac{40}{1} = \mathbf{40}$ …

$\mathbf{SAC} = \frac{STC}{q}$; $SAC_1 = \frac{STC_1}{q_1} = \frac{50}{0} = /$; $SAC_2 = \frac{STC_2}{q_2} = \frac{90}{1} = \mathbf{90}$ …

$\mathbf{SMC} = \frac{\Delta STC}{\Delta q}$; $SMC_1 = \frac{\Delta STC}{\Delta q} = \frac{90-50}{1-0} = \mathbf{40}$; $SMC_2 = \frac{\Delta STC}{\Delta q} = \frac{120-90}{2-1} = \mathbf{30}$ …

q	0	1	2	3	4	5	6	7	8	9	10
TFC	50	50	50	50	50	50	50	50	50	50	50
TVC	0	40	70	90	100	120	150	190	240	300	370
STC	50	90	120	140	150	170	200	240	290	350	420
AFC	—	**50**	**25**	**16.67**	**12.5**	**10**	**8.33**	**7.14**	**6.25**	**5.55**	**5**
AVC	—	**40**	**35**	**30**	**25**	**24**	**25**	**27.14**	**30**	**33.33**	**37**
SAC	—	**90**	**60**	**46.67**	**37.5**	**34**	**33.33**	**34.28**	**36.25**	**38.89**	**42**
SMC	—	**40**	**30**	**20**	**10**	**20**	**30**	**40**	**50**	**60**	**70**



We can derive per unit costs curves geometrically from the corresponding total cost curves with the same way as we did before in chapter 8 (see page 111).

d. The relationship among the **SMC**, **AVC**, and **SAC** curves:

The SMC, AVC, and SAC curves are initially decline, reaching their minimum points, and then they rise. However, the SMC reaches its minimum point before AVC and SAC curves. SMC is below AVC when AVC is falling, and is above AVC when AVC is rising. When SMC curve rises, it intersects both AVC and SAC curves at their minimum points.

2. a. Does the firm operate in the short-run or in the long-run?

$\mathbf{TC = 20q^2 + 60q + 500}$

The firm operates in the short-run, because a part of total cost is fixed, **TFC =** DZD **500**.

b. Determining all possible total, average, and marginal costs:

Total variable cost: $\mathbf{TVC = 20q^2 + 60q}$

Total fixed cost: **TFC = 500**

Average fixed cost: $\mathbf{AFC} = \frac{TFC}{q} = \frac{500}{q}$

Average variable cost: $\mathbf{AVC} = \frac{TVC}{q} = \frac{20q^2 + 60q}{q} = \mathbf{20q + 60}$

Average total cost: $\mathbf{SAC} = \frac{STC}{q} = \frac{20q^2 + 60q + 500}{q} = \mathbf{20q + 60} + \frac{500}{q}$

Marginal cost: $\mathbf{SMC} = \frac{dSTC}{dq} = \frac{dTVC}{dq} = \mathbf{40q + 60}$

c. Finding the amount of output **q** when the **SMC** intersects the **SAC**:

SMC = SAC => $40q + 60 = 20q + 60 + \frac{500}{q}$

=> $20q - \frac{500}{q} = 0$

=> $\frac{20q^2 - 500}{q} = 0$

=> $20q^2 - 500 = 0$

=> **q = 5 units**

d. The minimum-cost output: **q = 5 units**

3. a. Deriving the **STC** function, and calculating the amount of this cost of producing **100** units of output:

$q = 4LK$; $\bar{K} = 10$; $w = \$10$; $r = \$12$

$q = 4L\bar{K}$ => $q = 4L(10)$ => $q = 40L$ => $L = \frac{q}{40}$

$STC = wL + r\bar{K}$ => $STC = 10L + 12(10)$ => $STC = 10(\frac{q}{40}) + 120$ => $STC = \frac{1}{4}q + 120$

$q_0 = 100$ units => $STC = \frac{1}{4}(100) + 120$ => **STC = $145**

b. Find the **AFC**, **AVC**, **SAC**, and **SMC** functions:

$AFC = \frac{TFC}{q} = \frac{120}{q}$

$AVC = \frac{TVC}{q} = \frac{\frac{1}{4}q}{q} = \frac{1}{4}$

$SAC = \frac{STC}{q} = \frac{\frac{1}{4}q + 120}{q} = \frac{1}{4} + \frac{120}{q}$

$SMC = \frac{dSTC}{dq} = \frac{1}{4}$

c. Deriving the **LTC** function, and calculating the amount of this cost of producing **100** units of output:

We use Lagrange multiplier to solve for a cost-minimizing problem:

$\mathcal{L} = WL + rK + \lambda(q - f(L,K))$ => $\mathcal{L} = 10L + 12K + \lambda(q - 4LK)$

$\frac{d\mathcal{L}}{dL} = 0$ => $10 - 4\lambda K = 0$ => $10 = 4\lambda K$ …..(1)

$\frac{d\mathcal{L}}{dK} = 0$ => $12 - 4\lambda L = 0$ => $12 = 4\lambda L$ …..(2)

$\frac{d\mathcal{L}}{d\lambda} = 0$ => $q - 4LK = 0$ => $q = 4LK$ ….(3)

Dividing (**1**) over (**2**), we find:

$\frac{10}{12} = \frac{4\lambda K}{4\lambda L}$ => $\frac{5}{6} = \frac{K}{L}$ => $K = \frac{5}{6}L$ …(4)

Substituting (**4**) into (**3**), we find:

$q = 4L(\frac{5}{6}L)$ => $q = \frac{10}{3}L^2$ => $L^2 = \frac{3}{10}q$ => $L^2 = 0.3q$ => $L = 0.55q^{\frac{1}{2}}$

Substituting **L** into (**4**), we find: $K = \frac{5}{6}(0.55q^{\frac{1}{2}})$ => $K = 0.46q^{\frac{1}{2}}$

We have: LTC = wL + rK => LTC = 10L + 12K

$$\Rightarrow LTC = 10(\mathbf{0.55q^{1/2}}) + 12(\mathbf{0.46q^{1/2}})$$

$$\Rightarrow LTC = 5.5q^{1/2} + 5.5q^{1/2}$$

$$\Rightarrow \mathbf{LTC = 11q^{1/2}}$$

$\mathbf{q_0 = 100}$ units => $\mathbf{LTC_0} = 11(\mathbf{100})^{1/2}$ => **LTC = $110**

d. Finding the **LAC** and **LMC** functions:

$$\mathbf{LAC} = \frac{LTC}{q} = \frac{11q^{1/2}}{q} = \mathbf{11q^{-1/2}}$$

$$\mathbf{LMC} = \frac{dLTC}{dq} = \mathbf{\frac{11}{2}q^{-1/2}}$$

e. Does this company experience economies or diseconomies of scale?

- Both **LAC** and **LMC** curves are declining as output increases, because:

$$\frac{dLAC}{dq} = \mathbf{-\frac{11}{2}\ q^{-1.5} < 0}$$

$$\frac{dLMC}{dq} = \mathbf{-\frac{11}{4}\ q^{-1.5} < 0}$$

- The **LMC** curve is below **LAC** curve for all levels of output, because:

  LMC = ½LAC => LMC < LAC

Therefore, the production function exhibits *increasing return to scale (IRS)*, and the company experiences *economies of scale* at all levels of output.

4. a. Calculating the $\mathbf{MP_L}$ and the $\mathbf{MP_K}$:

$$\boldsymbol{q = 10L^{0.3}K^{0.7}}$$

$$\mathbf{MP_L} = \frac{dq}{dL} = 10(0.3)L^{-0.7}K^{0.7} = \mathbf{3(\frac{K}{L})^{0.7}}$$

$$\mathbf{MP_K} = \frac{dq}{dK} = 10(0.7)L^{0.3}K^{-0.3} = \mathbf{7(\frac{L}{K})^{0.3}}$$

b. Finding the firm's expansion path equation:

We use the formula of the Marginal Rate of Technical Substitution of labor for capital ($\mathbf{MRTS_{L,K}}$):

$$\mathbf{MRTS_{L,K}} = \frac{\mathbf{MP_L}}{\mathbf{MP_K}} = \frac{\mathbf{w}}{\mathbf{r}} \Rightarrow \frac{3L^{-0.7}K^{0.7}}{7L^{0.3}K^{-0.3}} = \frac{w}{r} \Rightarrow \frac{3K}{7L} = \frac{w}{r} \Rightarrow \mathbf{K = \frac{7w}{3r}L}$$

c. Derive the contingent demand functions for L and K:

The firm seeks to minimize the total costs of producing a given quantity of output $\mathbf{q_0}$.

Using Lagrange multiplier method: $\boldsymbol{L} = \mathbf{wL + rK + \lambda(q_0 - 10L^{0.3}K^{0.7})}$

$$\frac{dL}{dL} = 0 \Rightarrow w - 10(0.3)\lambda L^{-0.7}K^{0.7} = 0 \Rightarrow \mathbf{w = 3\lambda L^{-0.7}K^{0.7}} \text{ …..(1)}$$

$$\frac{dL}{dK} = 0 \Rightarrow r - 10(0.7)\lambda L^{0.3}K^{-0.3} = 0 \Rightarrow \mathbf{r = 7\lambda L^{0.3}K^{-0.3}} \text{ …..(2)}$$

$$\frac{dL}{d\lambda} = 0 \Rightarrow q_0 - 10L^{0.3}K^{0.7} = 0 \Rightarrow \mathbf{q_0 = 10L^{0.3}K^{0.7}} \text{ ….(3)}$$

Dividing (**1**) over (**2**), we find:

$\frac{w}{r} = \frac{3\lambda L^{-0.7}K^{0.7}}{7\lambda L^{0.3}K^{-0.3}}$ => $\frac{w}{r} = \frac{3K}{7L}$ **=> $K = \frac{7w}{3r}L$ …(4)** **(**the firm's expansion path equation**)**

Substituting (**4**) into (**3**), we find:

$q_0 = 10L^{0.3}K^{0.7}$ **=>** $q_0 = 10L^{0.3}(\frac{7w}{3r}L)^{0.7}$ **=>** $q_0 = 10L^{0.3}(\frac{7w}{3r})^{0.7}L^{0.7}$ **=>** $q_0 = 10(\frac{7w}{3r})^{0.7}L$

=> $L = \frac{q_0}{10}(\frac{3r}{7w})^{0.7}$ the contingent demand functions for labor.

Substituting **L** into (**4**), we find:

$K = \frac{7w}{3r}[\frac{q_0}{10}(\frac{3r}{7w})^{0.7}]$ **=>** $K = \frac{7w}{3r}[\frac{q_0}{10}(\frac{7w}{3r})^{-0.7}]$

=> $K = \frac{q_0}{10}(\frac{7w}{3r})^{0.3}$ the contingent demand functions for capital.

d. Calculating the amounts of **L** and **K:**

w = $10 ; **r = $20** ; **$q_0$ = 200 units**

We just substitute into the demand functions for labor and capital.

$L = \frac{q_0}{10}(\frac{3r}{7w})^{0.7}$ => $L = \frac{200}{10}[\frac{3(20)}{7(10)}]^{0.7}$ => **L = 17.95 units.**

$K = \frac{q_0}{10}(\frac{7w}{3r})^{0.3}$ => $K = \frac{200}{10}[\frac{7(10)}{3(20)}]^{0.3}$ => **K = 20.95 units.**

5. a. Calculating the rental rate **r**:

$q = 2.66L^{½}K^{¼}$; **w= $30** ; capital investment, **I= 50** ; interest rate, ***i*= 5%**; N= **10** years (linear depreciation)

r = the amount of interest rate on capital investment + the amount of depreciation

$r = I \text{x} i + I \text{x} \frac{1}{N}$ => $r = I(i + \frac{1}{N})$ => $r = 50(5\% + \frac{1}{10})$ => **r** = 50x15% => **r** = $**7.5**

b. Finding the iso-cost equation:

TC = wL + rK => $K = \frac{TC}{r} - \frac{w}{r}L$ => $K = \frac{TC}{7.5} - \frac{30}{7.5}L$ => $K = \frac{TC}{7.5} - 4L$

c. Determining the producer's equilibrium:

<u>**Case 1**</u>**:** Maximizing total output **q** with total costs **TC = $4500**.

$$\begin{cases} \frac{MP_L}{MP_K} = \frac{w}{r} \text{ …(1)} \\ \text{s. t. } TC = wL + rK \text{ …(2)} \end{cases}$$

(1) <=> $\frac{2.66(½)L^{-½}K^{¼}}{2.66(¼)L^{½}K^{-¾}} = \frac{w}{r}$ <=> $\frac{2K}{L} = \frac{30}{7.5}$ <=> **K = 2L …(3)**

Substituting (**3**) into (**2**), we find:

TC = wL + rK => **4500** = **30**L + **7.5**K **=>** 4500 = 30L + 7.5(**2L**) => 4500 = 45L **=> L_0** = **100** units.

Substituting **L_0** into (**3**), we find: K_0 = 2**(100) => K_0 = 200 units**.

q_0 = $2.66L^{½}K^{¼}$ => **q_0** = $2.66(100)^{½}(200)^{¼}$ => **q_0 = 100 units**

Case 2: Minimizing total costs **TC** to produce a total output **q = 100** units.

$\boldsymbol{L = wL + rK + \lambda(q_0 - f(L,K)) \Rightarrow L = 30L + 7.5K + \lambda(100 - 2.66L^{1/2}K^{1/4})}$

$\frac{dL}{dL} = 0 \Rightarrow 30 - 2.66(\frac{1}{2})\lambda L^{-1/2}K^{1/4} = 0 \Rightarrow \mathbf{30 = 1.33\lambda L^{-1/2}K^{1/4}}$ **…..(1)**

$\frac{dL}{dK} = 0 \Rightarrow 7.5 - 2.66(\frac{1}{4})\lambda L^{1/2}K^{-3/4} = 0 \Rightarrow \mathbf{7.5 = 0.665\lambda L^{1/2}K^{-3/4}}$ **…..(2)**

$\frac{dL}{d\lambda} = 0 \Rightarrow 100 - 2.66L^{1/2}K^{1/4} = 0 \Rightarrow \mathbf{100 = 2.66L^{1/2}K^{1/4}}$ **….(3)**

Dividing (**1**) over (**2**), we find:

$\frac{30}{7.5} = \frac{1.33\lambda L^{-1/2}K^{1/4}}{0.665\lambda L^{1/2}K^{-3/4}} \Rightarrow 4 = \frac{2K}{L} \Rightarrow \mathbf{K = 2L}$ **…(4)**

Substituting (**4**) into (**3**), we find:

$100 = 2.66L^{1/2}(\mathbf{2L})^{1/4} \Rightarrow 100 = 2.66L^{1/2}2^{1/4}L^{1/4} \Rightarrow 100 = 2.66L^{1/2}2^{1/4}L^{1/4} \Rightarrow 100 = 3.16L^{3/4}$

$\Rightarrow L^{3/4} = \frac{100}{3.16} \Rightarrow L = (\frac{100}{3.16})^{4/3} \Rightarrow \mathbf{L_0 = 100}$ **units**

Substituting $\mathbf{L_0}$ into **(4)**, we find: $K_0 = 2(\mathbf{100}) \Rightarrow \mathbf{K_0 = 200}$ **units**.

TC = wL + rK **=>** **TC =** 30**(100) +** 7.5**(200) =>** $\mathbf{TC_0}$ **= $4500**

We can see that the results found in the two cases are the same. Therefore, whether using the method of maximizing total output, or the method of minimizing total costs allows us to obtain the same results.

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