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Par

MOKHTARI Khalil

THÈME

Multivariable Control of Not Almost Strictly Positive Real Systems (ASPR)

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SAIT Belkacem	Professeur	Univ. Ferhat Abbas Sétif 1	Président
ABDELAZIZ Mourad	Professeur	Univ. Ferhat Abbas Sétif 1	Directeur de thèse
LABIOD Salim	Professeur	Université de Jijel	Examinateur
KARA Kamel	Professeur	Université de Blida	Examinateur
REFFAD Aicha	M.C.A.	Univ. Ferhat Abbas Sétif 1	Examinatrice
BENALLEGUE Abdelaziz	Professeur	Univ. Versailles Saint Quentin en Yvelines	Invité
EL HADRI Abdelhafid	M.C.A	Univ. Versailles Saint Quentin en Yvelines	Invité

To my parents

To my friends

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Abstract

Simple Adaptive Control (SAC) is one of the main configurations used in passivity-based adaptive systems. Since the problems of stability and robustness have been partially solved, several researchers have set the goal of improving performance on one hand and simplifying design and implementation on the other. The SAC finds its application in deterministic systems as well as in disturbed systems. SAC of non-Almost Strictly Positive Real (ASPR) SISO/MIMO systems is well studied. In the last decades, some applications of the SAC technique presented a design complexity issue arising from a large number of parameters and coefficients to select. This issue was recently decreased by the idea of the Decentralized Simple Adaptive Controller (DSAC), which considers only the diagonal of the time-varying gain matrices. Therefore, the decreased computational requirements of the DSAC facilitated real-time implementation.

In the first part of this thesis, we further improve the flexibility of the DSAC by considering the fractional derivative of the adaptive gains. The proposed new controller is called Fractional Order DSAC (FO-DSAC). Firstly, the ASPR conditions for the design of a stable fractional adaptive system are established. A fractional Order parallel feedforward compensator (FO-PFC) to realize an augmented ASPR system is then provided and a new ASPR-based FODSAC controller is proposed. The stability analysis of the proposed control scheme is presented and simulation example comparing the standard DSAC with the new FODSAC is provided, showing the performance of the proposed method.

In the seconde part of this thesis, a new Adaptive Synergetic Controller (ASC) is proposed as a solution for the problem of the design of the standard Synergetic Control law when the system parameters and dynamics are unknown. It is well known that the design of the SC law requires a thorough knowledge of the system parameters and dynamics. Such problem obstructs the synthesis of the SC law and the designer is prompted to pass through the estimation methods, which, in turn, poses a problem of increasing the computation time of the control algorithm. To cope with this problem, a solution is proposed by modifying the original SC law to develop an SAC-like adaptive SC law without the need of prior knowledge of the system. The stability of the proposed adaptive controller is formally proven via the Lyapunov approach. Experimental application to a quadrotor system is given to validate the theoretical results.

Résumé

La commande adaptative simple (SAC) est l'une des principales configurations utilisées dans les systémes adaptatifs basés sur la passivité. Les problèmes de stabilité et de robustesse étant partiellement résolus, plusieurs chercheurs se sont fixés pour objectif d'améliorer les performances d'une part et de simplifier la conception et la mise en uvre d'autre part. Le SAC trouve son application aussi bien dans les systèmes déterministes et dans les systèmes perturbés. Le SAC des systèmes SISO / MIMO non ASPR est bien étudié. Au cours des dernires décennies, certaines applications de la technique SAC ont présenté un problème de complexité de conception découlant d'un grand nombre de paramètres et coefficients sélectionner. Ce problème a été récemment résolu par l'idée du contrôleur adaptatif simple décentralisé (DSAC), qui ne considère que la diagonale des matrices de gain variant dans le temps. Par conséquent, la diminution des exigences de calcul du DSAC a facilité la mise en œuvre en temps réel.

Dans la premire partie de cette thèse, nous améliorons encore la flexibilité du DSAC en considérant la dérivée fractionnelle des gains adaptatifs. Le nouveau contrôleur proposé est appelé DSAC d'ordre fractionnaire (FODSAC). Premirement, les conditions ASPR pour la conception d'un systéme adaptatif fractionnaire stable sont établies. Un compensateur à action directe parallèle d'ordre fractionnaire (FOPFC) pour réaliser un système ASPR augmenté est alors fourni et un nouveau contrôleur FODSAC basé sur ASPR est proposé. L'analyse de stabilité du schéma de commande proposé est présentée et un exemple de simulation comparant le DSAC standard au nouveau FODSAC est fourni, montrant les performances de la méthode proposée.

Dans la seconde partie de cette thèse, un nouveau contrôleur adaptatif synergétique (ASC) est proposé comme solution au problème de la conception de la loi de commande synergétique standard lorsque les paramètres et la dynamique du système sont inconnus. Il est bien connu que la conception de la loi SC nécessite une connaissance approfondie des paramètres et de la dynamique du système. Tel problème entrave la synthèse de la loi SC et le concepteur est invité à passer par les méthodes d'estimation, ce qui, à son tour, pose un problème d'augmentation du temps de calcul de l'algorithme de commande. Pour faire face à ce problème, une solution est proposée en modifiant la loi du SC originale pour développer une loi de commande ASC sans avoir besoin d'une connaissance préalable de la dynamique du système. La stabilité du contrôleur adaptatif proposé est formellement prouvée via le Lyapunov et le principe d'invariance de Lasalle. Une application expérimentale sur un quadrirotor est donnée pour valider les résultats théoriques.

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Nomenclature

Notations

- $\mathbb{R}$ denote the field of real numbers.
- $\mathbb{R}^+$ denotes the set of non-negative real numbers
- $\mathbb{C}$ denote fields of complex numbers.
- $\mathbb{R}^n$ denotes the n dimensional Euclidean space.
- $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices with n being the number of rows and m the number of columns.
- $diag(a_1, ..., a_n)$ denote the diagonal matrix whose diagonal elements are $a_1, ..., a_n$.
- $col\{A_1, ..., A_m\}$ denote the block column matrix $col\{A_1, ..., A_m\} = [A_1^T, ..., A_m^T]^T$.
- $(a, b] = \{x \in \mathbb{R} : a < x \leq b\}$ and $[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$.
- If any given matrix $A \in \mathbb{R}^{n\times m}$ or $\mathbb{R}^{n\times n}$, then A^T denote its transpose.
- If $A \in \mathbb{R}^{n\times n}$, then $A = A^T > 0$ indicates that A is a positive definite symmetric (PDS) matrix.
- $G^*(s) \equiv G^T(-s)$
- If A is non-singular (i.e., $det(A) \neq 0$), then A^{-1} denotes its inverse.
- $tr\{A\}$ denotes the trace of a matrix A
- I denotes the identity matrix
- $\|\cdot\|$ denote the Euclidean norm.

Acronyms / Abbreviations

- ASC Adaptive Synergetic Control

- ASP Almost Strictly Passive
- ASPR Almost Strictly Positive Real
- DSAC Decentralized Simple Adaptive Control
- FODSAC Fractional Order Decentralized Simple Adaptive Control
- FOPFC Fractional order Parallel Feedforward Compensator
- FOTF Fractional Order Transfer Function
- PFC Parallel Feedforward Compensator
- PDS Positive Definite Symmetric
- PT Perfect Tracking
- SAC Simple Adaptive Control
- SC Synergetic Control
- SP Strictly Passive
- SPR Strictly Positive Real
- TF Transfer Function
- WASP W Almost Strictly Passive
- WSP W Strictly Passive

General Introduction

It is well-known, in the last decades results of control theory that the notion of passivity, for a class of nonlinear systems with the representation $\{A(x), B(x), C\}$, plays an important role in guaranteeing stability in adaptive control, for which, the plant is required to be strictly passive (SP) or strictly positive real (SPR) for linear time-invariant systems (LTI). Since most real-world systems are not inherently SP, the SP condition was mitigated for a class of systems with the representation $\{A(x), B(x), C\}$ called Almost SP (ASP) (Almost SPR in LTI systems), which they can be rendered SP via constant or dynamic, output feedback. Later, in attempting to define what classes of systems that satisfy the ASP conditions, it was shown that any *minimum-phase* LTI system with state-space realization $\{A, B, C\}$ is ASP if the product CB is *positive definite symmetric* (PDS) [1], [2]. However, the feasibility of adaptive control techniques in multivariable LTI systems was apparently limited by the standard SP conditions that imply that the product CB must be PDS and the required symmetry of CB seemed to be rather difficult to fulfill in practice. To this effect, Barkana et all [3] eliminated the symmetry condition of CB and extended to a class of WASP systems, that is, for any *minimum-phase* LTI system with state-space realization $\{A, B, C\}$ is WASP if the *not necessarily symmetric* positive definite product CB is diagonalizable. Recently, the applicability of these previous results for LTI systems were successfully expanded to nonstationary and nonlinear systems [4], where the sufficient conditions that allow nonstationary systems to become stable and strictly passive via static or dynamic output feedback were founded.

The SAC technique has attracted the attention of many researchers in the last few decades, due to its simplicity and the ease of implementation of its algorithm [5–12] . The SAC is a type of direct adaptive controller developed by Sobel et al [13]. One of the main advantages of this technique is that it does not require full state access or any estimator in the control loop, and the system parameters and dynamics are not necessarily needed to be known. The only requirement for the implementation of the SAC algorithm is that the plant to be controlled should satisfy the ASP conditions. Note that the ASP conditions are not satisfied by most of the systems in the real world and this is why Barkana and Kaufman [14, 15] extend the applicability of SAC to a class of non-ASP systems by introducing the so-called Parallel Feedforward Compensator (PFC) [16–20] with the realisation $\{A_f, B_f, C_f\}$. By using the latter parallel to the nonlinear system one can realise an augmented ASP having the representation $\{A_a(x), B_a(x), C_a\}$.

However, in some applications the SAC technique may still present a design complexity issue arising from the large number of parameters and coefficients to select. This design complexity has been mitigated by the recently proposed Modified simple adaptive control (MSAC) [21], Decentralized MSAC (DMSAC) and Decentralized SAC (DSAC) methodologies [22]. The DSAC methodology was recently developed based on the recent ASP results for nonlinear and nonstationary systems [4]. The Lyapunov proof of stability of DSAC is established using the recent aforementioned ASP condition.

In the first part of this thesis, for completeness of the previous results presented in [22], we further extend the stability theorem used in the previous DSAC by using the recently mitigated passivity conditions that do guarantee stability with adaptive model tracking with the not necessarily symmetric matrix CB. Using the recent Lyapunov Function for WASP nonlinear systems, under the WASP conditions with symmetric W, we expand the proof of stability of the DSAC.

As the DSAC technique has mitigated the design complexity issue of the standard SAC arising from the large number of parameters and coefficients to select, there are still some exceptional remedies on how to improve tracking performance (flexibility and accuracy). Fortunately, as numerous previous studies show, the introduction of fractional order calculus can considerably improve the performance of conventional controllers [23] [24]. There is another possibility of further improving the flexibility and the tracking performance of DSAC by utilizing the fractional order calculus. Several researchers have improved the performance of some adaptive control techniques using fractional calculus. For example, a Fractional MRAC was studied in [25] [26] in order to achieve better tracking performance. In [27], an adaptive fractional sliding mode controller is proposed to improve tracking performance of a class of systems with nonlinear disturbances. In [28], an adaptive fractional order terminal sliding mode controller is proposed to improve the control performances (convergence and precision) of a cable-driven manipulator. In [29], a fractional order adaptive backstepping controller is proposed for the improvement of performances of a class of fractional order nonlinear systems.

In this thesis, motivated by the discussions above, we propose a novel Fractional Order DSAC (FO-DSAC) approach for linear fractional order systems. Based on the DSAC scheme [22], we extend the study to fractional order case. Firstly, we try to establish the ASPR conditions for the design of a stable fractional adaptive system. We provide a fractional Order parallel feedforward compensator (FO-PFC) to realize an augmented ASPR system. Then we propose the ASPR-based FO-DSAC controller. The stability analysis of the proposed control scheme is presented and simulation example comparing the standard DSAC with the new FO-DSAC is provided, showing the performance of the proposed approach.

In the second part of this thesis, an Adaptive Synergetic Controller (ASC) for a class of nonlinear systems with the representation $A(x), B(x), C$ is proposed, based on the aforementioned SAC technique. The control of such class of nonlinear systems has continued to attract more researchers: many control techniques like synergetic, sliding mode control, optimal feedback and adaptive controllers [22, 30–35] have been investigated for such representation due to its importance since it appears in modeling of many real-world systems such as robotic and mechatronic systems, electric machines, power converters, etc. Moreover, in adaptive control theory the representation $\{A(x),\ B(x),\ C\}$ highlights useful symmetries and simplifies the theoretical analysis [36]: those symmetries could be otherwise lost in the general nonlinear representation. In non adaptive controllers, the most used techniques are nonlinear output feedback, synergetic and sliding mode controllers because of their capabilities in controlling nonlinear systems. Besides the sliding mode control, a synergetic control (SC) technique has recently attracted the attention of many researchers [37–44] since it shares with the SMC the same idea of forcing the closed-loop system to move on a desired manifold, but without the chattering phenomenon [45]. The SC theory was firstly introduced by the Russian researcher Kolesnikov [46] and it has been investigated in hysteretic systems [37], DC-DC boost converters [38], robot manipulators [41], fault tolerant system [44] and power system stabilisers [47–49]. However, the control design procedures of the SC and most of the proposed techniques in the literature require extensive knowledge of the system parameters and dynamics so that the control law can be designed: the reason why researchers are interested in developing adaptive techniques based on observers and estimators that can deal with systems with unknown (unmodelled) parameters (dynamics). For example, in [50] and [51] a fuzzy based adaptive control methods are proposed for nonlinear systems with unmodelled dynamics. In [37, 48], adaptive and optimal adaptive synergetic controllers are proposed under the idea of estimating the unknown system dynamic by using fuzzy logic. In [49] a Type-2 fuzzy system to approximate the unknown nonlinear dynamics for the design of a SC is introduced. One of the main adaptive control techniques used for this class of nonlinear systems is aforementioned SAC technique [30, 52, 53].

Taking into account the advantages of the SAC technique for the aforementioned class of nonlinear system when the system parameters and dynamics are unknown, a new adaptive synergetic controller is proposed in this chapter. Starting by defining a macrovariable to be a function of the augmented error instead of the system states, the SC law is developed. By handling this law, and after some development leading to extract an SAC-like structure, one proposes for such a class of systems an adaptive synergetic controller (ASC) that is independent of the system parameters and dynamics. Unlike the adaptive synergetic controllers presented in the literature, the proposed one does not require any prior knowledge of the system nor estimators. Furthermore, thanks to the structure of the ASC controller with an adequate choice of the lyapunov function, we were able to achieve the asymptotic stability of the adaptive system. In fact, the stability analysis of the standard SAC reveals in general a residual term in the derivative of the lyapunov function that affect negativity of such function. To cope with this problem, some researchers [4, 6, 22, 30, 54] have tried to directly eliminate this residual term by assuming that either this one may vanish by assuming that the variation of system parameters is slow compared with the control dynamics, or it is supposed to be bounded and in this case the stability is analysed by the domination of some quadratic and quartic terms on the residual one. In our study, for a clear discussion of the system stability, and with a mathematical sense we substitute the residual term by its Jacobian linear approximation and with an appropriate choice of the macro-variable one can achieve the asymptotic convergence of the system to a desired manifold. An experimental application to a quadrotor system is given to validate the proposed ASC scheme.

Thesis organization

This thesis is divided into two parts: part 1 includes chapter 1 and chapter 2, part 2 includes chapter 3 and chapter 4.

In chapter 1, we extend the stability theorem used in the previous DSAC by using the recently mitigated passivity conditions that do guarantee stability with adaptive model tracking with the not necessarily symmetric matrix CB. Using the new WASP conditions with symmetric W for nonlinear systems, we analytically expand the proof of stability of the DSAC.

In chapter 2, a passivity based fractional order decentralized simple adaptive control

(FODSAC) for linear fractional order systems is considered. Almost strictly positive real (ASPR) conditions for the design of a stable fractional adaptive system will be established. Further, a fractional Order parallel feedforward compensator (FO-PFC) to realize an augmented ASPR system will be provided and a FO-DSAC controller will be proposed. The stability analysis of the proposed FO-DSAC scheme will be presented.

In chapter 3 we presents our new approach, namely, the design of the ASC controller as well as the stability proof of the obtained adaptive system.

In chapter 4, through a quadrotor test-bench we experimentally validate the theoretical results of the proposed ASC and confirm the usefulness of the proposed method for real-world (physical) system.

Scientific Contributions

Journal papers

1. Khalil Mokhtari, Abdelhafid Elhadri, and Mourad Abdelaziz. ”A passivity-based simple adaptive synergetic control for a class of nonlinear systems.” International Journal of Adaptive Control and Signal Processing 33.9 (2019): 1359-1373.

2. Khalil Mokhtari, and Mourad Abdelaziz. ”On simple adaptive control of plants not satisfying almost strict passivity and positivity conditions: an introduction to parallel feedforward configuration.” International Journal of Computer Aided Engineering and Technology 11.2 (2019): 267-281.

3. Khalil Mokhtari, Mourad Abdelaziz and Abdelhafid El Hadri. Almost Positivity and Decentralized Simple Adaptive Control Design with a PFC in the fractional domain. (To be submitted soon).

4. Khalil Mokhtari, Abdelhafid El Hadri, Mourad Abdelaziz. A Passivity based Simple Adaptive Synergetic Control for a Class of Nonlinear System: Part 2. (To be submitted soon).

International Conference Papers

1. Khalil Mokhtari, Mourad Abdelaziz: Simple Adaptive Control of Plants not Satisfying Almost Strictly Positive Condition. International Conference on Automatic

control, Telecommunications and Signals (ICATS15). University BADJI Mokhtar - Annaba - Algeria - November 16-18, 2015

2. Khalil Mokhtari, Mourad Abdelaziz: Passivity-based Simple Adaptive Control for Quadrotor Helicopter in the Presence of Actuator Dynamics. The 8th International Conference on Modelling, Identifiation and Control (ICMIC 2016),15-17 November 2016, Algiers, Algeria.

3. Khalil Mokhtari, Mourad Abdelaziz, Abdelhafid Elhadri: Adaptive attitude control of Quadrotor under disturbances: Actual implementation and experimental results. 8th INTERNATIONAL CONFERENCE ON DEFENSE SYSTEMS: ARCHITECTURES AND TECHNOLOGIES (DAT2020) April 14-16, 2020, Constantine, Algeria (submitted).

4. Khalil MOKHTARI, Abdelhafid ELHADRI, Mourad ABDELAZIZ : Adaptive Synergetic Control of Quadrotor: Algorithms and Experimental Result. ICNPAA WORLD CONGRESS 2020, Czech Technical University in Prague, Prague, Czech Republic, June 23-26 , 2020 (Accepted).

5. Khalil MOKHTARI, Mourad ABDELAZIZ : A New Fractional Order Decentralized Simple Adaptive Control For Ordinary and fractional order ASPR systems. ICNPAA WORLD CONGRESS 2020, Czech Technical University in Prague, Prague, Czech Republic, June 23-26 , 2020 (Accepted).

6. Khalil Mokhtari, Mourad Abdelaziz. Simple Adaptive Control and Stability Analysis with ASPR Condition. Journee15 des Doctorants, Setif, Algerie

7. Khalil Mokhtari, Mourad Abdelaziz. Strict Passivity based Simple Adaptive Control for the Attitude Stabilization of the Quadrotor Helicopter. Journee16 des Doctorants, Setif, Algerie.

8. Khalil Mokhtari, Mourad Abdelaziz. Robust Adaptive Control System based on the Strictly Positive Real Condition. Doctoriales de lUFAS1, 19 Mai 2016.

Part I

Extensions in SAC theory

Chapter 1

Extension in DSAC with the WASP conditions

> “ *The less attachment to the world. The easier your life.* ”
>
> Umar ibn Al-Khattab

1.1 Introduction

Recently, the DSAC has been introduced as a new class of direct adaptive controllers. The DSAC methodology has been proposed on the basis of the recent development of sufficient conditions which allow non-stationary and nonlinear systems to become stable and SP via a static or dynamic output feedback. The formal proof of the stability of the DSAC was also developed on the basis of the standard Lyapunov function for ASP systems. In this chapter, we extend the stability analysis of the DSAC using the recently attenuated passivity conditions which guarantee the stability of the adaptive systems in the case where the CB product is not necessarily symmetrical. Using the new WASP conditions for nonlinear systems, we analytically develop the proof of the stability of the DSAC.

1.2 System and Definitions

In this section, we present the basic definitions of passivity and W-strict passivity properties for a class of nonlinear systems of the form 1.1 with not necessarily symmetric matrix CB. The following definitions and theorem will be exploited in the subsequent development. Consider a class of nonlinear systems described by the following formulation

$$\begin{aligned} \dot{x}\,(t) &= A(x)x\,(t) + B(x)u\,(t) \\ y\,(t) &= Cx\,(t) \end{aligned} \tag{1.1}$$

Where $y(t) \in \mathbb{R}^m$ is the output vector. $x = [x_1 \cdots x_n]^T \in \mathbb{R}^n$, $u = [u_1 \cdots u_m]^T \in \mathbb{R}^m$, $y = [y_1 \cdots y_m]^T \in \mathbb{R}^m$. Note that the representation $\{A(x), B(x), C\}$ of the nonlinear model 1.1 does not necessary to be known for the synthesis of the simple adaptive control law, sufficient informations are required to ensure that the Almost Strict Passivity conditions are satisfied, thus, stability of the closed loop system can be guaranteed.

Definition 1.1. *The nonlinear system $\{A(x)$,$B(x)$,$C\}$ with the state-space realization 1.1 is called uniformly strictly minimum-phase if its zero dynamics is uniformly asymptotically stable. In other words, if there exist two matrices $M(x)$ and $N(x)$ satisfying the following relations*

$$CM(x) = 0 \tag{1.2}$$

$$N(x)B(x) = 0 \tag{1.3}$$

$$N(x)M(x) = I_m \tag{1.4}$$

such that the resulting zero dynamics given by

$$\dot{z} = (\dot{N}(x) + N(x)A(x))M(x)z \tag{1.5}$$

is uniformly asymptotically stable

Definition 1.2. *The nonlinear system $\{A(x)$,$B(x)$,$C\}$ with the statespace realization 1.1 is called WSP if there exist two uniformly PDS matrices, $P(x)$, $Q(x)$ and an appropriate PDS matrix $W(x)$ such that the following two relations are simultaneously satisfied*

$$\dot{P}(x) + P(x)A(x) + A^T(x)P(x) = -Q(x) \tag{1.6}$$

$$P(x)B(x) = C^T(x)W^T(x) \tag{1.7}$$

In addition, W should satisfies one of the following conditions:

(C1) the product $W(x)CB(x) = R(x)$ renders $R(x)$ uniformly PDS.

(C2) The PDS matrix $W(x)$ that renders the matrix $R(x)$ in (C1) PDS satisfies the condition $\dot{W}(x) \leq 0$ along the trajectories of the controlled system [4].

Relation 1.6 is an algebraic Lyapunov equation which means that a WSP system is asymptotically stable, the second relation 1.7 shows that

$$B^T PB = B^T C^T W^T = (WCB)^T = WCB \tag{1.8}$$

which implies that the product WCB is PDS.

Definition 1.3. *A nonlinear system $\{A(x), B(x), C\}$ with the square state-space realization 1.1 is called WASP if there exists a positive definit constant output feedback matrix $\widetilde{K}_e$ such that the resulting closed-loop system with the state-space realization $\{(A(x) - B(x)\widetilde{K}_e C), B(x), C\}$ is WSP. In other words, there exists two PDS matrices $P(x)$ and $Q(x)$ such that the closed-loop system*

$$\begin{aligned} \dot{x}(t) &= [A(x) - B(x)\widetilde{K}_e C]x(t) \\ y\,(t) &= Cx\,(t) \end{aligned} \tag{1.9}$$

simultaneously satisfies the following WSP relations

$$\dot{P}(x) + P(x)A_{cl}(x) + A_{cl}^T(x)P(x) = -Q(x) \tag{1.10}$$

$$P(x)B(x) = C^T(x)W^T(x) \tag{1.11}$$

Where $A_{cl}(x) = [A(x) - B(x)\widetilde{K}_e C]$.

Theorem 1.1. *Any uniformly strictly minimum-phase nonlinear system $\{A(x), B(x), C\}$ with the state-space realization 1.1 where all system matrices are uniformly bounded, and if any one of conditions C1 and C2 is satisfied, is WASP.*

Proof. See [4]. □

1.3 Model Following with DSAC

The DSAC methodology is based on the standard Simple Adaptive Control (SAC) theory [55], the DSAC strategy is meant to perform model tracking, the plant output is required to track the outputs of an ideal stable plant called "model" that serves to generate the "reference" trajectory. The model does not need any prior knowledge about the plant and it can be of any "large" or "lower" order. The model is

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \tag{1.12}$$

where $x_m(t) \in \mathbb{R}^{n_m}$ and $y_m(t) \in \mathbb{R}^m$ represent the reference model state vector and output vector respectively, $u_m(t) \in \mathbb{R}^{p_m}$ is the model command input. A_m, B_m, and C_m are the model matrices with appropriate dimensions. The standard SAC algorithm [54] monitors the output tracking error

$$e_y(t) = y_m(t) - y(t) \tag{1.13}$$

Based on the available model variables, x_m and u_m, the adaptive control signal is defined as

$$u(t) = K_e(t)\, e_y(t) + K_x(t)\, x_m(t) + K_u(t)\, u_m(t) \tag{1.14}$$

Where $K_e(t)$ is the feedback stabilizing control gain which is needed to guarantee the stability of the closed-loop system, $K_x(t)$ and $K_u(t)$ are appropriate feedforward control gains, and they perform to minimize the output tracking error which allow good tracking without requiring large values of the stabilizing gain $K_e(t)$. The SAC gains are a combination of proportional and integral gains as follows

$$K_e(t) = K_{p_e}(t) + K_{I_e}(t) \tag{1.15}$$

$$K_x(t) = K_{p_x}(t) + K_{I_x}(t) \tag{1.16}$$

$$K_u(t) = K_{p_u}(t) + K_{I_u}(t) \tag{1.17}$$

Note that only the adaptive integral gains are necessary to guarantee the stability of the adaptive control system, proportional adaptive gains are added to increase the convergence speed of the adaptive system. So, a low tracking error can be obtained in a very short time.

1.3.1 Adaptation Law of DSAC

In the DSAC algorithm, the components of the proportional and integral gains of the stabilizing control gain in 1.15 are updated by the following adaptive law [22]

$$K_{p_e}(t) = diag\{e_y e_y^T\}\Gamma_{P_e} \tag{1.18}$$

$$\dot{K}_{I_e}(t) = diag\{e_y e_y^T\}\Gamma_{I_e} \tag{1.19}$$

where $K_{p_e}(t),\ K_{I_e}(t) \in \mathbb{R}^{m\times m}$ and $diag\{A\}$ with $A \in \mathbb{R}^{n\times n}$ denotes the diagonalization operation defined as follows

$$diag\{A\} = \begin{bmatrix} a_{1,1} & 0 & \cdots & 0 \\ 0 & a_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{n,n} \end{bmatrix} \tag{1.20}$$

The feedforward control gain components $K_{p_x}(t),\ K_{I_x}(t) \in \mathbb{R}^{m\times n_m}$ and $K_{P_u}(t),\ K_{I_u}(t) \in \mathbb{R}^{m\times n_{p_m}}$ are updated as follows

$$K_{p_x}(t) = R^T diag\{Re_y x_m^T\}\Gamma_{P_x} \tag{1.21}$$

$$\dot{K}_{I_x}(t) = R^T diag\{Re_y x_m^T\}\Gamma_{I_x} \tag{1.22}$$

$$K_{p_u}(t) = T^T diag\{Te_y u_m^T\}\Gamma_{P_u} \tag{1.23}$$

$$\dot{K}_{I_u}(t) = T^T diag\{Te_y u_m^T\}\Gamma_{I_u} \tag{1.24}$$

with

$$R = \begin{bmatrix} I_m \\ I_m \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{n_m\times m},\ \ T = \begin{bmatrix} I_m \\ I_m \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{p_m\times m} \tag{1.25}$$

where $\Gamma_{P_e},\ \Gamma_{I_e} \in \mathbb{R}^{m\times m},\ \Gamma_{P_x},\ \Gamma_{I_x} \in \mathbb{R}^{n_m\times n_m}$ and $\Gamma_{P_u},\ \Gamma_{I_u} \in \mathbb{R}^{p_m\times p_m}$ are time-invariant diagonal weighting matrices that control the rate of adaptation. Defining $K(t) = \begin{bmatrix} K_e(t) & K_x(t) & K_u(t) \end{bmatrix} = K_I(t) + K_P(t)$ and $r^T(t) = \begin{bmatrix} e_y^T & x_m^T & u_m^T \end{bmatrix}$, the total adaptive control algorithm can be written as the following form

$$u(t) = K(t)\, r(t) \tag{1.26}$$

where $K(t) \in \mathbb{R}^{m\times(m+n_m+p_m)}$ and $r(t) \in \mathbb{R}^{m+n_m+p_m}$. The total integral and proportional adaptive control gains $K_I(t), K_P(t) \in \mathbb{R}^{m\times(m+n_m+p_m)}$, are updated as follows

$$K_P(t) = S^T diag\{Se_y r^T\}\Gamma_P \tag{1.27}$$

$$\dot{K}_I(t) = S^T diag\{Se_y r^T\}\Gamma_I \tag{1.28}$$

where $\Gamma_P, \Gamma_I \in \mathbb{R}^{(m+n_m+p_m)\times(m+n_m+p_m)}$, and the scaling matrix S is given by

$$S = \begin{bmatrix} I_m \\ I_m \\ \vdots \\ I_m \end{bmatrix} \in \mathbb{R}^{(m+n_m+p_m)\times m} \tag{1.29}$$

1.3.2 Ideal control and state trajectory

When perfect output tracking occurs between the system 1.1 and the ideal model 1.12, one can define the following ideal system dynamic and an ideal control input:

$$\begin{aligned} \dot{x}^*(t) &= A(x^*)\,x^*(t) + B(x^*)\,u^*(t) \\ y^*(t) &= Cx^*(t) \end{aligned} \tag{1.30}$$

$$u^*(t) = \widetilde{K}_x x_m(t) + \widetilde{K}_u u_m(t) + \widetilde{K}_e e_a(t) \tag{1.31}$$

Where $\widetilde{K}_e$ is the ideal feedback stabilizing gain matrix, $\widetilde{K}_x$ and $\widetilde{K}_u$ are the ideal feedforward control gain matrices. When the ideal system is affected by the ideal control input signal $u^*(t)$ then, perfect output tracking error occur. Thus, we can get

$$e_y(t) = y_m(t) - y(t) = 0 \tag{1.32}$$

$$u^*(t) = \widetilde{K}_x x_m(t) + \widetilde{K}_u u_m(t) \tag{1.33}$$

$$y^*(t) = Cx^*(t) = y_m(t) \tag{1.34}$$

$$e_y(t) = Cx^*(t) - Cx(t) = Ce_x(t) \tag{1.35}$$

And the state error

$$e_x(t) = x^*(t) - x(t) \tag{1.36}$$

Taking the time derivative of 1.36, we get

$$\begin{aligned} \dot{e}_x(t) &= \dot{x}^*(t) - \dot{x}(t) \\ &= A^*(x^*)x^* + B^*(x^*)u^* - A(x)x(t) - B(x)u(t) \end{aligned} \tag{1.37}$$

Adding and subtracting $A(x)x^*$, replacing $u(t)$ and u^* by 1.26 and 1.33 respectively and adding and subtracting $B(x)\widetilde{K}_e e_y(t)$ and rearranging, gives

$$\begin{aligned} \dot{e}_x(t) &= A(x)e_x(t) - B(x)K(t)r(t) + B^*(x^*)\widetilde{K}_x x_m(t) + B^*(x^*)\widetilde{K}_u u_m(t) \\ &\quad + [A^*(x^*) - A(x)]x^* + B(x)\widetilde{K}_e e_y(t) - B(x)\widetilde{K}_e e_y(t) \end{aligned} \tag{1.38}$$

Adding and subtracting $B(x)u^*(t)$ and replacing $e_y(t)$ by 1.35 and rearranging, results in

$$\begin{aligned}\dot{e}_x(t) &= [A(x) - B(x)\widetilde{K}_eC]e_x(t) - B(x)(K(t) - \widetilde{K})r(t) + [A^*(x^*) - A(x)]x^*(t)\\ &+[B^*(x^*) - B(x)]\widetilde{K}_x x_m(t) + [B^*(x^*) - B(x)]\widetilde{K}_u u_m(t) \qquad (1.39)\end{aligned}$$

Equation 1.39 can be rewritten as

$$\begin{aligned}\dot{e}_x(t) &= [A(x) - B(x)\widetilde{K}_eC]e_x(t) - B(x)(K(t) - \widetilde{K})r(t)\\ &+[A^*(x^*) - A(x)]x^*(t) + [B^*(x^*) - B(x)]u^*(t) \qquad (1.40)\end{aligned}$$

Where $\widetilde{K} \in \mathbb{R}^{m\times(m+n_m+p_m)}$ defined as

$$\widetilde{K} = [\widetilde{K}_e \quad \widetilde{K}_x \quad \widetilde{K}_u] \qquad (1.41)$$

Finally, substituting $K(t) = K_P(t) + K_I(t)$ yields

$$\begin{aligned}\dot{e}_x(t) &= [A(x) - B(x)\widetilde{K}_eC]e_x(t) - B(x)(K_I(t) - \widetilde{K})r(t) - BK_P(t)r\\ &+[A^*(x^*) - A(x)]x^*(t) + [B^*(x^*) - B(x)]u^*(t) \qquad (1.42)\end{aligned}$$

1.4 Stability Analysis

In this section, we extend the standard theorem of stability used in DSAC [22] with the new WASP conditions for a class of systems of the form 1.1 with not necessarily symmetric matrix CB.

Theorem 1.2. *Under the WASP conditions with symmetric W, the Adaptive Control law represented by 1.14-1.24 ensures the boundedness of the system signals under closed-loop operation, and results in asymptotic convergence of the state and output tracking errors, in the sense that*

$$\|e_y\| \to 0 \quad \textit{and} \quad \|e_x\| \to 0 \quad \textit{as} \quad t \to \infty \qquad (1.43)$$

where $\|\cdot\| \to 0$ *denotes the standard Euclidean norm of a vector*

Proof. Let $V \in \mathbb{R}$ be a positive definite function and continuously differentiable

$$V(t) = e_x^T(t)\,P(x)\,e_x(t) + tr\left\{W[K_I(t) - \widetilde{K}]\Gamma_I^{-1}[K_I(t) - \widetilde{K}]^T\right\} \qquad (1.44)$$

Taking the time derivative of 1.44 results in

$$\begin{aligned}\dot{V}(t) &= e_x^T(t)\,\dot{P}(x)\,e_x(t) + \dot{e}_x^T(t)\,P(x)\,e_x(t) + e_x^T(t)\,P(x)\,\dot{e}_x(t)\\ &+tr\{W\dot{K}_I(t)\,\mathit{\Gamma}^{-1}(K_I(t) - \tilde{K})^T + tr\{W(K_I(t) - \tilde{K})\mathit{\Gamma}^{-1}\dot{K}_I^T(t)\}\\ &+tr\left\{\dot{W}[K_I(t) - \widetilde{K}]\Gamma_I^{-1}[K_I(t) - \widetilde{K}]^T\right\} \qquad (1.45)\end{aligned}$$

Substituting $\dot{e}_x$ from 1.42 into 1.45 gives

$$\begin{aligned}\dot{V}(t) &= e_x^T(t)\,\dot{P}(x)\,e_x(t)+\{[A(x)-B(x)\widetilde{K}_eC]e_x(t)-B(x)(K_I(t)-\widetilde{K})r(t)\\&-BK_P(t)r+[A^*(x^*)-A(x)]x^*(t)+[B^*(x^*)-B(x)]u^*(t)\}^T P(x)\,e_x(t)\\&+e_x^T(t)\,P\{[A(x)-B(x)\widetilde{K}_eC]e_x(t)-B(x)(K_I(t)-\widetilde{K})r(t)-BK_P(t)r\\&+[A^*(x^*)-A(x)]x^*(t)+[B^*(x^*)-B(x)]u^*(t)\}\\&+tr\{W\dot{K}_I(t)\,\mathit{\Gamma}^{-1}(K_I(t)-\tilde{K})^T+tr\{W(K_I(t)-\tilde{K})\mathit{\Gamma}^{-1}\dot{K}_I^T(t)\}\\&+tr\left\{\dot{W}[K_I(t)-\widetilde{K}]\Gamma_I^{-1}[K_I(t)-\widetilde{K}]^T\right\}\end{aligned}\tag{1.46}$$

Substituting e_y from 1.35, K_P from 1.27, $\dot{K}_I$ from 1.28 into 1.46 results in

$$\begin{aligned}\dot{V}(t) &= e_x^T(t)\,\dot{P}(x)\,e_x(t)+e_x^T(t)\,(A(x)-B(x)\widetilde{K}_eC)^T P(x)e_x(t)\\&+\{[A^*(x^*)-A(x)]x^*(t)+[B^*(x^*)-B(x)]u^*(t)\}^T P(x)e_x(t)\\&-[B(x)S^T diag[SCe_x(t)r^T(t)]\Gamma_P r(t)]^T\times P(x)e_x(t)\\&-r^T(K_I(t)-\widetilde{K})^T B^T(x)P(x)e_x(t)+e_x^T(t)\,P(x)[A(x)-B(x)\widetilde{K}_eC]e_x(t)\\&+e_x^T(t)\,P(x)\{[A^*(x^*)-A(x)]x^*(t)+[B^*(x^*)-B(x)]u^*(t)\}\\&-e_x^T(t)\,P(x)B(x)S^T diag[SCe_x(t)r^T(t)]\times\Gamma_P r(t)\\&-e_x^T(t)\,P(x)B(x)(K_I(t)-\widetilde{K})r(t)+tr\{WS^T diag[SCe_x(t)r^T(t)]\Gamma_I\\&\times\Gamma_I^{-1}(K_I(t)-\tilde{K})^T\}+tr\{W(K_I(t)-\tilde{K})\Gamma_I^{-1}\times\Gamma_I diag[SCe_x(t)r^T(t)]S\}\\&+tr\left\{\dot{W}[K_I(t)-\widetilde{K}]\Gamma_I^{-1}[K_I(t)-\widetilde{K}]^T\right\}\end{aligned}\tag{1.47}$$

Rearranging, using the WSP conditions 1.10 and 1.11, 1.47 can be simplified to

$$\begin{aligned}\dot{V}(t) &= -e_x^T(t)\,Qe_x(t)-2e_x^T(t)\,C^TW^T\times S^T diag[SCe_x(t)r^T(t)]\Gamma_P r(t)\\&-r^T(t)(K_I(t)-\widetilde{K})^T WCe_x(t)-e_x^T(t)\,C^TW^T(K_I(t)-\widetilde{K})r(t)\\&+e_x^T(t)\,P(x)\{[A^*(x^*)-A(x)]x^*(t)+[B^*(x^*)-B(x)]u^*(t)\}\\&+\{[A^*(x^*)-A(x)]x^*(t)+[B^*(x^*)-B(x)]u^*(t)\}^T P(x)e_x(t)\\&+tr\{WS^T diag[SCe_x(t)r^T(t)]\times(K_I(t)-\tilde{K})^T\}+tr\{W(K_I(t)-\tilde{K})\\&\times diag[SCe_x(t)r^T(t)]S\}+tr\left\{\dot{W}[K_I(t)-\widetilde{K}]\Gamma_I^{-1}[K_I(t)-\widetilde{K}]^T\right\}\end{aligned}\tag{1.48}$$

The following terms cancels each other

$$tr\{WS^T diag[SCe_x(t)r^T(t)](K_I(t)-\tilde{K})^T\}-r^T(t)(K_I(t)-\widetilde{K})^T WCe_x(t)=0 \tag{1.49}$$

and similarly

$$tr\{W(K_I(t)-\tilde{K})diag[SCe_x(t)r^T(t)]S\}-e_x^T(t)\,C^TW^T(K_I(t)-\widetilde{K})r(t)=0 \tag{1.50}$$

Thus, 1.48 can be simplified to

$$\begin{aligned} \dot{V}(t) \;=\; & -e_x^T(t)\,Qe_x(t) - 2e_x^T(t)\,C^TW^T \times S^T diag[SCe_x(t)r^T(t)]\Gamma_P r(t) \\ & +e_x^T(t)\,P(x)\{[A^*(x^*) - A(x)]x^*(t) + [B^*(x^*) - B(x)]u^*(t)\} \\ & +\{[A^*(x^*) - A(x)]x^*(t) + [B^*(x^*) - B(x)]u^*(t)\}^T P(x)e_x(t) \\ & +tr\left\{\dot{W}[K_I(t) - \widetilde{K}]\Gamma_I^{-1}[K_I(t) - \widetilde{K}]^T\right\} \end{aligned} \tag{1.51}$$

According to [22], for those nonlinear systems with parameters that vary slowly in comparison with the control dynamics, one may assume that

$$A(x) = A^*(x^*) \text{ and } B(x) = B^*(x^*) \tag{1.52}$$

In this case, 1.51 can be simplified to

$$\begin{aligned} \dot{V}(t) \;=\; & -e_x^T(t)\,Qe_x(t) - 2e_x^T(t)\,C^TW^T \times S^T diag[SCe_x(t)r^T(t)]\Gamma_P r(t) \\ & -M(x) \end{aligned} \tag{1.53}$$

where $M(x) = tr\left\{\dot{W}[K_I(t) - \widetilde{K}]\Gamma_I^{-1}[K_I(t) - \widetilde{K}]^T\right\}$. Note that $M(x)$ equal to zero if the condition (C1) is satisfied or nonnegative-definit if (C2) is satisfied.

The Lyapunov derivative $\dot{V}(t)$ in 1.53 is negative definite with respect to $e_x(t)$ but only semidefinite with respect to the state-space $\{e_x(t), K_I(t)\}$. According to Lyapunov stability theory, all dynamic values are bounded. Also, based on LaSalle's Invariance Principle [56], all states, errors and adaptive gains are bounded, all system trajectories reache asymptotically the domain defined by $\dot{V}(t) \equiv 0$. Since $\dot{V}(t)$ is negative definite in $e_x(t)$, then, the system ends with $e_x(t) \equiv 0$ which implies $e_a(t) \equiv 0$. Therefore, the adaptive control system demonstrates asymptotic convergence of the state and output error and boundedness of the adaptive gains. □

Remark 1.1. *When disturbances are considered, the well-know σ terms [57] can be included to avoid divergence of the integral adaptive control gains. By adding the σ terms the time-varying integral adaptive control gains are obtained as follows*

$$\dot{K}_{I_e}(t) = diag\{e_y e_y^T\}\Gamma_{I_e} - \sigma_e K_{I_e}(t) \tag{1.54}$$

$$\dot{K}_{I_x}(t) = R^T[diag\{Re_y x_m^T\}\Gamma_{I_x} - diag\{\sigma_x R K_{I_x}(t)\}] \tag{1.55}$$

$$\dot{K}_{I_u}(t) = T^T[diag\{Re_y u_m^T\}\Gamma_{I_u} - diag\{\sigma_u T K_{I_u}(t)\}] \tag{1.56}$$

and similarly

$$\dot{K}_I(t) = S^T[diag\{Se_y r^T(t)\}\Gamma_I - diag\{\sigma_I S K_I(t)\}] \tag{1.57}$$

where $\sigma_e \in \mathbb{R}^{(m\times m)}, \sigma_x \in \mathbb{R}^{(n_m\times n_m)}, \sigma_u \in \mathbb{R}^{(p_m\times p_m)}$, and $\sigma_I \in \mathbb{R}^{(m+n_m+p_m)\times(m+n_m+p_m)}$ are small positive coefficient matrices. With this modification, the Lyapunov derivative function becomes

$$\begin{aligned}\dot{V}(t) &= -e_x^T(t)\,Qe_x(t) - 2e_x^T(t)\,C^T W^T \times S^T diag[SCe_x(t)r^T(t)]\Gamma_P r(t) - M(x)\\ &\quad -2tr\{WS^T diag[\sigma_I S K_I(t)\}\Gamma_I^{-1}(K_I(t) - \tilde{K})^T\}\end{aligned} \tag{1.58}$$

Thus, according to Lyapunov stability theory and LaSalle invariance principle, the application of the DSAC algorithm with the σ terms results in bounded error tracking.

1.5 Application to quadrotor vehicle

For the application of the proposed ASC, the following dynamic of the quadrotor with respect to its attitude (i.e. Pitch, roll and yaw) [58] is considered:

$$J\dot{\omega} = J\omega \times \omega + u \tag{1.59}$$

$$\omega = N(a)\dot{a} \tag{1.60}$$

Where $\times$ represents the cross product, $J \in R^{3\times3}$ and ω denote the inertia matrix and angular velocity respectively, $u \in R^3$ denote the quadrotor control vector. $a = (\phi\ \theta\ \psi)^T \in R^3$ presents the three Euler angles of the quadrotor with $-\pi/2 < \theta < \pi/2$ and $-\pi/2 < \phi < \pi/2$, $N(a)$ is defined as:

$$N(a) = \begin{pmatrix} 0 & \cos(\phi) & -\sin(\phi) \\ 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \frac{\sin(\phi)}{\cos(\theta)} & \frac{\cos(\phi)}{\cos(\theta)} \end{pmatrix}^{-1} \tag{1.61}$$

Taking the time derivative of (1.60) to get

$$\dot{\omega} = \dot{N}(a)\dot{a} + N(a)\ddot{a} \tag{1.62}$$

Substituting (1.62) and (1.60) into (1.59) and rearranging gives

$$J\dot{N}(a)\dot{a} + JN(a)\ddot{a} - JN(a)\dot{a} \times N(a)\dot{a} = u \tag{1.63}$$

recall that $JN(a)\dot{a}\times$ can be represented as a skew symmetric matrix, thus one have

$$JN(a)\ddot{a} + \left\{J\dot{N}(a) - \{JN(a)\dot{a}\times\}N(a)\right\}\dot{a} = u \tag{1.64}$$

Equation (1.64) can be rewritten as follows

$$M(a)\ddot{a} + L(a, \dot{a})\dot{a} = u \tag{1.65}$$

where $M(a) = JN(a)$ and $L(a, \dot{a}) = \{J\dot{N}(a) - \{JN(a)\dot{a}\times\}N(a)\}$. From (1.65), taking $x_1 = a$ and $x_2 = \dot{a}$, the nonlinear state space model of the form (1.1) can be obtained with,

$$A(x) = \begin{bmatrix} 0 & I_3 \\ 0 & -M(x_1)^{-1}L(x_1, x_2) \end{bmatrix}, \; B(x) = \begin{bmatrix} 0 \\ M(x_1)^{-1} \end{bmatrix} \tag{1.66}$$

$$C = \begin{bmatrix} I_3 & I_3 \end{bmatrix}, \; x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \tag{1.67}$$

In this particular case, by choosing $W = I_3$ one can easily show that the product WCB is PDS, as follows

$$WC_qB_q = I_3 \begin{bmatrix} I_3 & I_3 \end{bmatrix} \begin{bmatrix} 0 \\ M^{-1}(x) \end{bmatrix} = M^{-1}(x) > 0 \tag{1.68}$$

In this case, since CB is symmetric, then W is chosen to be the unity matrix, thus, the WSP condition coincides with the standard SP condition.

A simple selection of matrices that satisfy (1.2-1.4) are

$$M = \begin{bmatrix} I_3 \\ -I_3 \end{bmatrix} \qquad N = \begin{bmatrix} I_3 & 0 \end{bmatrix} \tag{1.69}$$

Computing

$$\dot{z} = NA_q(x, \dot{x})Mz = -I_3z = -z \tag{1.70}$$

which shows that the zero dynamics is stable and the quadrotor nonlinear dynamics is minimum-phase, therefore, based on theorem 1, the system is WASP or simply ASP since CB is symmetric. Thus, the application of the DSAC control methodology would result in a stable closed-loop system.

1.6 Simulation results

The DSAC controller parameters are, $\Gamma_{p_e} = \Gamma_{p_\xi} = I_3, \Gamma_{p_u} = 1e^6 I_3$ and $\Gamma_{I_e} = \Gamma_{I_\xi} = 100I_3, \Gamma_{I_u} = 1e^{-2}I_3$, the reference model is chosen as $G_m(s) = 1/(s^2 + 3s + 2)$ for the attitude angles (i.e. roll, pitch and yaw) of the Quadrotor. The initial conditions are set to zero. A sinusoidal trajectory is chosen for the roll and pitch angles. The desired yaw angle is 0.6 rad. Figure 1.1 shows the evolution of the Quadrotor. One can see that the proposed controller was able to stabilize the angles to their desired values. The efficiency of the proposed controller is verified, and the results are perfect.

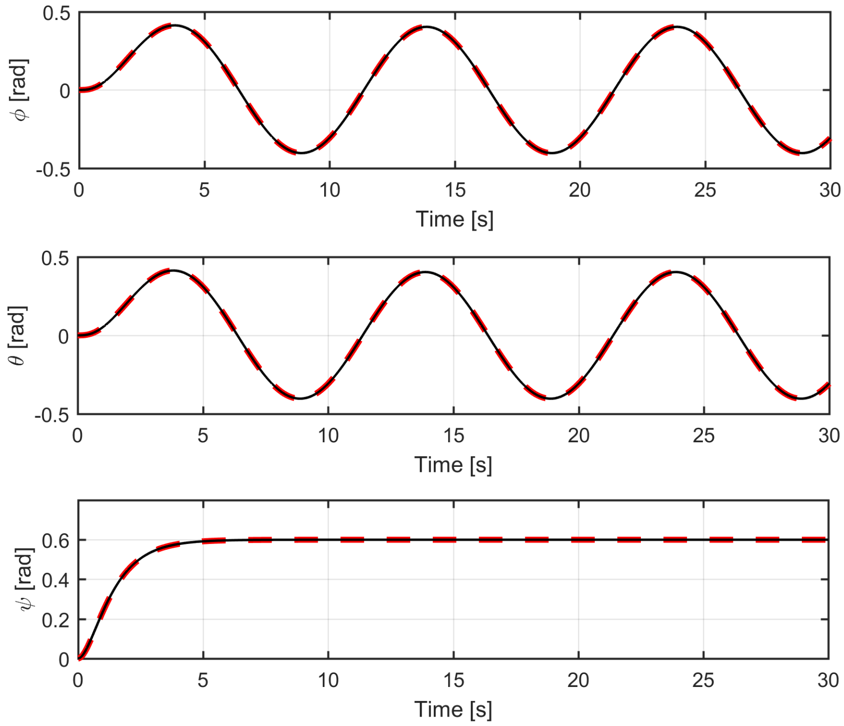

Figure 1.1: Evolution of the Quadrotor attitude angles: desired trajectory (red curve), actual trajectory (blue curve)

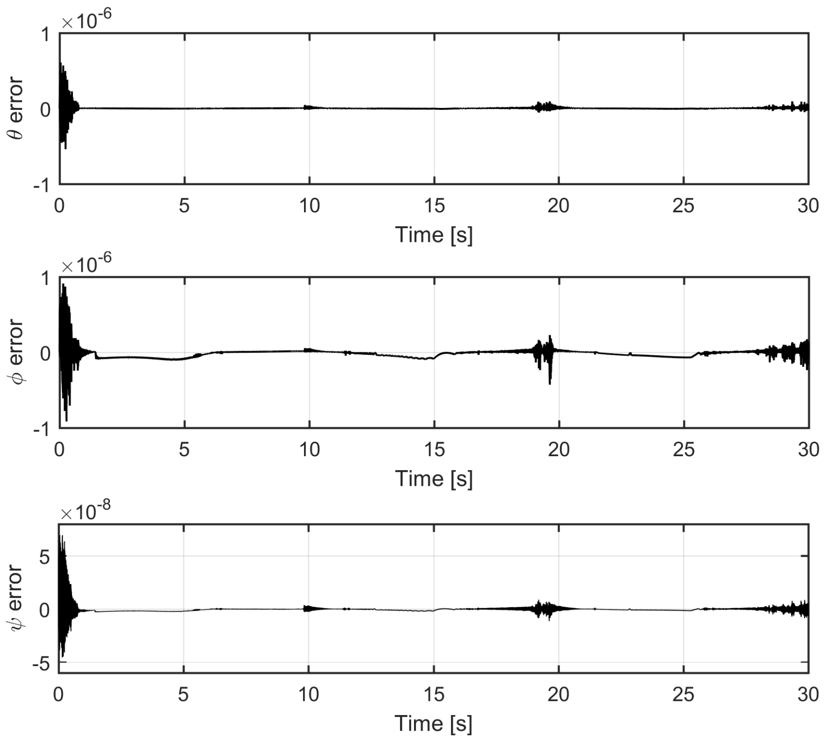

Figure 1.2: Tracking errors of the Quadrotor attitude angles

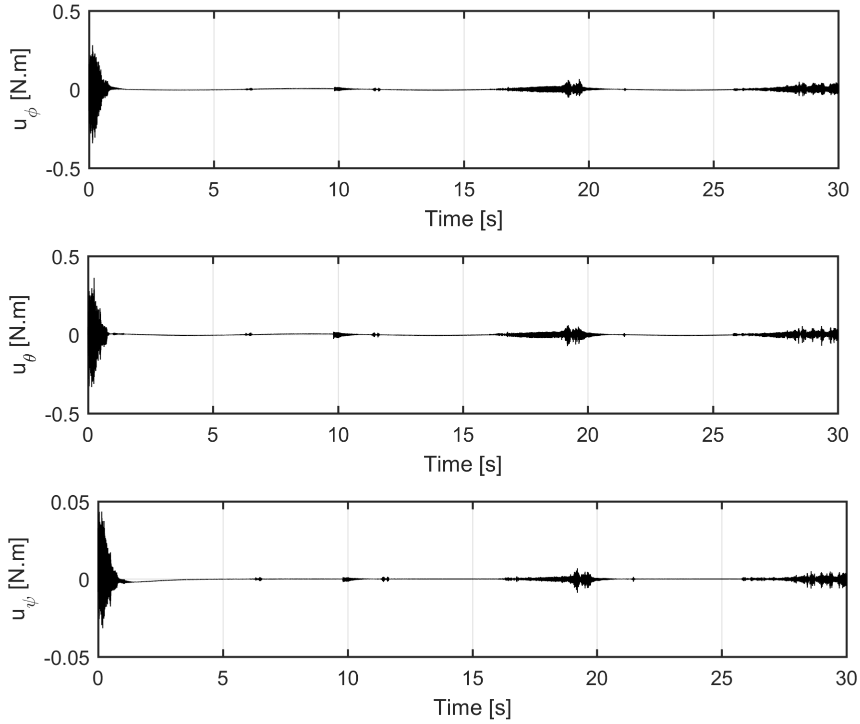

Figure 1.3: Control inputs of the Quadrotor

Figure 1.2 and figure 1.3 represent the tracking errors and control signals of the quadrotor, respectively. The tracking errors are very small and the convergence to zero is obtained as expected. It can be clearly seen that the control signals applied to the quadrotor in figure 1.3 is acceptable in value and physically realizable, and this is very important when moving from simulation toward real application.

1.7 Conclusion

An extension in decentralized simple adaptive control with the new relaxed W-Passivity conditions is presented in this chapter. A formal proof of stability of the DSAC is established using the new WASP theorem for square nonlinear systems with not necessarily symmetric matrix CB. An application of the proposed DSAC to a Quadrotor system is given in order to show the effectiveness of the DSAC method.

Chapter 2

Extension of the ASP and DSAC to the Fractional domain

> *To be alone means that you avoid bad company. But to have a true friend is better than being alone.*
>
> Umar ibn al-Khaab

2.1 Introduction

In this chapter, one further improve the flexibility of the DSAC controller by considering the fractional derivative of its adaptive gains. First, the ASPR conditions for the design of a stable fractional adaptive system will be established. A fractional order PFC to realize an augmented ASPR system is then introduced, and a new FODSAC controller will be proposed. The stability analysis of the proposed control loop is presented and a simulation example comparing the standard DSAC to the new FODSAC is provided, showing the performance of the proposed method.

2.2 Preparation

In this section we recall and extend some existing definitions and lemmas about fractional calculus and almost passivity theory. The extended results will be exploited in the subsequent development.

2.2.1 Fractional calculus

Definition 2.1 (Caputo Fractional Derivative [59]). *The Caputo fractional derivative of order $\alpha \in \mathbb{R}^+$ on the half axis $\mathbb{R}^+$ is defined as follows*

$$ {}_{t_0}^{C}D_t^{\alpha} x(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{x^n(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \; t > t_0 \tag{2.1} $$

with $n = \min\{k \in \mathbb{N}/k > \alpha\}, \alpha > 0$. Γ is the well known Gamma function.

Lemma 2.1. *[60] Let $x(t) \in \mathbb{R}^n$ be a differentiable vector. Then, for any time instant $t \geqslant t_0$*

$$ \frac{1}{2} {}_{t_0}^{C}D_t^{\alpha} \left[x^T(t)x(t) \right] \leqslant x^T(t) {}_{t_0}^{C}D_t^{\alpha} x(t), \; \forall \alpha \in (0,1) \tag{2.2} $$

Lemma 2.2. *[61] Let $x(t) \in \mathbb{R}^n$ be a vector of differentiable functions. Then, for any time instant $t \geqslant t_0$, the following relationship holds*

$$ \frac{1}{2} {}_{t_0}^{C}D^{\alpha} \left(x^T(t)Px(t) \right) \leqslant x^T(t)P \, {}_{t_0}^{C}D^{\alpha} x(t), \; \forall \alpha \in (0,1] \tag{2.3} $$

where $P \in \mathbb{R}^{n\times n}$ is a constant positive definit symmetric matrix.

Lemma 2.3. *[61] Let $A(t) \in \mathbb{R}^{m\times n}$ be differentiable. Then, $\forall t \geqslant t_0$, the following relationship holds*

$$ {}_{t_0}^{C}D_t^{\alpha} \left[tr\left\{ A^T(t)A(t) \right\} \right] \leqslant 2tr\left\{ A^T(t) {}_{t_0}^{C}D_t^{\alpha} A(t) \right\}, \forall \alpha \in (0,1]. \tag{2.4} $$

Remark 2.1. We remark that lemma 2.3 is limited to calculating the fractional derivative of the trace of a rectangular matrix and its transpose only. Below, we extend the results to the case where a positive definite diagonal matrix appears between this product.

Lemma 2.4. *Let $A(t) \in \mathbb{R}^{m\times n}$ be differentiable, let $W > 0$ be a diagonal matrix and $W \neq \lambda I$. Then, $\forall \alpha \in (0,1]$, $\forall t \geqslant t_0$, the following relationship holds*

$$ {}_{t_0}^{C}D_t^{\alpha} \left[tr\left\{ A^T(t)WA(t) \right\} \right] \leqslant 2tr\left\{ A^T(t)W \, {}_{t_0}^{C}D_t^{\alpha} A(t) \right\}. \tag{2.5} $$

Proof. See Appendix A □

2.2.2 Stability of fractional order systems

A recent results [61] regarding the stability analysis of FOS when the fractional derivative of the Lyapunov function is negative semidefinite can be stated in theorem 2.1. A FOS using the Caputo derivative can be expressed in a general form as:

$$ {}_{t_0}^{C}D_t^{\alpha} x(t) = f(x,t) \tag{2.6} $$

In this study we will consider $\alpha \in (0,1)$

Definition 2.2. *A continuous function $\gamma : [0, t) \to [0, \infty)$ is said to belong to class-K if it is strictly increasing and $\gamma(0) = 0$ [62].*

Theorem 2.1. *[61] Let $x = 0$ be an equilibrium point for the non autonomous FOS (2.6). Assume that there exists a continuous Lyapunov function $V(x(t), t)$ and a scalar class-K function $\gamma_1(\cdot)$ such that, $\forall x \neq 0$*

$$\gamma_1(\|x(t\|) \leqslant V(x(t), t) \tag{2.7}$$

and

$${}_{t_0}^{C}D_t^{\beta} V(x(t), t) \leqslant 0, \ \text{with } \beta \in (0, 1] \tag{2.8}$$

then the origin of the system 2.6 is Lyapunov stable. If, furthermore, there is a scalar class-K function $\gamma_2(\cdot)$ satisfying

$$V(x(t), t) \leqslant \gamma_2(\|x(t)\|) \tag{2.9}$$

then the origin of the system 2.6 is Lyapunov uniformly stable.

2.2.3 Almost Strict Passivity Extension to Fractional Order System

In this section we present and generalize some of the existing results of the concept of almost passivity theory from ordinary to fractional order LTI systems. Consider the controllable and observable LTI system

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u_p(t) \\ y_p(t) &= C_p x_p(t) + D_p u_p(t) \end{aligned} \tag{2.10}$$

where $x_p(t) \in \mathbb{R}^n$ and $y_p(t) \in \mathbb{R}^m$ represent the state vector and output vector respectively, $u_p(t) \in \mathbb{R}^m$ is the control input. $A_p \in \mathbb{R}^{n \times n}$, $B_p \in \mathbb{R}^{n \times m}$, $C_p \in \mathbb{R}^{m \times n}$ and $D_p \in \mathbb{R}^{m \times m}$.

Definition 2.3. *[63] An $m \times m$ transfer function matrix $G_p(s)$ is called SPR if: (i) All elements of $G_p(s)$ are analytic in $Re(s) \geqslant 0$, (ii) $G_p(s)$ is real for real s, (iii) $G_p(s) + G_p^{T*}(s) > 0$ for $Re(s) \geqslant 0$ and finite s or ($G_p(s) + G_p^{T*}(s) > \varepsilon I$ for some $\varepsilon > 0$ when $G_p(s)$ is proper).*

It was shown that if (2.10) is a minimal realization of $G_p(s)$ which is SPR, then one can ensure the following relations [64]

$$PA_p + A_p^T P = -Q - L^T L < 0 \tag{2.11}$$

$$PB_p = C_p^T - L^T W \tag{2.12}$$

$$D_p + D_p^T = W^T W \tag{2.13}$$

Solving (2.12) for L and substituting this and (2.13) into (2.11) gives

$$PA_p + A_p^T P + [PB_p - C_p^T][D_p + D_p^T]^{-1}[B_p^T P - C_p] = -Q < 0 \tag{2.14}$$

where $L \in \mathbb{R}^{m\times n}$, $W \in \mathbb{R}^{m\times m}$,and where $P \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$ are positive definite matrices. Also C_p and B_p are assumed to be of maximal rank. For strictly proper systems when $D_p = 0$, we get $W = 0$, and including $L^T L$ in Q, thus, the SPR relations become

$$PA_p + A_p^T P = -Q < 0 \tag{2.15}$$

$$PB_p = C_p^T \tag{2.16}$$

In [65], it was also shown that the classical SPR definitions and concepts apply to FOS as well. Based on these results we have the following definitions and lemmas.

Definition 2.4. *A FO LTI system with the state-space representation*

$$\begin{aligned} &{}_{t_0}^{C}D_t^{\alpha} x_s(t) = A_s x_s(t) + B_s u(t) \\ &y_s(t) = C_s x_s(t) \end{aligned} \tag{2.17}$$

where $A_s \in \mathbb{R}^{n\times n}$, $B_s \in \mathbb{R}^{n\times m}$ and $C_s \in \mathbb{R}^{m\times n}$ is called Strictly Passive (SP) and its transfer function matrix $G_s(s) = C(s^\alpha I - A)^{-1}B$ Strictly Positive Real (SPR) if it simultaneously satisfies the time-domain relations (2.15)-(2.16).

Definition 2.5. *Let $G_s(s)$ be an $m \times m$ fractional order transfer function matrix. Let $\{A_s, B_s, C_s\}$ be the minimal realization of $G_s(s)$, Assume that there exists a positive definite constant gain matrix, $\tilde{K}_e$, such that the closed-loop system*

$$\begin{aligned} &{}_{t_0}^{C}D_t^{\alpha} x_s(t) = (A_s - B_s\tilde{K}_e C_s)x_s(t) + B_s u(t) \\ &y_s(t) = C_s x_s(t) \end{aligned} \tag{2.18}$$

is Strictly passive and its transfer function SPR. Then, the original FO system $\{A_s, B_s, C_s\}$ is called ASP and its transfer function matrix $G_s(s)$ ASPR.

Lemma 2.5. *[65] Let a fractional order transfer function matrix $G_a(s)$ be ASPR and let $\tilde{K}_e$ be a constant positive gain that satisfies 2.18. Then the closed loop denoted by $G_c(s)$ is SPR for any gain K_e that satisfies $K_e > \tilde{K}_e$.*

Remark 2.2. The fractional order ASPR plant is not necessarily stable, actually, the fictitious gain K_e will stabilize it.

Lemma 2.6. *Any proper (but not strictly proper) and strictly minimum-phase $m \times m$ fractional order transfer matrix $G_s(s)$ with the realization $\{A_s, B_s, C_s, D_s\}$ is ASPR.*

Proof. The detailed proof in [66] for integer system case apply for FOS as well, as shown in [65], any ASPR plant must also be proper which completes the proof. □

Lemma 2.7. *[66] Let $G(s)$ be any $m \times m$ transfer matrix of arbitrary MacMillan degree. $G(s)$ is not necessarily stable or minimum-phase. Let $H(s)$ be any dynamic stabilizing controller. Then*

$$G_a(s) = G(s) + H^{-1}(s) \tag{2.19}$$

is ASPR if the MacMillan degree of $G(s)$ is p/p or $(p-m)/p$, for any p.

2.3 Realization of ASPR fractional order system

Consider a strictly proper non ASPR FO-LTI system given by

$$G : \begin{cases} {}_{t_0}^{C}D_t^{\alpha} x_p(t) = A_p x_p(t) + B_p u(t) \\ y_p(t) = C_p x_p(t) \end{cases} \tag{2.20}$$

where $x_p(t) = col\{x_1, \cdots, x_{n_p}\} \in \mathbb{R}^{n_p}$, $y_p(t) = col\{y_1, \cdots, y_m\} \in \mathbb{R}^m$ and $u_p(t) = col\{u_1, \cdots, u_m\} \in \mathbb{R}^m$. A_p, B_p and C_p are bounded matrix with appropriate dimension. In order for this system to satisfy the ASPR conditions, we consider augmentation with a FO-PFC of the form

$$F : \begin{cases} {}_{t_0}^{C}D_t^{\alpha} x_f(t) = A_f x_f(t) + B_f u(t) \\ y_f(t) = C_f x_f(t) + D_f u(t) \end{cases} \tag{2.21}$$

The parallel interconnection with this FO-PFC is shown in figure 2.1. The resulting augmented system $G_a = G + F$ is expressed as

$$G_a : \begin{cases} {}_{t_0}^{C}D_t^{\alpha} x_a(t) = A_a x_a(t) + B_a u(t) \\ y_a(t) = C_a x_a(t) + D_f u(t) \end{cases} \tag{2.22}$$

where $x_a = col\{x_p, x_f\} \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $A_a = diag\{A_p \ A_f\}$, $B_a = col\{B_p \ \ B_f\}$ and $C_a = [C_p \ \ C_f]$.

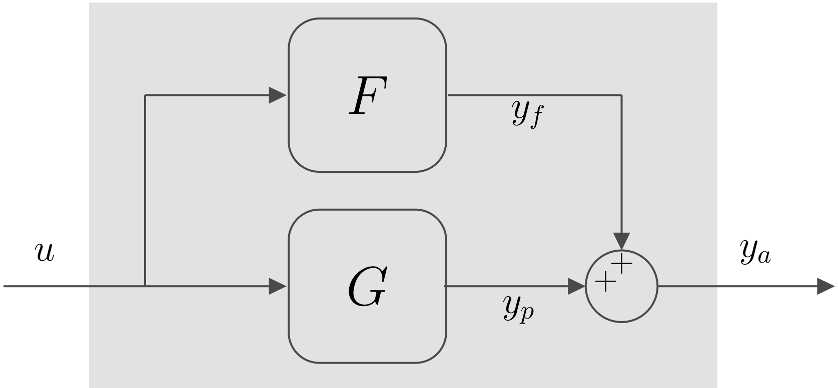

Figure 2.1: Augmented Fractional order system with a FO-PFC

The FO-PFC is designed so that the augmented FO system is ASPR. Concerning the existence and design of the FO-PFC, we have the following theorem

Theorem 2.2. *Assume that the FO LTI system (2.20) can be stabilized by a linear feedback dynamic of the form*

$$\begin{aligned}
&{}_{t_0}^{C}D_t^{\alpha} x_d\,(t) = A_d x_d\,(t) + B_d y_p\,(t) \\
&y_d\,(t) = C_d x_d\,(t) + D_d y_p(t) \\
&u(t) = -y_d\,(t)\,.
\end{aligned} \tag{2.23}$$

The inverse system of (2.23) with $u(t)$ as input and $y_f(t) = -y_p$ as output is given by

$$\begin{aligned}
&{}_{t_0}^{C}D_t^{\alpha} x_f\,(t) = (A_d - B_d D_d^{-1})C_d - B_d D_d^{-1} u(t) \\
&y_f\,(t) = D_d^{-1} C_d x_f + D_d^{-1} u(t)
\end{aligned} \tag{2.24}$$

Consider an augmentation with the inverse system as a PFC, as shown in figure 2.1. Then the augmented FO system is minimum phase.

Proof. Using the control input $u(t) = -y_d(t)$, one obtains the closed loop system ${}_{t_0}^{C}D_t^{\alpha} x_0 = A_0 x_0$, where $x_0 = col\{x_p, x_d\}$ and

$$A_0 = \begin{bmatrix} A_p - B_p D_d C_p & -B_p C_d \\ B_d C_p & A_d \end{bmatrix} \tag{2.25}$$

which is a stable system by assumption.

Now, consider the inverse of (2.23) given in (2.24), and consider the parallel interconnection (figure 2.1) with the inverse system as a FO-PFC, the augmented open loop system can then be given by (2.22), where $A_a = diag\{A_p \;\; A_d - B_d D_d^{-1} C_d\}$, $B_a = col\{B_p \; B_d D_d^{-1}\}$, $C_a = [C_p \;\; D_d^{-1} C_d]$ and $D_f = D_d^{-1}$ with $y_a(t) = y_p(t) + y_f(t)$.

The zeros of this augmented FOS are the eigenvalues of the matrix $A_1 = A_a - B_a D_f^{-1} C_a$ [67], substituting the corresponding matrices to obtain

$$A_1 = \begin{bmatrix} A_p - B_p D_d C_p & -B_p C_d \\ B_d C_p & A_d \end{bmatrix} \tag{2.26}$$

By Comparison of (2.26) with (2.25) we find $A_1 = A_0$, thus the zeros of the augmented FOS are stable and therefore is minimum phase. $\square$

This theorem indicates that using the inverse of a stabilizing controller as a FO-PFC makes the resulting augmented system minimum phase. Furthermore, since $deg[F(s)] = p/p$ (proper) then by lemma 2.7 we obtain $deg[G_a(s) = (p+n)/(p+n)]$, therefore, using lemma 2.6 the augmented FO system is ASP.

We focus in the subsequent developments on designing controller for proper and strictly proper systems (i.e. for both cases $D = 0$ and $D > 0$). Thus, a more general proof for theorem (2.2) when $D = 0$ can be given. If G can be stabilized by a linear stabilizing dynamic F such that the closed loop is $G_{cl} = (I + FG)^{-1}F = (G + F^{-1})^{-1} = G_a^{-1}$, then $\exists K_e > 0$ (sufficiently large) such that $G_1 = G_{cl} + K_e$ is SPR. Here $G_1^{-1} = (G_{cl} + K_e)^{-1} = (G_a^{-1} + K_e)^{-1} = (I + G_a K_e)^{-1} G_a$ is also SPR, therefore $G_a = G + F^{-1}$ is ASPR.

2.4 Fractional adaptive controller: formulation and design

Assumption 2.1. *There exist a known FO-PFC of the form (2.21) that can render the augmented system (2.22) ASPR, and the resulting strictly proper augmented system can be made SPR via constant feedback, in other words the augmented system (2.22) satisfies the SPR conditions 2.11 - 2.13 .*

Remark 2.3. *We call any minimum phase system with relative degree $r \in (0\ ,1)$ ASPR.*

Under assumption 2.1, there exist a static feedback $\tilde{K}_e$ such that the resulting augmented closed-loop system

$$\begin{aligned} &{}_{t_0}^{C}D_t^{\alpha} x_a(t) = (A_a - B_a[I + \tilde{K}_e D_f]^{-1} \tilde{K}_e C_a) x_a(t) + B_a[I + \tilde{K}_e D_f]^{-1} u_d(t) \\ &y_a\,(t) = [I + D_f \tilde{K}_e]^{-1} C_a x_a\,(t) + D_f [I + D_f \tilde{K}_e]^{-1} u_d(t) \end{aligned} \tag{2.27}$$

satisfies relations (2.11 - 2.13) .

Thus, if the control input

$$u(t) = -\tilde{K}_e y_a(t) + u_d(t) \tag{2.28}$$

can be designed, then one can obtain a stable control system.

Now, we want to design under assumption 2.1 a fractional *adaptive feedback* control system based on the DSAC methods. The control objective is shown below.

2.4.1 Control objectif

Consider the augmented system defined in (2.22), if we define the augmented output to be controlled

$$y_a(t) = y_p(t) + y_f(t) \tag{2.29}$$

Then, the objective is to design an adaptive control signal $u(t) = K(t)r(t)$ (to be defined subsequently) based on the proposed FO-DSAC adaptation laws, which makes the augmented system output (2.29) *track* the output of a given ideal fractional order model reference which leads to the boundedness of all signals in the closed-loop system. The model reference is given by

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} x_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \tag{2.30}$$

where $A_m \in \mathbb{R}^{n_m \times n_m}$, $B_m \in \mathbb{R}^{n_m \times q_m}$ and $C_m \in \mathbb{R}^{m \times n_m}$. $x_m(t) = col\{x_{m_1}, \ldots, x_{m_{n_m}}\} \in \mathbb{R}^{n_m}$, $y_m(t) = col\{y_{m_1}, \ldots, y_{m_m}\} \in \mathbb{R}^m$ and $u_m(t) = col\{u_{m_1}, \ldots, u_{m_{q_m}}\} \in \mathbb{R}^{q_m}$ represent the model state vector, output vector and command input respectively. For this model the order n_m and the number of inputs q_m are both assumed to be multiples of m and thus satisfying $n_m = k_n m$ and $q_m = k_q m$.

Remark 2.4. This model can be much lower than the order of the plant and not necessarily square, it is permissible to have $n = dim(x_a) >> dim(x_m) = n_m$.

Remark 2.5. We define Perfect Tracking (PT) the case where $y_a(t)$ perfectly track $y_m(t)$ with zero steady state error, that is $y_m(t) = y_a(t)$. The PT possibility of occurrence will be discussed subsequently.

2.4.2 Algorithm and Adaptation Law of the FO-DSAC

As in the standard SAC, the FO-DSAC algorithm monitors the augmented output tracking error

$$e_a(t) = y_m(t) - y_a(t) \tag{2.31}$$

the adaptive control signal to be designed has the following form

$$u(t) = K_e\,(t)\,e_a\,(t) + K_x\,(t)\,x_m\,(t) + K_u\,(t)\,u_m\,(t) \tag{2.32}$$

where $K_e\,(t)$ is the main feedback gain that should guarantee the stability of the closed-loop system under assumption 2.1. $K_x\,(t)$ and $K_u\,(t)$ are additional feedforward gains, they allow good tracking without requiring large values of $K_e\,(t)$. Each of these control gains is a combination of proportional and integral gains defined as

$$K_e(t) = K_{p_e}(t) + K_{I_e}(t) \tag{2.33}$$

$$K_x(t) = K_{p_x}(t) + K_{I_x}(t) \tag{2.34}$$

$$K_u(t) = K_{p_u}(t) + K_{I_u}(t) \tag{2.35}$$

The components of the stabilizing control gain in (2.33) are adaptively computed as follow

$$K_{p_e}(t) = diag\{e_a e_a^T\}\Gamma_{P_e} \tag{2.36}$$

$${}_{t_0}^{C}D_t^{\alpha} K_{I_e}(t) = diag\{e_a e_a^T\}\Gamma_{I_e} \tag{2.37}$$

where $K_{p_e}(t)$, $K_{I_e}(t)\in\mathbb{R}^{m\times m}$.
The feedforward control gain components $K_{p_x}(t)$, $K_{I_x}(t) \in \mathbb{R}^{m\times n_m}$ and $K_{P_u}(t)$, $K_{I_u}(t) \in \mathbb{R}^{m\times q_m}$ are updated as follows

$$K_{p_x}(t) = R^T diag\{Re_a x_m^T\}\Gamma_{P_x} \tag{2.38}$$

$${}_{t_0}^{C}D_t^{\alpha} K_{I_x}(t) = R^T diag\{Re_a x_m^T\}\Gamma_{I_x} \tag{2.39}$$

$$K_{p_u}(t) = T^T diag\{Te_a u_m^T\}\Gamma_{P_u} \tag{2.40}$$

$${}_{t_0}^{C}D_t^{\alpha} K_{I_u}(t) = T^T diag\{Te_a u_m^T\}\Gamma_{I_u} \tag{2.41}$$

with $R = col\{I_m, I_m, ..., I_m\} \in \mathbb{R}^{n_m\times m}$, $T = col\{I_m, I_m, ..., I_m\} \in \mathbb{R}^{q_m\times m}$, where Γ_{P_e}, $\Gamma_{I_e} \in \mathbb{R}^{m\times m}$, Γ_{P_x}, $\Gamma_{I_x} \in \mathbb{R}^{n_m\times n_m}$ and Γ_{P_u}, $\Gamma_{I_u} \in \mathbb{R}^{q_m\times q_m}$ are time-invariant diagonal weighting matrices that control the rate of adaptation. Defining $K\,(t) \triangleq [K_e(t), K_x(t), K_u(t)] = K_I\,(t) + K_P\,(t)$ and $r(t) = col\{e_a(t), x_m(t), u_m(t)\}$, the adaptive control algorithm can be rewritten as

$$u\,(t) = K\,(t)\,r\,(t) \tag{2.42}$$

where $K(t) \in \mathbb{R}^{m\times(m+n_m+q_m)}$ and $r(t) \in \mathbb{R}^{m+n_m+q_m}$. The total integral and proportional adaptive control gains $K_I(t), K_P(t) \in \mathbb{R}^{m\times(m+n_m+q_m)}$, are updated as follows

$$K_P(t) = S^T diag\{Se_a r^T\}\Gamma_P \tag{2.43}$$

$$ {}_{t_0}^{C}D_t^{\alpha} K_I(t) = S^T diag\{Se_a r^T\}\Gamma_I \tag{2.44}$$

with the scaling matrix $S = col\{I_m, I_m, ..., I_m\} \in \mathbb{R}^{(m+n_m+q_m)\times m}$, $\Gamma_P = diag\{\Gamma_{p_e} \quad \Gamma_{p_x} \quad \Gamma_{p_u}\} \in \mathbb{R}^{(m+n_m+q_m)\times(m+n_m+q_m)}$ and $\Gamma_I = diag\{\Gamma_{I_e} \quad \Gamma_{I_x} \quad \Gamma_{I_u}\} \in \mathbb{R}^{(m+n_m+q_m)\times(m+n_m+q_m)}$.

2.4.3 Ideal control and state trajectories

When PT occure, in this case the system will have reached a bounded ideal state trajectories denoted by $x_a^*(t) \in \mathbb{R}^n$, and then it is asked to move along them. Mathematically,

$$ y_a(t) = y_a^*(t) = C_a x_a^*(t) = C_m x_m(t) = y_m \tag{2.45}$$

$$ e_a(t) = y_m(t) - y_a(t) = y_m(t) - y_a^*(t) = 0 \tag{2.46}$$

and the ideal plant

$$ \begin{aligned} {}_{t_0}^{C}D_t^{\alpha} x_a^*(t) &= A_a x_a^*(t) + B_a u_p^*(t) \\ y_a^*(t) &= C_a x_a^*(t) + D_f u^*(t) \end{aligned} \tag{2.47}$$

the ideal control $u_p^*(t)$ that would keep the system (2.47) moving along $x_a^*(t)$ is defined as

$$ u_p^*(t) = \widetilde{K}_x x_m(t) + \widetilde{K}_u u_m(t) \tag{2.48}$$

where $\widetilde{K}_x$ and $\widetilde{K}_u$ denote the ideal feedforward gains.
Concerning the existance of $x^*(t)$, we have the following lemma.

Lemma 2.8. *There exist a matrices $X_1 \in \mathbb{R}^{n\times n_m}$ and $X_2 \in \mathbb{R}^{n\times q_m}$ such that the following ideal trajectories*

$$ x^*(t) = X_1 x_m(t) + X_2 u_m(t) \tag{2.49}$$

which makes a relation between the system's state and model's state, exist. Furthermore, define $\widetilde{K}_x = D^{-1}(C_m - C_a X_1)$ and $\widetilde{K}_u = -D^{-1}C_a X_2$, then $y_a^*(t) = y_m(t)$ is satisfied and thus PT is allowed.

Proof. Equation (2.49) consist of more variable than equation and then it has at least a solution for X_1 and X_2. Substituting (2.48) and (2.49) in (2.47.b) gives $y_a^*(t) = y_m(t)$. Thus, PT is achievable. □

Define a state error $e_x(t) \in \mathbb{R}^n$

$$ e_x(t) = x_a^*(t) - x_a(t) \tag{2.50}$$

thus (2.46) can be rewritten as

$$e_a(t) = C_a x_a^*(t) + D_a u^*(t) - C_a x_a(t) - D_a u(t) \tag{2.51}$$

Adding and subtracting $D_a\widetilde{K}_e e_a$, substituting $u(t)$ and $u^*(t)$ from (2.42) and (2.48) to get

$$e_a(t) = C_a e_x + D_a\widetilde{K}_x x_m + D_a\widetilde{K}_u u_m + D_a\widetilde{K}_e e_a - D_a\widetilde{K}_e e_a - D_a K r \tag{2.52}$$

Define $\widetilde{K}(t) \triangleq [\widetilde{K}_e, \widetilde{K}_x, \widetilde{K}_u] \in \mathbb{R}^{m\times(m+n_m+q_m)}$, and rearranging to get,

$$\begin{aligned} e_a(t) &= C_a e_x - D_a(K(t) - \widetilde{K})r - D_a\widetilde{K}_e e_a \\ &= (I + D_a\widetilde{K}_e)^{-1} C_a e_x - (I + D_a\widetilde{K}_e)^{-1} D_a(K(t) - \widetilde{K})r \\ e_a(t) &= \overline{C}_a e_x - \overline{D}_a(K(t) - \widetilde{K})r \end{aligned} \tag{2.53}$$

The error equation can be then derived by taking the Caputo fractional derivative of (2.50)

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} e_x &= {}_{t_0}^{C}D_t^{\alpha} x_a^* - {}_{t_0}^{C}D_t^{\alpha} x_a(t) \\ &= A_a x_a^* + B_a u_p^* - A_a x_a - B_a u_p \\ &= A_a e_x + B_a(u_p(t) - u_p^*(t)) \end{aligned} \tag{2.54}$$

Adding and subtracting $B_a\widetilde{K}_e e_a$, substituting e_a from (2.53), $u(t)$ and $u^*(t)$ from (2.42) and (2.48) to get

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} e_x(t) &= A_a e_x - B_a(K(t) - \widetilde{K})r - B_a\widetilde{K}_e[(I + D_a\widetilde{K}_e)^{-1} C_a e_x \\ &\quad -(I + D_a\widetilde{K}_e)^{-1} D_a(K(t) - \widetilde{K})r] \\ {}_{t_0}^{C}D_t^{\alpha} e_x(t) &= A_a e_x - B_a\widetilde{K}_e(I + D_a\widetilde{K}_e)^{-1} C_a e_x \\ &\quad -\{B_a[I - \widetilde{K}_e(I + D_a\widetilde{K}_e)^{-1} D_a]\}(K(t) - \widetilde{K})r \end{aligned} \tag{2.55}$$

Using binomial inverse theorem $(A+UBV)^{-1} = A^{-1} - A^{-1}UB(B+BVA^{-1}UB)^{-1}BVA^{-1}$ from Woodbury matrix identity for the case where $A = I_A$ and $B = I_B$, one obtain

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} e_x(t) &= A_a e_x - B_a\widetilde{K}_e(I + D_a\widetilde{K}_e)^{-1} C_a e_x \\ &\quad -B_a(I + \widetilde{K}_e D_a)^{-1}(K(t) - \widetilde{K})r \end{aligned} \tag{2.56}$$

Finally, let $\overline{A}_a = A_a - B_a\widetilde{K}_e(I + D_a\widetilde{K}_e)^{-1}C_a$ and $\overline{B} = B_a(I + \widetilde{K}_e D_a)^{-1}$, thus, one can write

$${}_{t_0}^{C}D_t^{\alpha} e_x(t) = \overline{A}_a e_x - \overline{B}_a(K(t) - \widetilde{K})r(t) \tag{2.57}$$

Equation (2.57) represents the error equation for the overall fractional closed loop system with PFC.

2.4.4 Stability analysis

Consider the obtained adaptive system described by (2.57) and (2.44). Let $V(e_x, K_I) \in \mathbb{R}$ be a PDS function given by

$$V(e_x, K_I) = e_x^T(t)Pe_x(t) + tr\{(K_I(t) - \widetilde{K})\Gamma_I^{-1}(K_I(t) - \widetilde{K})\} \tag{2.58}$$

where $\Gamma_I > 0$ and $(K_I(t) - \widetilde{K}) \in \mathbb{R}^{m \times (m+n_m+q_m)}$.
Here, we try to examine the stability of the system (2.57) and (2.44) in one hand, and to show the usefulness of lemma (2.4) in the other hand.
Taking the Caputo fractional derivative of (2.58) we get:

$$^C_{t_0}D^\alpha_t V(e_x, K_I) \leqslant {}^C_{t_0}D^\alpha_t[e_x^T(t)Pe_x(t)] + {}^C_{t_0}D^\alpha_t[tr\{(K_I(t) - \widetilde{K})\Gamma_I^{-1}(K_I(t) - \widetilde{K})\}] \tag{2.59}$$

Applying lemma (2.2) and the new lemma (2.4) gives

$$^C_{t_0}D^\alpha_t V(e_x, K_I) \leqslant 2e_x^T(t)P\,{}^C_{t_0}D^\alpha_t e_x(t) + 2tr\{(K_I(t) - \widetilde{K})\Gamma_I^{-1}\,{}^C_{t_0}D^\alpha_t(K_I(t) - \widetilde{K})\} \tag{2.60}$$

Substituting 2.57 into 2.60 gives

$$\begin{aligned} {}^C_{t_0}D^\alpha_t V(e_x, K_I) &\leqslant 2e_x^T(t)P\overline{A}_a e_x(t) - 2e_x^T(t)P\overline{B}_a(K(t) - \widetilde{K})r(t) \\ &\quad +2tr\left\{(K_I(t) - \widetilde{K})\Gamma_I^{-1}{}^C_{t_0}D^\alpha_t K_I^T(t)\right\} \\ &\leqslant e_x^T(t)(P\overline{A}_a + \overline{A}_a P)e_x(t) - 2e_x^T(t)P\overline{B}_a(K(t) - \widetilde{K})r(t) \\ &\quad +2tr\left\{(K_I(t) - \widetilde{K})\Gamma_I^{-1}{}^C_{t_0}D^\alpha_t K_I^T(t)\right\} \end{aligned} \tag{2.61}$$

Let $\delta(t) = (K(t) - \widetilde{K})$, substituting ${}^C_{t_0}D^\alpha_t K_I(t)$ from (2.44) into the above equation to obtain

$$\begin{aligned} {}^C_{t_0}D^\alpha_t V(e_x, K_I) &\leqslant e_x^T(t)(P\overline{A}_a + \overline{A}_a P)e_x(t) - 2e_x^T(t)P\overline{B}_a\delta(t)r(t) \\ &\quad +2tr\left\{(K_I(t) - \widetilde{K})diag(Se_a(t)r^T(t))S\right\} \end{aligned} \tag{2.62}$$

using passivity relation to obtain

$$\begin{aligned} {}^C_{t_0}D^\alpha_t V(e_x, K_I) &\leqslant -e_x^T(t)Qe_x(t) - e_x^T(t)L^T Le_x(t) \\ &\quad -2e_x^T(t)P\overline{B}_a\delta(t)r(t) \\ &\quad +2tr\left\{(K_I(t) - \widetilde{K})diag(Se_a(t)r^T(t))S\right\} \end{aligned} \tag{2.63}$$

Using $K_I(t) = K(t) - K_P(t)$, and obtain

$$\begin{aligned} {}^C_{t_0}D^\alpha_t V(e_x, K_I) &\leqslant -e_x^T(t)Qe_x(t) - e_x^T(t)L^T Le_x(t) \\ &\quad -2e_x^T(t)P\overline{B}_a\delta(t)r(t) \\ &\quad +2tr\{\delta(t)diag(Se_a(t)r^T(t))S\} \\ &\quad -2tr\{S^T diag(Se_a(t)r^T(t))T_P diag(Se_a(t)r^T(t))S\} \end{aligned} \tag{2.64}$$

Using the relation $(PB = C^T - L^T W)$ one get

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} V(e_x, K_I) \leqslant & -e_x^T(t) Q e_x(t) - e_x^T(t) L^T L e_x(t) \\ & -2e_x^T(t) C^T \delta(t) r(t) + 2e_x^T(t) L^T W \delta(t) r(t) \\ & +2tr\{\delta(t) diag(Se_a(t) r^T(t)) S\} \\ & -2tr\{S^T diag(Se_a(t) r^T(t)) T_P diag(Se_a(t) r^T(t)) S\} \end{aligned} \tag{2.65}$$

we have $e_a^T = e_x^T \overline{C}_a^T + \overline{D}_f \delta r$, therefore, the third and the fifth terms cancel each other, thus one get

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} V(e_x, K_I) \leqslant & -e_x^T(t) Q e_x(t) - e_x^T(t) L^T L e_x(t) \\ & -2r^T \delta^T(t) D_f^T \delta(t) r + 2e_x^T(t) L^T W \delta(t) r(t) \\ & -2tr\{S^T diag(Se_a(t) r^T(t)) T_P diag(Se_a(t) r^T(t)) S\} \end{aligned} \tag{2.66}$$

using $(D_f^T = W^T W - D_f)$ one obtain

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} V(e_x, K_I) \leqslant & -e_x^T(t) Q e_x(t) - e_x^T(t) L^T L e_x(t) - 2r^T \delta^T(t) W^T W \delta(t) r \\ & +2r^T \delta^T(t) D_f \delta(t) r(t) + 2e_x^T(t) L^T W \delta(t) r(t) \\ & -2tr\{S^T diag(Se_a(t) r^T(t)) T_P diag(Se_a(t) r^T(t)) S\} \end{aligned} \tag{2.67}$$

rearranging,

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} V(e_x, K_I) \leqslant & -e_x^T(t) Q e_x(t) - \{L e_x(t) - W \delta(t) r(t)\}^T \{L e_x(t) - W \delta(t) r(t)\} \\ & -r^T \delta^T(t) W^T W \delta(t) r(t) + 2r^T \delta^T(t) D_f \delta(t) r(t) \\ & -2tr\{S^T diag(Se_a(t) r^T(t)) T_P diag(Se_a(t) r^T(t)) S\} \end{aligned} \tag{2.68}$$

we have $2r^T \delta^T(t) D_f \delta(t) r(t) = 2r^T(t) \delta^T(t) W^T W \delta(t) r(t) - 2r^T(t) \delta^T(t) D_f^T \delta(t) r(t)$. Substituting this into the above equation to get

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} V(e_x, K_I) \leqslant & -e_x^T(t) Q e_x(t) - \{L e_x(t) - W \delta(t) r(t)\}^T \{L e_x(t) - W \delta(t) r(t)\} \\ & +r^T \delta^T(t) W^T W \delta(t) r(t) - 2r^T \delta^T(t) D_f \delta(t) r(t) \\ & -2tr\{S^T diag(Se_a(t) r^T(t)) T_P diag(Se_a(t) r^T(t)) S\} \end{aligned} \tag{2.69}$$

rearranging,

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} V(e_x, K_I) \leqslant & -e_x^T(t) Q e_x(t) - \{L e_x(t) - W \delta(t) r(t)\}^T \{L e_x(t) - W \delta(t) r(t)\} \\ & -2tr\{S^T diag(Se_a(t) r^T(t)) T_P diag(Se_a(t) r^T(t)) S\} \\ & -r^T \delta^T(t) [D_f^T - D_f] \delta(t) r(t) \end{aligned} \tag{2.70}$$

Finally,

$$\begin{aligned}{}_{t_0}^{C}D_t^{\alpha}V(e_x, K_I) \;\leqslant\; & -e_x^T(t)Qe_x(t) - \{Le_x(t) - W\delta(t)r(t)\}^T\{Le_x(t) - W\delta(t)r(t)\} \\ & -2tr\{S^T diag(Se_a(t)r^T(t))T_P diag(Se_a(t)r^T(t))S\} \end{aligned} \quad (2.71)$$

For strictly proper systems we have $L = 0$ and $W = 0$, thus,

$$\begin{aligned}{}_{t_0}^{C}D_t^{\alpha}V(e_x, K_I) \;\leqslant\; & -e_x^T(t)Qe_x(t) \\ & -2tr\{S^T diag(Se_a(t)r^T(t))T_P diag(Se_a(t)r^T(t))S\} \end{aligned} \quad (2.72)$$

From this result, the Caputo fractional derivative of ${}_{t_0}^{C}D_t^{\alpha}V(e_x, K_I)$ is negative semidefinite. Thus, using theorem (2.1), we can conclude that all the signals in the augmented fractional control system are bounded, and the origin of the system (2.57) and (2.44) is uniformly stable in the Lyapunov sense.

Finally, we have the following theorem

Theorem 2.3. *Under the Assumptions (2.1), and remark (2.3) all the signals in the resulting augmented closed loop FO system with the control input (2.32) and the fractional adaptation laws (2.33-2.41) are uniformly bounded.*

2.5 Simulation examples

2.5.1 Example 1:

Consider the following fractional order system:

$$G(s) = \frac{3}{2s^{1.2} + s^{0.6}} \quad (2.73)$$

One can see that the relative degree of the above system is 1.2, therefore, it is not ASPR. One considers the following FOPFC in order to realize an augmented ASPR system:

$$F(s) = \frac{0.01}{0.61177s^{0.6} + 0.467} \quad (2.74)$$

The augmented system is:

$$G_a(s) = \frac{0.002s^{1.2} + 1.8363s^{0.6} + 1.401}{1.2235s^{1.8} + 1.5458s^{1.2} + 0.467s^{0.6}} \quad (2.75)$$

The augmented system is minimum phase and of relative degree 0.6, therefore, by assumption 2.3 is ASPR. The state space form of the above FOTF is:

$$\begin{aligned} {}_{t_0}^{C}D_t^{0.6} &= \begin{bmatrix} -1.2634 & -0.3817 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) \\ y &= \begin{bmatrix} 0.0016 & 1.5008 & 10145 \end{bmatrix} x(t) \end{aligned} \tag{2.76}$$

2.5.2 Example 2:

Consider the following fractional order system:

$$G(s) = \frac{5s^{0.4}}{2s^{0.4}+1} \tag{2.77}$$

One can see that the above system is not minimum phase, therefore, it is not ASPR. One considers the following FOPFC with $\alpha = 0.4$ in order to realize an augmented ASPR system:

$$F(s) = \frac{0.000305s^{0.4}}{0.00014372s^{0.8} + 7.564e-05s^{0.4} + 0.0124} \tag{2.78}$$

The augmented system is:

$$G_a(s) = \frac{0.00071858s^{1.2} + 0.0009882s^{0.8} + 0.062305s^{0.4}}{0.00028743s^{1.2} + 0.000295s^{0.8} + 0.024876s^{0.4} + 0.0124} \tag{2.79}$$

The augmented system is minimum phase and of relative degree 0, therefore, by assumption 2.3 is ASPR. The state space form of the above FOTF is:

$$\begin{aligned} {}_{t_0}^{C}D_t^{0.6} &= \begin{bmatrix} -1.0263 & -86.5444 & -43.1406 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) \\ y &= \begin{bmatrix} 0.8722 & 0.4032 & -107.8516 \end{bmatrix} x(t) + 2.5u(t) \end{aligned} \tag{2.80}$$

2.5.3 Simulation results

The above two examples are simulated through MATLAB in order to verify the performances of the proposed FODSAC controller. The controller parameters used for these tests are: $T_I = diag(5,5,5)$, $T_p = diag(5,5,5)$. The adaptive control gains were initialized with $K_e = K_x = K_u = 0$. The model reference is chosen as $G_m(s) = 1/(1s^{0.5}+1)$.

The simulation results of the two examples are presented in figure 2.2 and 2.3. Compared with the the classical DSAC, thanks to the fractional operator, the proposed ASC controller managed to improve the tracking performances.

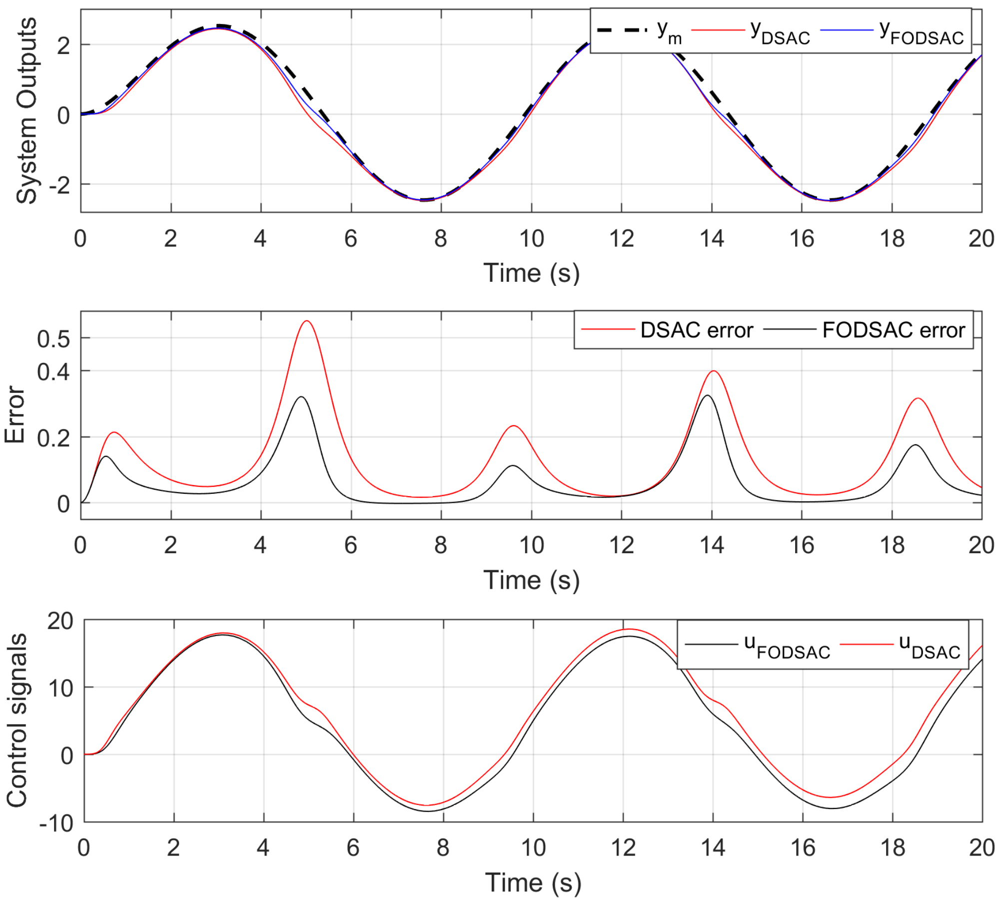

Figure 2.2: Simulation results of example 1: The proposed FODSAC compared with DSAC

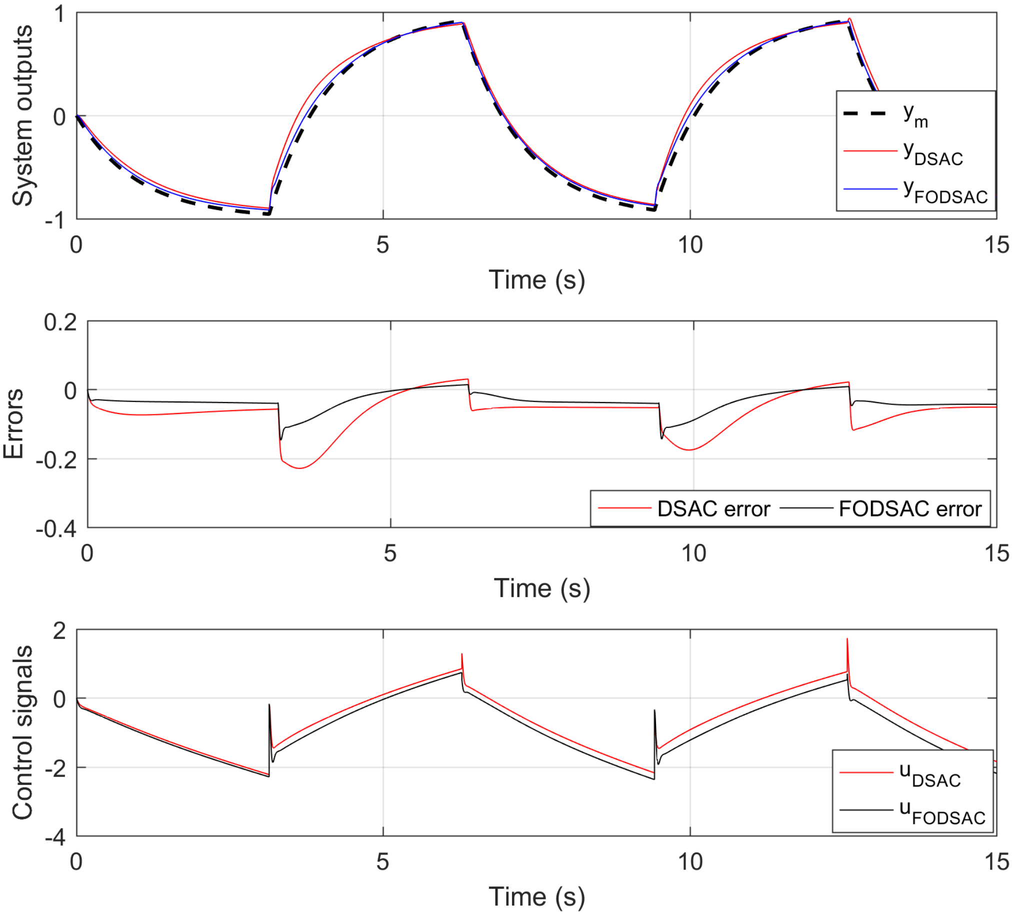

Figure 2.3: Simulation results of example 2: The proposed FODSAC compared with DSAC

2.6 Conclusion

In this chapter we considered an almost passivity based fractional order Decentralized simple adaptive control design for linear fractional-order systems. We first presented a lemma related to the Caputo fractional derivative of the trace function where a Positive-definite diagonal matrix appears between a product of a rectangular matrix and its transpose. Thereafter, the almost passivity conditions for the design of the adaptive system were established. A new fractional order simple adaptive controller was proposed and the formal proof of the stability of the passivity-based fractional adaptive system was established. A numerical simulation showing the effectiveness of the proposed method was given.

Part II

Design and experimental validation of an Adaptive Synergetic Controller (ASC)

Chapter 3

Adaptive Synergetic Control Design

> “ *Once you eliminate the impossible, whatever remains, no matter how improbable, must be the truth.* ”
>
> Arthur Conan Doyle

3.1 Introduction

In this chapter, a simple adaptive synergetic controller ensuring the asymptotic convergence of the system to a desired manifold is proposed as a solution to the design of the Synergetic Control where the system parameters and dynamics are unknown. It is well known that the design of the synergetic control (SC) law requires a thorough knowledge of the system parameters and dynamics. Such problem obstructs the synthesis of the SC law and the designer is prompted to pass through the estimation methods, which, in turn, poses a problem of increasing the computation time of the control algorithm. To cope with this problem, a solution is proposed by modifying the original SC law to develop an SAC-like adaptive SC law without the need of prior knowledge of the system. The stability of the proposed adaptive controller is formally proven via the Lyapunov approach.

3.2 Preliminaries

This section presents some used notations and specifies fundamentals on the synergetic control technique and Simple Adaptive Control method.

3.2.1 Synergetic control

Consider a class of nonlinear systems of the form:

$$\begin{aligned} \dot{x}\,(t) &= A(x)x\,(t) + B(x)u\,(t) \\ y\,(t) &= Cx\,(t) \end{aligned} \tag{3.1}$$

where $x \in \mathbb{R}^n$ is the system states, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$ represent the control input and system output respectively.

The synergetic controller design procedure for the system (3.1) consists of the following steps:

1. Define a macro-variable φ for constructing a manifold M given as follows:

$$M = \{x : \varphi(x) = 0,\, \varphi \in \mathbb{R}^m\} \tag{3.2}$$

the macro-variable φ is a function of the system states.

2. Synthesize a control law that would force the system to exponentially reach the desired manifold M with a dynamic evolution of φ which can be stated as:

$$T\dot{\varphi}(x) + \varphi(x) = 0 \tag{3.3}$$

where the $T = T^T > 0$ specifies the convergence rate of the system states. The dynamic evolution (3.3) will make the macro-variable φ and its derivative $\dot{\varphi}$ reach zero.

3. Calculation of the control law u_{SC} by solving the system (3.1) with the evolution condition (3.3) as follows:

$$T\dot{\varphi}(x) + \varphi(x) \;\; = \;\; T\varphi_x(x)\left(A(x)(x) + B(x)u_{SC}(t)\right) + \varphi(x) = 0$$

where $\varphi_x(x) = \partial\varphi(x)/\partial x$. The resulting synergetic control law u_{SC} can be expressed as:

$$u_{SC}(t) \;\; = \;\; -\left(T\varphi_x(x)B(x)\right)^{-1}\varphi_x(x)A(x)x(t) - \left(T\varphi_x(x)B(x)\right)^{-1}\varphi(x) \tag{3.4}$$

Note that the control law (3.4) requires the knowledge of the system parameters.

3.2.2 Simple Adaptive Control

Consider the system given by equation (3.1). For the design of the SAC control law, the system realisation $\{A(x), B(x), C\}$ does not necessary need to be known but just requires the system (3.1) to be ASP. The Almost strict passivity of a system of the form (3.1) is defined as follows [4, 30]:

Definition 3.1. *A nonlinear system* $\{A(x), B(x), C\}$ *is called SP if there exists two matrices* $P(x) > 0$ *and* $Q(x) > 0$ *such that the system satisfies simultaneously the following relations [30, 54]:*

$$\dot{P}(x) + P(x)A(x) + A^T(x)P(x) = -Q(x) < 0 \tag{3.5}$$

$$P(x)B(x) = C^T \tag{3.6}$$

Relation (3.5) means that an SP system is asymptotically stable, and relation (3.6) shows that $CB(x) = (CB(x))^T > 0$ *.*

Definition 3.2. *For the nonlinear system* $\{A(x), B(x), C\}$ *if there exists a constant feedback matrix* $\widetilde{K}_e > 0$ *such that the resulting closed-loop system with the state-space realisation* $\{(A(x) - B(x)\widetilde{K}_e C), B(x), C\}$ *is SP, then the open-loop system* $\{A(x), B(x), C\}$ *is called ASP.*

Since most real world systems are not inherently ASP, then, one needs the following lemma [54]:

Lemma 3.1. *Let the system* $G : \{A(x), B(x), C\}$ *be a stabilisable nonlinear system and let* F *be any stabilising linear controller for* G*. Assume that the inverse* F^{-1} *(exist, proper or strictly proper) is added as parallel feedforward along* G*, such that the augmented system* $G_a = G + F^{-1}$ *has the form* $G_a : \{A_a(x), B_a(x), C_a\}$*, in this case,* G_a *is called ASP. That is there exists some constant feedback matrix* $\widetilde{K}_e > 0$ *such that the augmented closed-loop system* $G_a^{cl} : \{(A_a(x) - B_a(x)\widetilde{K}_e C_a), B_a(x), C_a\}$ *satisfies relations (3.5) and (3.6) [4, 54]*

Now, consider the system (3.1) and assume it is ASP. The principle of the SAC algorithm is to design a control signal $u(t)$ which forces the outputs of the system to track the outputs of an ideal linear model of the form:

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \tag{3.7}$$

where $x(t) \in \mathbb{R}^{n_m}$ is the model state vector, $u_m(t) \in \mathbb{R}^{q_m}$ is the command and $y_m(t) \in \mathbb{R}^m$ is the model output signal. A_m, B_m and C_m are the model matrices with appropriate dimensions.

Therefore, to specify the control objective, one define the output tracking error $e_a(t) \in \mathbb{R}^m$ as follows:

$$e_a(t) \triangleq y_m(t) - y_a(t) \tag{3.8}$$

The SAC control law that would force the outputs of the plant to asymptotically track the outputs of the reference model is given as [52]:

$$u_{SAC}(t) = K_e(t)e_a(t) + K_x(t)x_m(t) + K_u(t)u_m(t) \tag{3.9}$$

where $K_e(t) \in \mathbb{R}^{m\times m}$, $K_x(t) \in \mathbb{R}^{m\times n_m}$ and $K_u(t) \in \mathbb{R}^{m\times q_m}$ are adaptive gains.

By defining $K(t) = [K_e(t)\ K_x(t)\ K_u(t)] \in \mathbb{R}^{m\times(m+n_m+q_m)}$ and $r^T(t) = [e_a^T(t)\ x_m^T(t)\ u_m^T(t)] \in \mathbb{R}^{(m+n_m+q_m)}$, the adaptive controller (3.9) can be written into matrix form as follows:

$$u_{SAC}(t) = K(t)r(t) \tag{3.10}$$

According to the SAC theory, the adaptive gain $K(t)$ is a combination of proportional gain $K_P(t)$ and an integral gain $K_I(t)$:

$$K(t) = K_P(t) + K_I(t) \tag{3.11}$$

where $K_p(t) = [K_{p_e}(t)\ K_{p_x}(t)\ K_{p_u}(t)] \in \mathbb{R}^{m\times(m+n_m+q_m)}$ and $K_I(t) = [K_{I_e}(t)\ K_{I_x}(t)\ K_{I_u}(t)] \in \mathbb{R}^{m\times(m+n_m+q_m)}$, they are adapted as follows:

$$K_P(t) = e_a(t)r^T(t)\mathrm{T}_p \tag{3.12}$$

$$\dot{K}_I(t) = e_a(t)r^T(t)\mathrm{T}_I \tag{3.13}$$

with, $T_p = diag(T_{p_e}, T_{P_x}, T_{p_u}) \geqslant 0 \in \mathbb{R}^{(m+n_m+q_m)\times(m+n_m+q_m)}$ and $T_I = diag(T_{I_e}, T_{I_x}, T_{I_u}) > 0 \in \mathbb{R}^{(m+n_m+q_m)\times(m+n_m+q_m)}$ are diagonal weighting matrices that define the adaptation rates of the control gains.

In the next section, the proposed ASC will be presented. One imposes the following assumption:

Assumption 3.1. *There exist a PFC mentioned in lemma (3.1) that can render the augmented system ASP.*

3.3 The proposed adaptive synergetic control

The whole basic structure of the proposed control method is shown in Fig. 3.1. The controller design procedures will be shown in the subsequent developments.

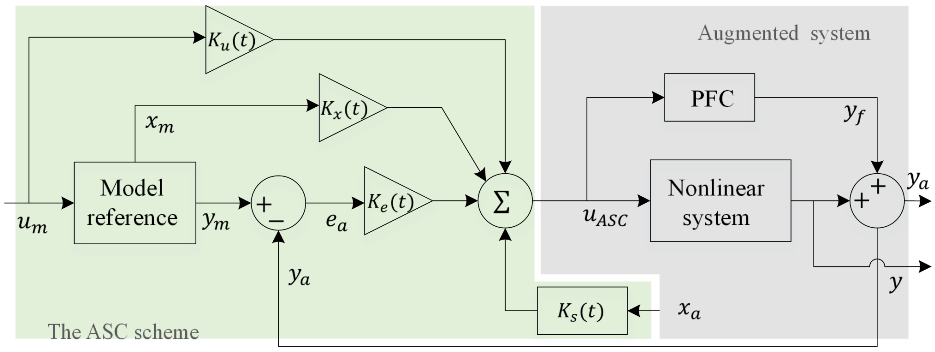

Figure 3.1: The proposed Adaptive Synergetic Control Scheme

3.3.1 Problem statement

Consider the class of nonlinear system (3.1) with $CB = 0$ and assume it is not ASP. The matrices $A(x)$ and $B(x)$ are not needed to be known. To fulfill the ASP conditions required by the SAC, one consider under assumption (3.1) an augmentation with a PFC of the form:

$$\begin{aligned} \dot{x}_f(t) &= A_f x_f(t) + B_f u(t) \\ y_f(t) &= C_f x_f(t) \end{aligned} \tag{3.14}$$

where $A_f = diag(a_{f_1}, ..., a_{f_m}) \in \mathbb{R}^{m\times m}$, $B_f = diag(b_{f_1}, ..., b_{f_m}) \in \mathbb{R}^{m\times m}$ and $C_f = diag(c_{f_1}, ..., c_{f_m}) \in \mathbb{R}^{m\times m}$. Where the elements $c_{f_i}, b_{f_i} > 0$, $\forall\, i = 1, ..., m$. The augmented system is:

$$\begin{aligned} \dot{x}_a(t) &= A_a(x_a)x_a(t) + B_a(x_a)u(t) \\ y_a(t) &= C_a x_a(t) \end{aligned} \tag{3.15}$$

where $x_a = col\{x, x_f\} \in \mathbb{R}^{n_a}$, $y_a \in \mathbb{R}^m$, $A_a = diag[A\ A_f]$, $B_a = col\{B, B_f\}$, $C_a = [C\ C_f]$.

Lemma 3.2. *For the class of nonlinear system (3.1) with $CB = 0$, $C_aB_a = (C_aB_a)^T > 0$ hold true.*

Proof. The proof is straightforward, we have $C_aB_a = [C\ C_f][B^T\ B_f^T]^T = C_fB_f$. Since $C_fB_f = (C_fB_f)^T > 0$, then $C_aB_a = (C_aB_a)^T > 0$. □

The control objective is to design without prior knowledge of the system parameters and dynamics, an adaptive synergetic control law that would force the outputs of the system (3.15) to track the outputs of the model (3.7). For this purpose, instead of using an estimate of unknown parameters for the adaptation process, one try to bring out a synergetic like SAC structure.

3.3.2 Adaptive Synergetic Control design

To develop the adaptive SC law for the case of augmented system (3.15), the design method considers the following two steps:

Step 1: Define the macro-variable φ as function of augmented error e_a that is defined as $e_a = y_m - y_a$, and the manifold M_{e_a} defined as:

$$M_{e_a} = \{e_a : \varphi(e_a) = 0,\ \varphi \in \mathbb{R}^m\} \tag{3.16}$$

By choosing $\varphi(e_a) = e_a(t)$, the SC law can be obtained by solving the system (3.15) with the evolution $T\dot{\varphi}(e_a) + \varphi(e_a) = 0$, which gives

$$T\dot{e}_a + e_a = 0 \tag{3.17}$$

Substituting $\dot{e}_a = \dot{y}_m - \dot{y}_a$, $y_m = C_m x_m$ and $y_a = C_a x_a$ into (3.17) gives

$$\begin{aligned} T\dot{e}_a + e_a \;&=\; TC_m A_m x_m + TC_m B_m u_m \\ &\quad -TC_a A_a(x_a)x_a - TC_a B_a(x_a)u_{SC} + e_a = 0 \end{aligned} \tag{3.18}$$

from (3.18) one can extract $u_{sc}(t)$ to obtain

$$\begin{aligned} u_{SC}(t) \;&=\; -\left(TC_a B_a(x_a)\right)^{-1} TC_a A_a(x_a)x_a \\ &\quad + \left(TC_a B_a(x_a)\right)^{-1} TC_m A_m x_m \\ &\quad + \left(TC_a B_a(x_a)\right)^{-1} TC_m B_m u_m \\ &\quad + \left(TC_a B_a(x_a)\right)^{-1} e_a \end{aligned} \tag{3.19}$$

Define $k_1 = -\left(TC_a B_a(x_a)\right)^{-1} TC_a A_a(x_a) \in \mathbb{R}^{m\times n_a}$, $k_2 = \left(TC_a B_a(x_a)\right)^{-1} TC_m A_m \in \mathbb{R}^{m\times n_m}$, $k_3 = \left(TC_a B_a(x_a)\right)^{-1} TC_m B_m \in \mathbb{R}^{m\times q_m}$, and $k_4 = \left(TC_a B_a(x_a)\right)^{-1}$, then one can write

$$u_{SC}(t) = k_1 x_a + k_2 x_m + k_3 u_m + k_4 e_a \tag{3.20}$$

Step 2: As the parameters of the control (3.20) are unknown, only x_a is supposed to be available, one consider similar to the adaptation law of standard SAC gains, the nonlinear adaptive gains $K_x(t), K_u(t), K_e(t)$ and $K_s(t)$ and propose the following ASC law:

$$u_{ASC}(t) = K_e(t)e_a + K_x(t)x_m + K_u(t)u_m + K_s(t)x_a \tag{3.21}$$

where $K_e \in \mathbb{R}^{m\times m}$, $K_x \in \mathbb{R}^{m\times n_m}$, $K_u \in \mathbb{R}^{m\times q_m}$ and $K_s \in \mathbb{R}^{m\times n_a}$. Note that the parameters k_1, k_2, k_3 and k_4 of (3.20) are target to which the adaptive gains in (3.21) try to converge so that the system asymptotically reaches the desired manifold (3.16).

Defining $K_a(t) = [K_e(t)\ K_x(t)\ K_u(t)\ K_s(t)] \in \mathbb{R}^{m\times(m+n_m+q_m+n_a)}$ and $r_a^T(t) = [e_a^T(t)\ x_m^T(t)\ u_m^T(t)\ x_a^T(t)] \in \mathbb{R}^{(m+n_m+q_m+n_a)}$, the final ASC law can be expressed as

$$u_{ASC}(t) = K_a(t)r_a(t) \tag{3.22}$$

with the adaptation law

$$K_a(t) = K_P(t) + K_I(t) \tag{3.23}$$

$$K_P(t) = e_a(t)r_a^T(t)\mathrm{T}_p \tag{3.24}$$

$$\dot{K}_I(t) = e_a(t)r_a^T(t)\mathrm{T}_I \tag{3.25}$$

where $T_p = T_P^T \geqslant 0 \in \mathbb{R}^{(m+n_m+q_m+n_a)\times(m+n_m+q_m+n_a)}$ and $T_I = T_I^T > 0 \in \mathbb{R}^{(m+n_m+q_m+n_a)\times(m+n_m+q_m+n_a)}$. They are defined as

$$\begin{aligned} T_p &= diag(T_{p_e}, T_{P_x}, T_{p_u}, T_{p_s}) \\ T_I &= diag(T_{I_e}, T_{I_x}, T_{I_u}, T_{I_s}) \end{aligned} \tag{3.26}$$

3.3.3 Error dynamic, ideal control and state trajectories

Let t^* define the moment where the system output $y_a(t)$ perfectly match the output of the ideal model $y_m(t)$ (i.e.,$y_a(t) = y_m(t)$), in that case, the system will have reached a bounded ideal trajectories denoted by $x_a^*(t) \in \mathbb{R}^n$ (for details about the existence of such ideal trajectories refer to [54, 68], and then it is asked to move along them (i.e., $\forall t > t^*, x_a = x_a^*$). Thus, one have

$$y_a^*(t) = C_a x_a^*(t) = C_m x_m = y_m \tag{3.27}$$

$$e_a(t) = y_m(t) - y_a(t) = 0 \tag{3.28}$$

for the perfect tracking, define the ideal plant

$$\begin{aligned} \dot{x}_a^*(t) &= A_a(x^*)x_a^*(t) + B_a(x^*)u_p^*(t) \\ y_a^*(t) &= C_a x_a^*(t) \end{aligned} \tag{3.29}$$

The ideal control that would keep the system (3.29) moving along $x_a^*(t)$ is defined as:

$$u^*\left(t\right) = \widetilde{K}_x x_m\left(t\right) + \widetilde{K}_u u_m\left(t\right) + \widetilde{K}_s x_a\left(t\right). \tag{3.30}$$

where $\widetilde{K}_x \in R^{m\times n_m}$, $\widetilde{K}_s \in R^{m\times n_a}$ and $\widetilde{K}_u \in R^{m\times q_m}$ denote the ideal feedforward gains. Define a state error $e_x(t) \in \mathbb{R}^{n_a}$

$$e_x(t) = x_a^*(t) - x_a(t) \tag{3.31}$$

thus (3.28) can be rewritten as

$$e_a(t) = C_a x_a^*(t) - C_a x_a(t) = C_a e_x(t) \tag{3.32}$$

The error equation can be then derived by taking the time derivative of (3.31)

$$\dot{e}_x = A_a(x_a^*)x_a^* + B_a(x_a^*)u^* - A_a(x_a) - B_a(x_a)u \tag{3.33}$$

Adding and subtracting $A_a(x_a)x^*$ and $B_a(x_a)u^*$ to obtain

$$\begin{aligned}\dot{e}_x \;&=\; A_a(x_a)e_x + \left(A_a(x_a^*) - A_a(x_a)\right)x^* \\ &\quad + \left(B_a(x_a^*) - B_a(x_a)\right)u^* + B_a(x_a)(u^* - u)\end{aligned} \tag{3.34}$$

Adding and subtracting $B_a(x_a)\widetilde{K}_e e_a$ and substituting $e_a = C_a e_x$ gives

$$\begin{aligned}\dot{e}_x \;&=\; (A_a(x_a) - B_a(x_a)\widetilde{K}_e C_a)e_x \\ &\quad +(A_a(x_a^*) - A_a(x_a))x^* + (B_a(x_a^*) - B_a(x_a))u^* \\ &\quad +B_a(x_a)(u^* - u) + B_a(x_a)\widetilde{K}_e e_a\end{aligned} \tag{3.35}$$

Substituting u^* from (3.30) and u from (3.22) into (3.35) gives

$$\begin{aligned}\dot{e}_x \;&=\; (A_a(x_a) - B_a(x_a)\widetilde{K}_e C_a)e_x \\ &\quad +(A_a(x_a^*) - A_a(x_a))x^* + B_a(x_a)\widetilde{K}_x x_m \\ &\quad +B_a(x_a)\widetilde{K}_u u_m + B_a(x_a)\widetilde{K}_s x_a + B_a(x_a)\widetilde{K}_e e_a \\ &\quad -B_a(x_a)K_a r_a + (B_a(x_a^*) - B_a(x_a))u^*\end{aligned} \tag{3.36}$$

Finally, substituting $K_a(t)$ from (3.23) and rearranging to obtain

$$\dot{e}_x = A_c e_x - B_a(x_a)K_p r - B_a(x_a)(K_I - \widetilde{K}_a)r + R \tag{3.37}$$

where $A_c = A_a(x_a) - B_a(x_a)\widetilde{K}_e C_a$, $\widetilde{K}_a = [\widetilde{K}_e \ \widetilde{K}_x \ \widetilde{K}_u \ \widetilde{K}_s]$ and the residual term $R = (A_a(x_a^*) - A_a(x_a))x^* + (B_a(x_a^*) - B_a(x_a))u^*$. Let $F(x) = A(x)x^* + B(x)u^*$ and $F(x^*) = A(x^*)x^* + B(x^*)u^*$ thus, R can be rewritten as

$$R = F(x^*) - F(x) \tag{3.38}$$

Equation (3.37) represents the error dynamic of the overall augmented closed loop system with PFC.

The following assumption and remark are needed before proceeding to the stability analysis:

Assumption 3.2. *The Jacobian linear approximation of $F(x)$ at x^* is bounded.*

Remark 3.1. *The parameters k_1, k_2, k_3 and k_4 of (3.20) are therefore, respectively, the ideal gains $\widetilde{K}_s, \widetilde{K}_x, \widetilde{K}_u$ and $\widetilde{K}_e$ that the adaptive gain K_s, K_x, K_u and K_e in (3.21) try to reach in order for the system to exponentially reach the desired manifold (3.16).*

3.3.4 Stability analysis

Theorem 3.1. *Under assumptions (3.1-3.2), the application of the adaptive synergetic control (3.22) with the adaptation laws (3.24-3.25) to the nonlinear system (3.15) results in asymptotic convergence of the state and output error and boundedness of the adaptive gains.*

Proof. Let $V(e_x, \varphi, K_I) \in \mathbb{R}$ be a differentiable Lyapunov function defined as:

$$\begin{aligned} V(e_x, \varphi, K_I) &= e_x^T P(x_a) e_x + \varphi(e_a)^T \varphi(e_a) \\ &\quad + tr\{(K_I - \widetilde{K}) T_I^{-1} (K_I - \widetilde{K})^T\} \\ &\quad + tr\{(K_I - \widetilde{K}) C_a B_a T_I^{-1} (K_I - \widetilde{K})^T\} \end{aligned} \tag{3.39}$$

Where $P(x_a) > 0$. Note that $V(0, 0, \tilde{K}) = 0$, $V(e_x, \varphi, K_I) > 0 \ \forall \ \{e_x, \varphi, K_I\} \neq \{0, 0, \widetilde{K}\}$ and $V(e_x, \varphi, K_I) \to \infty$ if $||e_x|| \to \infty$, $||\varphi|| \to \infty$ or $||K_I|| \to \infty$. The time derivative of $V(e_x, \varphi, K_I)$ is

$$\begin{aligned} \dot{V} &= \dot{e}_x^T P(x_a) e_x + e_x^T \dot{P}(x_a) e_x + e_x^T P(x_a) \dot{e}_x \\ &\quad + 2\varphi^T(e_a)\dot{\varphi}(e_a) + 2tr\{(K_I - \widetilde{K}) T_I^{-1} \dot{K}_I^T\} \\ &\quad + 2tr\{(K_I - \widetilde{K}) C_a B_a T_I^{-1} \dot{K}_I^T\} \end{aligned} \tag{3.40}$$

where $V \equiv V(e_x, \varphi, K_I)$. Substituting $\dot{e}_x$ from (3.37), K_p from (3.24), $\dot{K}_I$ from (3.25) and using passivity relation (3.5) to get

$$\begin{aligned}\dot{V} &= -e_x^T Q(x_a)e_x - 2e_x^T P(x_a)B_a(x_a)e_a r_a^T T_p r_a \\ &\quad -2e_x^T P(x_a)B_a(x_a)(K_I - \widetilde{K})r_a + 2e_x^T P(x_a)R \\ &\quad +2\varphi^T(e_a)\dot{\varphi}(e_a) + 2tr\{(K_I - \widetilde{K}_a)r_a e_a^T\} \\ &\quad +2tr\{(K_I - \widetilde{K})C_a B_a r_a e_a^T\}\end{aligned} \tag{3.41}$$

Where $Q(x_a) > 0$. In the relation $\dot{\varphi}(e_a) = \dot{e}_a = \dot{y}_m - \dot{y}_a$, substituting $y_m = C_m x_m$ and $y_a = C_a x_a$ to get $\dot{\varphi}(e_a) = C_m A_m x_m + C_m B_m u_m - C_a A_a x_a - C_a B_a K_a r_a$, by adding and subtracting $C_a B_a \widetilde{K}_a r_a$ and by remark (3.1) one get $\dot{\varphi}(e_a) = -T^{-1}\varphi(e_a) - C_a B_a(K_a - \widetilde{K}_a)r_a$, substituting this into equation (3.41) to get

$$\begin{aligned}\dot{V} &= -e_x^T Q(x_a)e_x - 2e_x^T P(x_a)B_a(x_a)e_a r_a^T T_p r_a \\ &\quad -2e_x^T P(x_a)B_a(x_a)(K_I - \widetilde{K}_a)r_a - 2e_a^T T^{-1}e_a \\ &\quad -2e_x^T C_a^T C_a B_a(K_a - \widetilde{K}_a)r_a + 2tr\{(K_I - \widetilde{K}_a)r_a e_a^T\} \\ &\quad +2tr\{(K_I - \widetilde{K})C_a B_a r_a e_a^T\} + 2e_x^T P(x_a)R\end{aligned} \tag{3.42}$$

using $K_a(t) = K_p + K_I$ and substituting K_p from (3.24) to get,

$$\begin{aligned}\dot{V} &= -e_x^T Q(x_a)e_x - 2e_x^T P(x_a)B_a(x_a)e_a r_a^T T_p r_a \\ &\quad -2e_x^T P(x_a)B_a(x_a)(K_I - \widetilde{K}_a)r_a - 2e_a^T T^{-1}e_a \\ &\quad -2e_x^T C_a^T C_a B_a(K_I - \widetilde{K}_a)r_a + 2e_x^T P(x_a)R \\ &\quad -2e_x^T C_a^T C_a B_a e_a r_a^T T_p r_a + 2tr\{(K_I - \widetilde{K}_a)r_a e_a^T\} \\ &\quad +2tr\{(K_I - \widetilde{K}_a)C_a B_a r_a e_a^T\}\end{aligned} \tag{3.43}$$

knowing that $tr\{\alpha^T\beta\} = \alpha^T\beta$, and $tr\{AB\} = tr\{BA\}$, the third and eighth, the fifth and ninth terms cancel each other, thus, one get by substituting $P(x_a)B_a(x_a) = C_a^T$ and $e_a^T = e_x^T C_a^T$

$$\begin{aligned}\dot{V} &= -e_x^T Q(x_a)e_x - 2e_a^T e_a r_a^T T_p r_a - 2e_a^T T^{-1}e_a \\ &\quad +2e_x^T P(x_a)R - 2e_a^T C_a B_a e_a r_a^T T_p r_a\end{aligned} \tag{3.44}$$

The residual term R that appears in the derivative of the lyapunov function (3.44) may affect negativity. In general, in the standard SAC [4, 6, 22, 30, 54], researchers have tried to directly eliminate this residual term by assuming that this term may vanish under the assumption that the variation of system parameters is slow compared with the control dynamics. Or, it is supposed to be bounded and the stability is analysed by the domination of some quadratic and quartic terms on the residual one. In the present study and with the help of the ASC structure (3.47) one try to avoid these philosophical suppositions that seem to be ambiguous. For a clear discussion of the system stability, and with a mathematical sense we try to substitute the residual term by its Jacobian linear approximation in order to achieve the asymptotic convergence of the states.

Recall that $R = F(x^*) - F(x)$, using the Jacobian linear approximation to $F(x)$ at x^* to obtain,

$$F(x) \approx F(x^*) + J_F(x^*)(x - x^*) \approx F(x^*) - J_F(x^*)e_x \tag{3.45}$$

substituting the above equation into R to get

$$R = J_F(x^*)e_x \tag{3.46}$$

substituting the above into (3.44) and rearranging to get

$$\begin{aligned} \dot{V} \;\; = \;\; & -2e_a^T e_a r_a^T T_p r_a - 2e_a^T C_a B_a e_a r_a^T T_p r_a \\ & -e_x^T [Q(x_a) + 2C_a^T T^{-1} C_a - 2P(x_a) J_F(x^*)] e_x \end{aligned} \tag{3.47}$$

Under assumption (3.2), the term $[Q(x_a) + C_a^T T^{-1} C_a - P(x_a)J_F(x^*)]$ becomes positive under an appropriate choice of T^{-1}. It can be checked if the eigenvalues of T^{-1} are taken quite high, i.e T is positive with small eigenvalues and thus T can be found considering the extreme values of $A(x_a)$ and $B(x_a)$. Also, according to the SAC theory, selecting higher values of T_e is sufficient in order for the systems to returns immediately to the stable region [30]. Indeed, one can show that the choice of T^{-1} is in fact related to the choice of T_e and that T^{-1} is indirectly included in the control law (3.21). From (3.19) one can have $u_{sc} = k_1 x_a + k_2 x_m + k_3 u_m + k_4 T^{-1} e_a$. After replacing k_1, k_2, k_3 and k_4 by the adaptive gains one get $u_{asc} = K_e T^{-1} e_a + K_x x_m + K_u u_m + K_s x_a$. Substituting $K_e = e_a e_a^T T_e$ to get $u_{asc} = e_a e_a^T T_e T^{-1} e_a + K_x x_m + K_u u_m + K_s x_a$. It is obvious that T can be merged with T_e since both the two matrices are chosen for the fast adaptation rate. Thus, the ASC law ends with (3.21).

After having developed the residual term R to better analyze the negativity of $\dot{V}$, and thus to be able to conclude on the stability of the system and the asymptotic convergence of the gains, one can now have the third term of $\dot{V}$ in (47) negative. It follows that $\dot{V}$ is negative definite with respect to e_x and negative semi-definite with respect to the state-space $\{e_x, K_I\}$. Thus, according to the Lyapunov stability theory, all signals in the closed loop adaptive system with the ASC law (3.22,3.24-3.25) are bounded.

Furthermore, according to ([54], p. 43) and the new theorem of stability [69, 70], for a Lyapunov derivative of the form (3.47) the entire state $[e_x, \varphi, K_I]$ ultimately reaches the domain $\Omega_f = \Omega_0 \cap \Omega$, where the domain $\Omega = \dot{V}([e_x, \varphi, K_I(t)] \equiv 0$ and $\Omega_0 = \{[e_x, \varphi, K_I] | V([e_x, \varphi, K_I(t)], t) \leq V([e_{x0}, \varphi_0, K_{I_0}], 0)\}$ is the domain that contains the trajectories of the system. Because $\dot{V}([e_x, \varphi, K_I(t)], t) < 0$ in e_x, the system ends with $e_x \equiv 0 \Rightarrow e_x(t) = 0 \Rightarrow e_a(t) = 0$. Thus, asymptotic stability of the state and output tracking errors is guaranteed. □

3.4 Application to quadrotor system

For the application of the proposed ASC, the following nonlinear state space of the quadrotor in used:

$$A(x) = \begin{bmatrix} 0 & I_3 \\ 0 & -M(x_1)^{-1}L(x_1, x_2) \end{bmatrix}, \; B(x) = \begin{bmatrix} 0 \\ M(x_1)^{-1} \end{bmatrix} \tag{3.48}$$

$$C = \begin{bmatrix} I_3 & 0_3 \end{bmatrix}, \; x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \tag{3.49}$$

where $M(x_1)$ and $L(x_1, x_2)$ are defined in section (1.5)

3.4.1 Numerical simulation

Two simulation tests are presented in order to verify the performances of the proposed ASC controller, i) stabilisation test and ii) variable references test. The controller parameters used for these tests are: $T_I = diag(2e6, 1, 1, 10)$, $T_p = diag(1, 1, 1, 10)$. The adaptive control gains were initialized with $K_e = K_x = K_u = K_s = 0$. We just started by such a selection of T_I and T_p, and all initial gains are set to zero because one have no knowledge about the system, only x_a is available. While the parameters of adaptation T_x, T_u, T_s could have been independently selected to improve the tracking performance, T_e should be selected high in order to ensure the rapid convergence to the stability region. Thanks to the adaptation mechanism, the controller will find the closest gain values that perform asymptotic perfect tracking at the end of the steepest descent minimization process from any initial gain values . The model reference is chosen as $G_m(s) = 1/(0.1s + 1)$. The selected matrices of the PFC are $A_f = diag(-5.92, -7.615, -5.92)$, $B_f = diag(1, 1, 1)$, and $C_f = diag(1.113, 0.7615, 1.113)$. The inertia matrix $J = diag(0.0104, 0.0104, 0.0196)$

is used for the stabilisation test. For the second test we increased the inertia to 400% than the nominal value and the reference inputs were set to $200sin(2\pi/5t)$ for the pitch angle and $-200sin(2\pi/5t)$ for the roll angle. In both simulation tests, the initial conditions were set to $\phi_0 = 5.8°$ and $\theta_0 = -5.8°$.

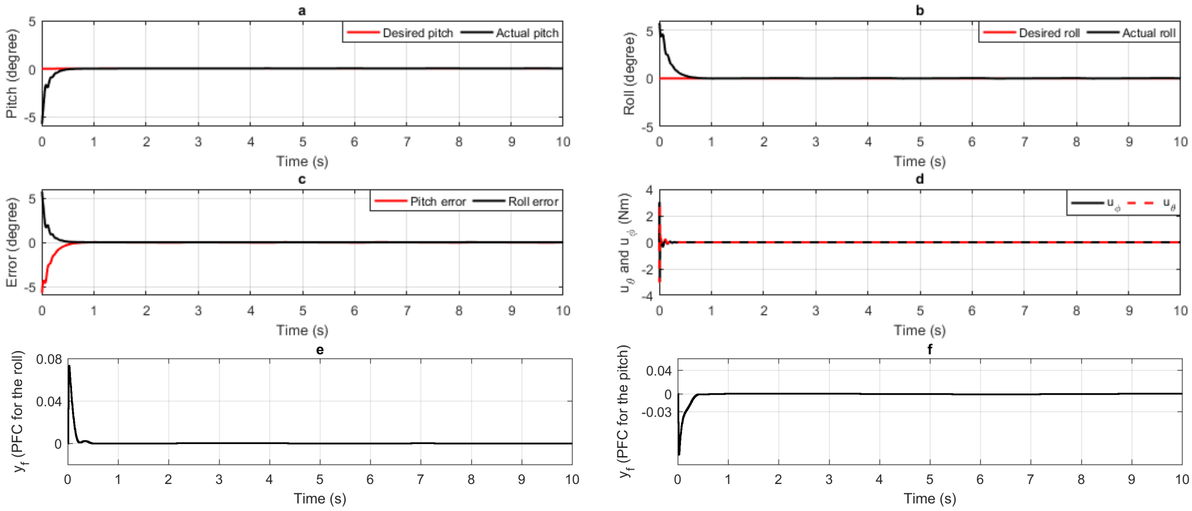

Figure 3.2: Simulation results of the stabilization test: a) pitch angle, b) roll angle, c) error, d) control signals, e) PFC output (for the roll), f) PFC output (for the pitch)

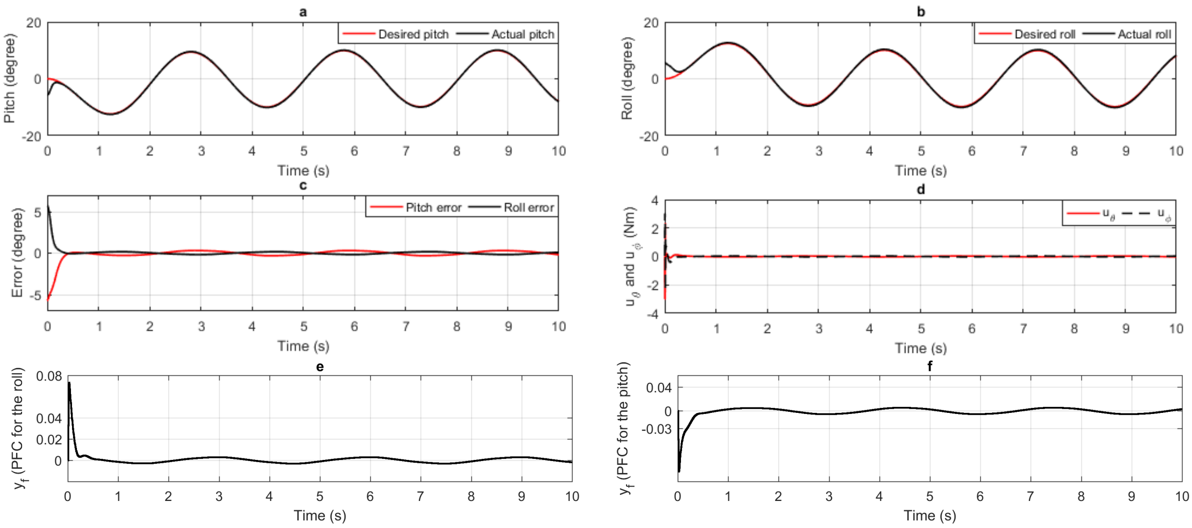

Figure 3.3: Simulation results of the tracking test: a) pitch angle, b) roll angle, c) error, and d) control signals, e) PFC output (for the roll), f) PFC output (for the pitch)

The simulation results of the first test are presented in figure 3.2. A good stabilization of the roll and pitch angles (figure 3.2a-b) is obtained. The errors is very small (figure

3.2c) and the control signal is practically realisable (figure 3.2d). The outputs $y_f(t)$ of the PFC are shown in figure 3.2e-f, one can observe that $y_f(t)$ is small enough and thus satisfies $y_a(t) \approx y(t)$. For the second test, the obtained results are shown in figure 3.3. The proposed ASC controller managed to maintain the roll and pitch angles tracking the desired trajectories even after increasing the inertia value to 400% from its nominal value (figure 3.3a-b). One can remark that the tracking error (figure 3.3c) is relatively small and the control inputs (figure 3.3d) do not require much effort in order to achieve the objective of minimizing the error. The outputs $y_f(t)$ of the PFC are shown in figure 3.3e-f, again, one can observe that $y_f(t)$ is small enough and thus satisfies $y_a(t) \approx y(t)$.

3.5 Conclusion

In this chapter, an adaptive synergetic controller for a class of nonlinear systems without requiring knowledge of the system parameters and dynamics is proposed. Derived from the synergetic control theory, a SAC-like adaptive synergetic control structure is synthesised. The stability analysis of the closed loop adaptive system is formally established. To validate the theoretical results, the proposed algorithm is tested on an experimental quadrotor system without prior information about the system. The obtained experimental results confirm the effectiveness and robustness of the adaptive controller. For practical applications, the proposed controller only requires a PFC in order to satisfy the almost passivity condition of the system. The latter becomes a constraint and therefore it presents a challenge to be addressed in the future for the purpose of relaxing the passivity conditions and having an implementable controller without PFC.

Chapter 4

Experimental Validation

> *The less attachment to the world. The easier your life.*
>
> Umar ibn Al-Khattab

4.1 Introduction

One of the most used research platform for experimental tests is probably the quadrotor aerial robot. Basically, this is often due to their simple structure and low cost. For this, numerous universities have designed their own quadrotor and a curiously open source projects are developing.

The best way to use such platforms is to start from an open source project. The use of this type of project provides a functional solution with the ability to make any kinds of changes, both hardware and software. One of the most famous open-hardware and open-software multirotor projects is DIY drone project [2]. The open source software code named ArduCopter is a generic customized code that can be used for many type of aerial multirotor robots. The hardware is developed and marketed by 3DR [3] and a set of platforms are available, see Figure 4.3. The software development follows the evolution of material and two types of autopilots are available, the APM2.6 and PixHawk, that can be used on any platform. The cheaper solution was selected based on DIY Quad and APM2.6 autopilot. The cost of all hardware needed do not exceed 800 Euros. Figure 4.2 presents the developed DIY drone.

This chapter presents the experimental used test-bench (see, Figure 4.3). The effectiveness and the performances of the proposed ASC in this thesis are validated by performing many experiments using this platform.

4.2 Test-bench presentation

Experiments were done based on the open-hardware and open-software DIY drone project. We have used the platform shown in Figure 4.3. It is a test-bench with DIY Quad used for indoor tests. Specially, to validate the developed adaptive controllers for attitude control. In this experiment, due to the quality of the measurements, one consider only the roll and pitch motions.
The test bench for attitude control is composed of a support and the DIY Quad. The DIY Quad is a Quadcopter equipped with:

1. Holder with rotation ball joint
2. Quad frame
3. Pixhawk autopilot
4. Four electronic speed controllers (ESCs)
5. Four brushless motors with propellers
6. Graupner RC receiver
7. 3DR Telemetry Radios
8. u-blox GPS with compass
9. Power distribution board and LiPo battery
10. APM Power module for current consumption and battery voltage measurements

In addition, the RC transmitter is used to perform manual control and fly mode selection. The open source ground station (QGroundControl or Misson Planner) is used to visualize telemetry data and to configure flight mode parameters and to calibrate sensors. See Figure 4.2 for Pixhawk 2 mounting with different other components.

The Pixhawk autopilot is based on a 32-bit STM32F427 Cortex M4F core, 168 Mhz/256 KB RAM/2 MB Flash and 32 bit STM32F103 failsafe co-processor. The embedded system is equipped with a 6 DoF Accelerometer/Gyro MPU-6000 and an ST Micro

Figure 4.1: First developed DIY quadrotor

LSM303D 3-axis accelerometer/magnetometer. The pixhawk is also equipped with a microSD card for high-rate logging over extended periods of time. The main loop operating frequency of the firmware and the acquisition frequency of accelerometer and gyros measurements are both 400Hz. Thus, stability will be better and the measurements will be more reliable than using the APM2.6 which operates at 100hz.

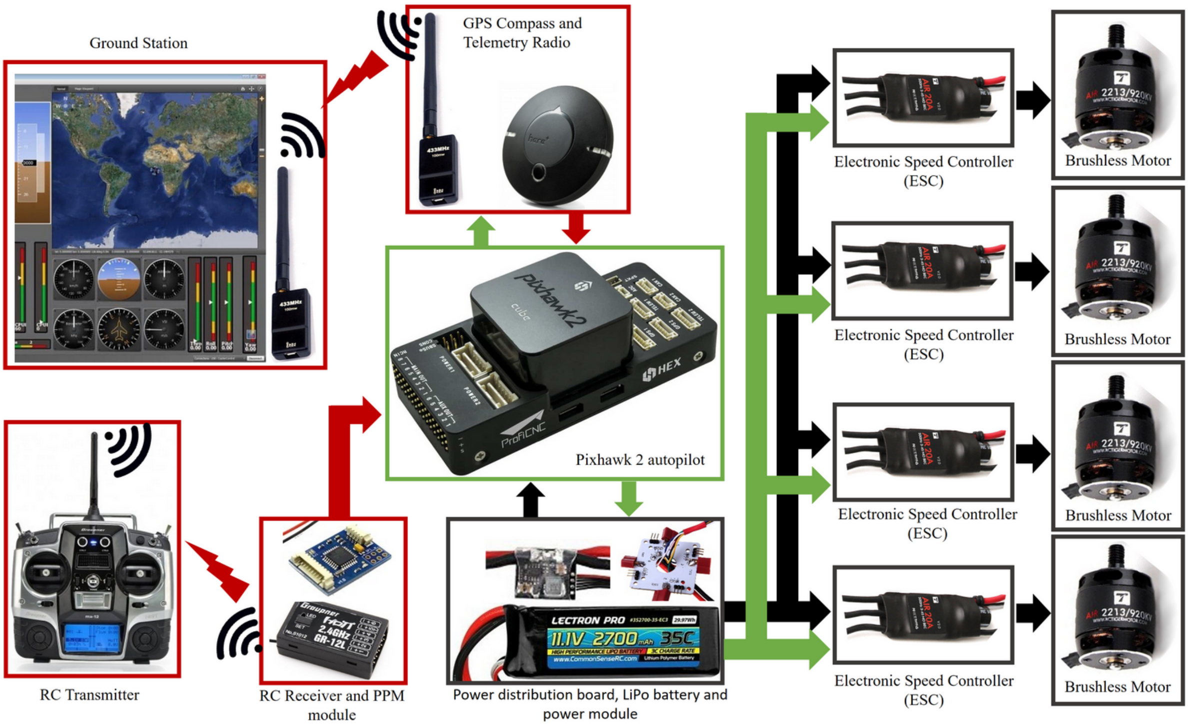

Figure 4.2: Pixhawk 2 mounting

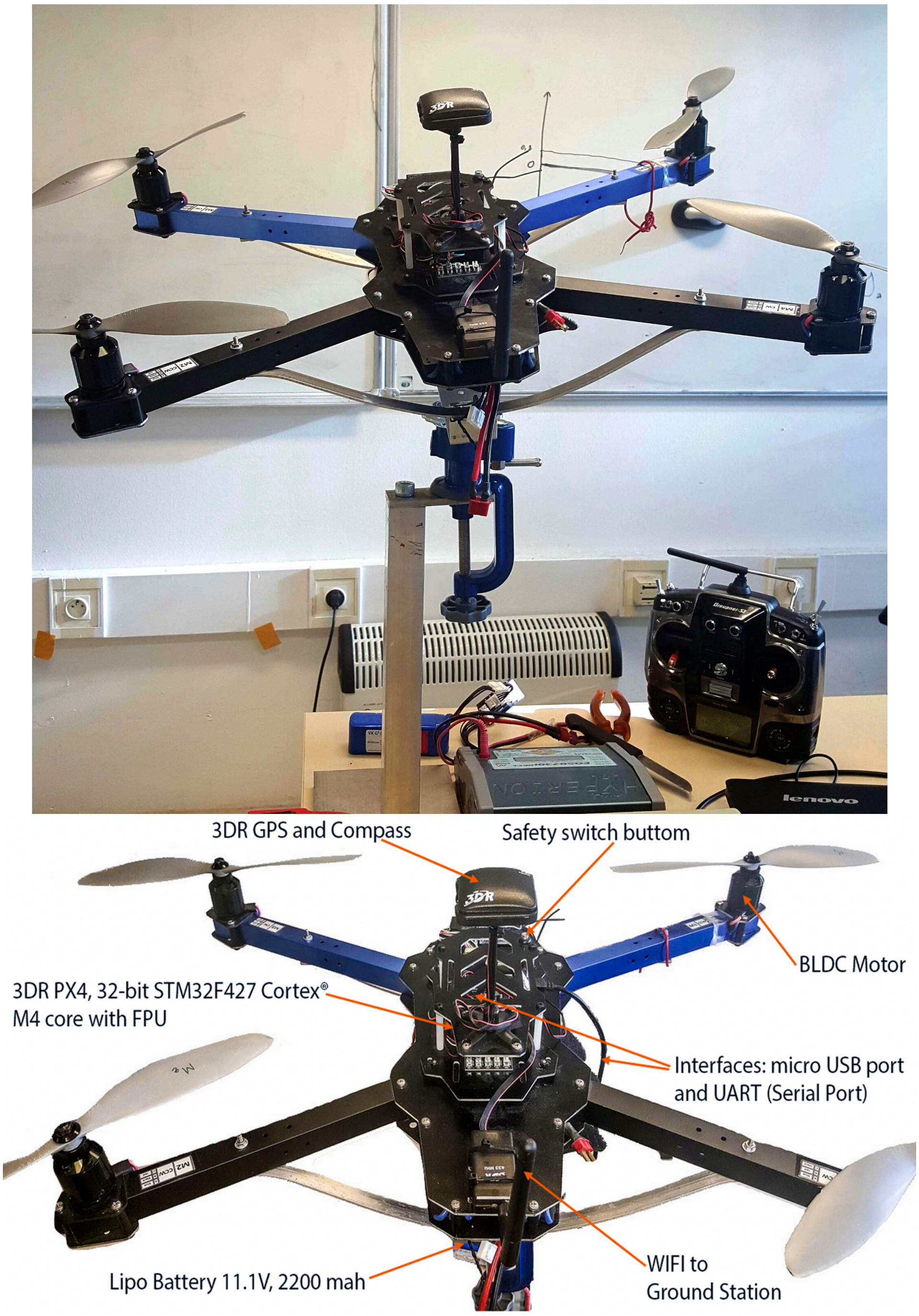

Figure 4.3: Test bench Quad equipped with the Pixhawk autopilot

4.3 Experimental validation of the proposed ASC

In this experiment, due to the quality of the measurements, one consider only the roll and pitch motions. The adaptive synergetic controller is designed in Matlab / Simulink, by using the PX4 Support from Embedded Coder one can generate the C / C ++ code from Simulink and then deploy the code into the micro controller.

The ASC controller is applied to this quadrotor without prior knowledge of its parameters. The same controller parameters above used for the simulation are used in this experiments. The adaptive control gains were again initialized with $K_e = K_x = K_u = K_s = 0$. If one has full knowledge about the system, one could shorten the cycle of adaptation and further improve the performance after a first test, when better initial gains K conditions that makes the system SP could be used. The figures 4.4, 4.5 and 4.7 below illustrate the experimental results. Three test were done in order to show the effectiveness of the proposed controller: i) Stabilisation test (figure 4.4), ii) Stabilisation with hand push perturbation test (figure 4.5) and iii) Variable reference test by using the "Graupner SJ mx-20" remote control (figure 4.7).

For the first test, the initial conditions were set to $\phi_0 = -8.5°$ and $\theta_0 = -9°$, a good stabilisation (figure 4.4a-b) of the quadrotor with a very small error (figure 4.4c) is obtained for both roll and pitch angles. For the second test, a hand push perturbations were introduced at the ends of the quadrotor arms at $t \in [10s, 65s]$ in order to show the ability of the proposed controller to dealing with such perturbations. As shown in figure 4.5a-b, despite these perturbation, the ASC controller managed to maintain the stability of the quadrotor without much effort (figure 4.5d). The history of the adaptation gains $K_e(t), K_x(t), K_u(t)$ and $K_s(t)$ corresponding to this second test is shown in Figure 4.6, one can see the rapid adaptation of the gains which aim to minimize the error. Also, figure 4.6 shows that the adaptive gains do converge towards their ideal values that satisfy the asymptotic tracking, which confirms the theoretical results. For the third test, the ASC controller was put into a tough challenge where a variable reference was given by the remote control and the variation becomes fast when $t > 75s$ (figure 4.7). A good tracking is obtained for both roll and pitch angles as shown in figure 4.7a-b. The tracking error is relatively small and again the controller only demands minimum values (figure 4.7d) in order to achieve the objective of minimizing the error.

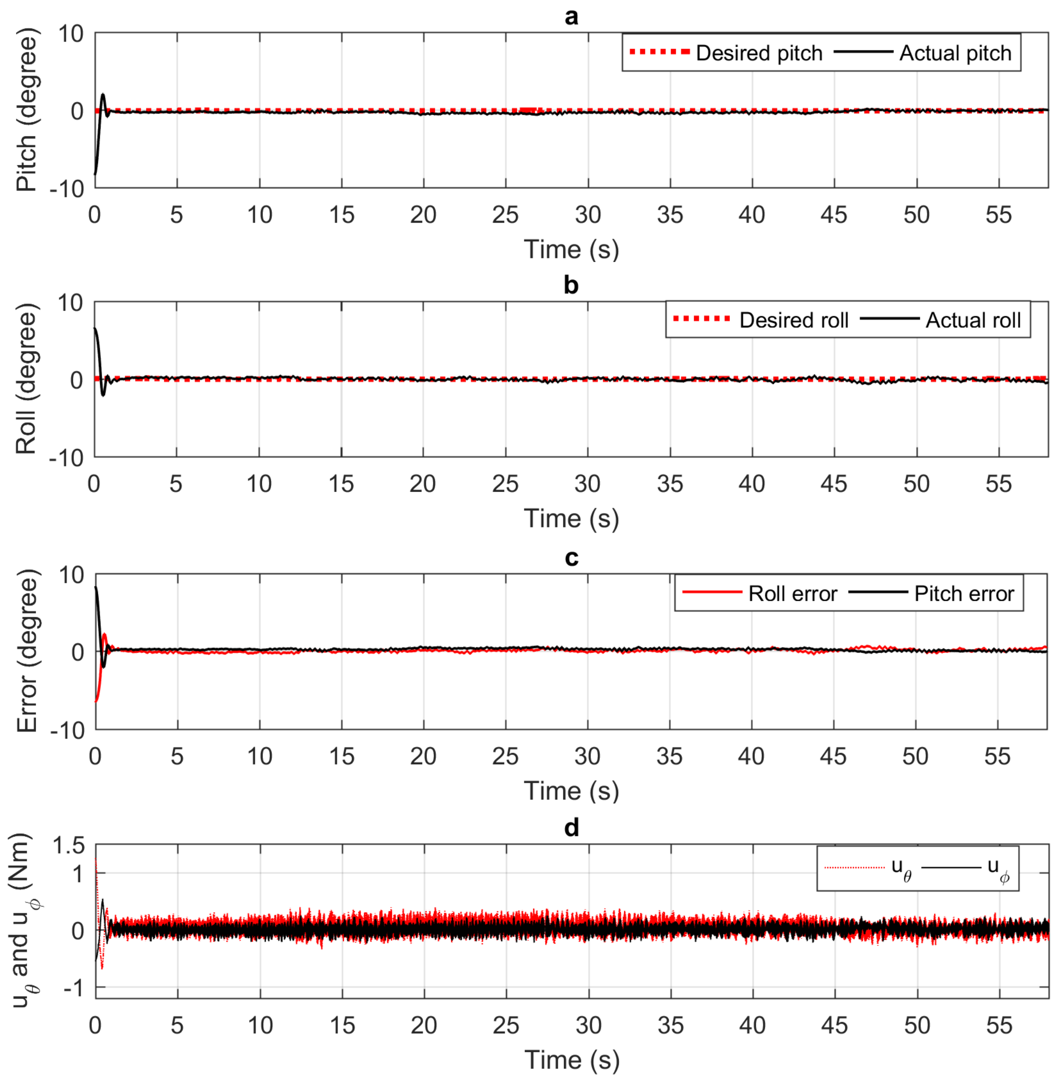

Figure 4.4: Experimental results of the stabilization test: a) pitch angle, b) roll angle, c) error, and d) control signals

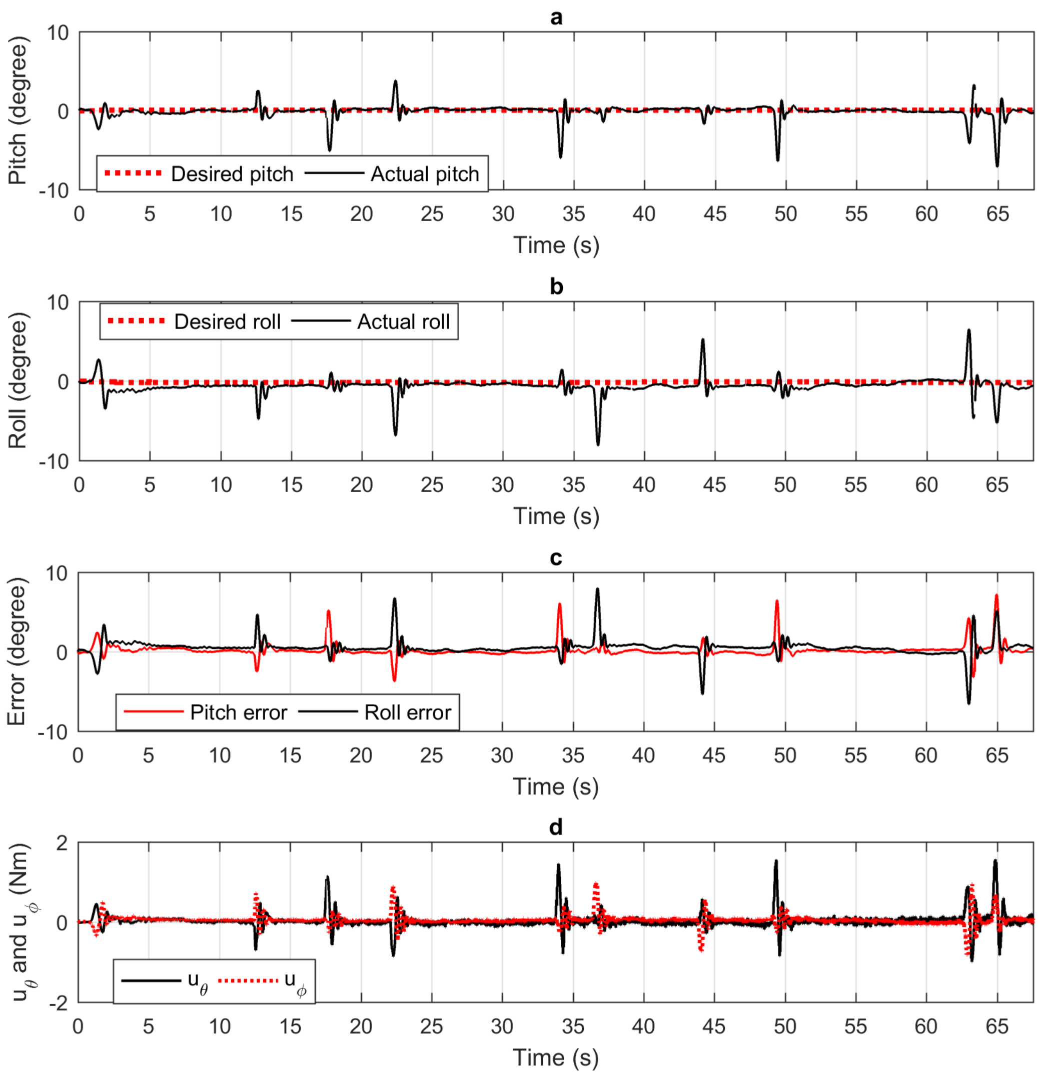

Figure 4.5: Experimental results of the stabilization with hand push perturbation test: a) pitch angle, b) roll angle, c) error, and d) control signals

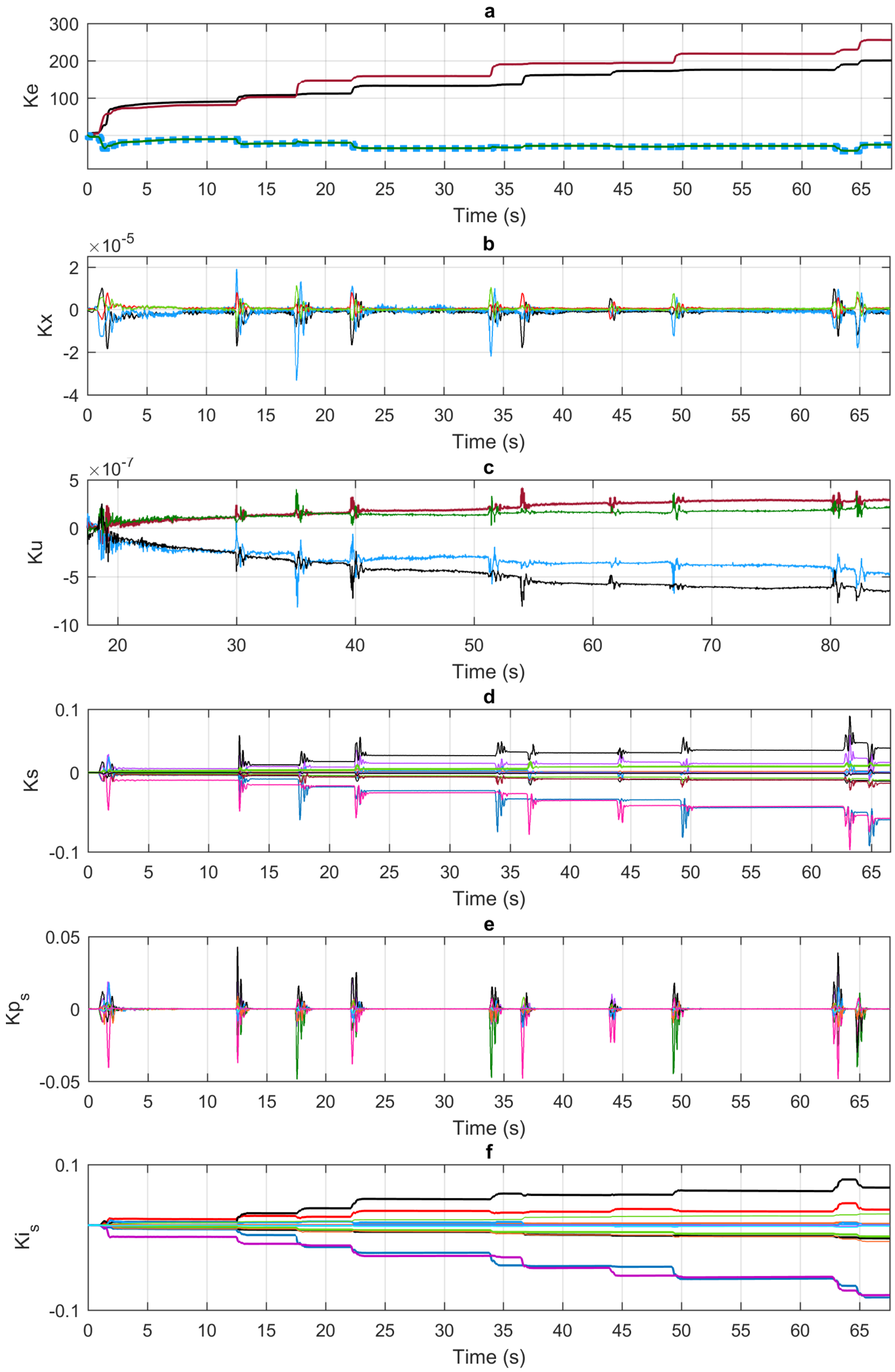

Figure 4.6: Adaptation gains history of the experimental adaptive gains: a) K_e, b) K_x, c) K_u, and d) K_s

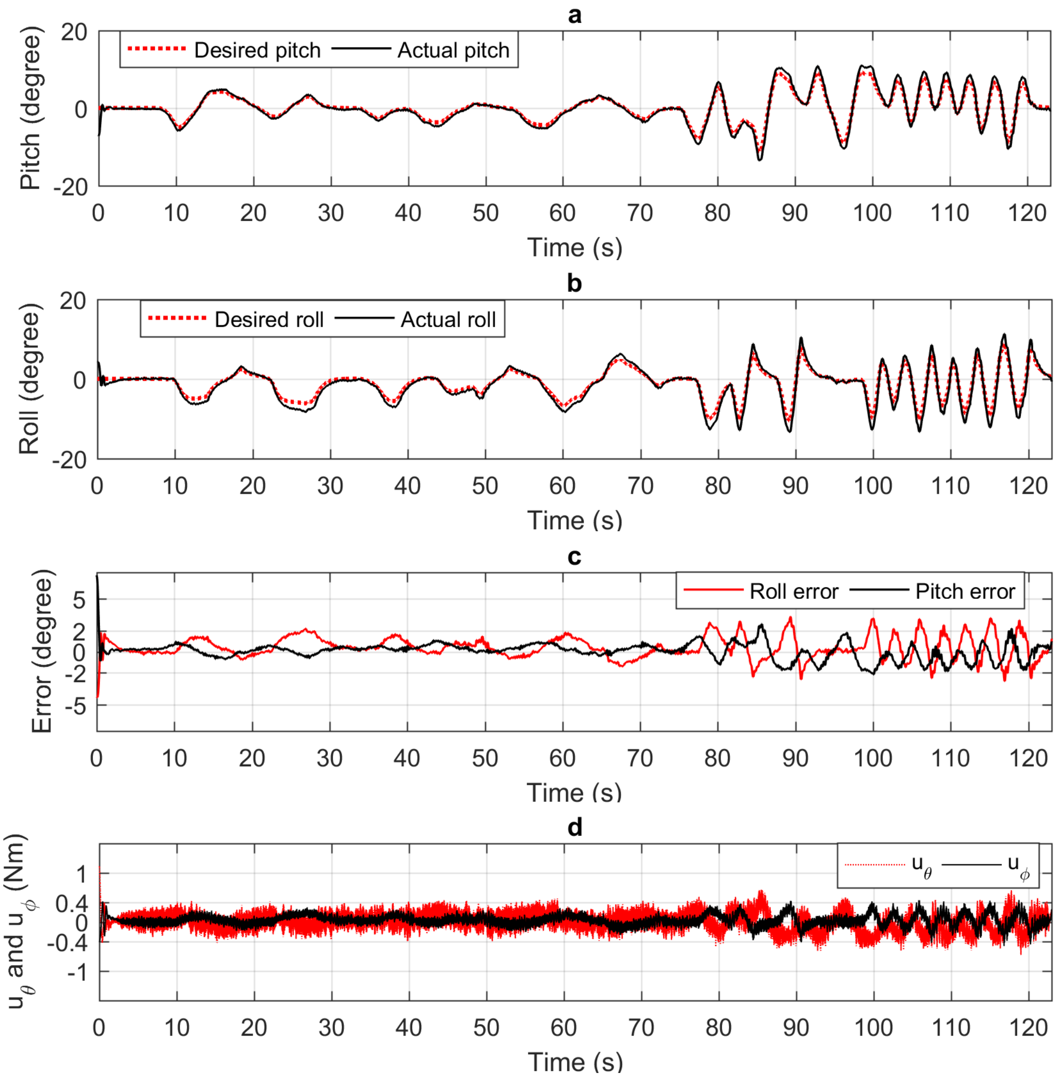

Figure 4.7: Experimental results of the tracking test of a variable references: a) pitch angle, b) roll angle, c) error, and d) control signals

4.4 Conclusion

The Unmanned Aerial Vehicles markets are in exponential developments. Especially, due to widening of the civil application fields and high potentials of aerial robotics. Generally, it is difficult to utilise commercial aerial robot for research purposes, due to the fact that researchers should develop their own flight controller so that they can access all the sensor information they need and control the robot properly. Such problem is solved by the emergence of open source projects in this domain. Probably, one of the most known open source project in aerial robots community is ArduCopter project. Sponsored by 3DRobotics, the project offer a complete open source solution to control various types of aerial robots. For this reason, we used this project as a start point to make experimental research.

ArduCopter project was modified and used successfully to validate research theoretical results. Indeed, the proposed ASC was validated using DIY Quadcopter hardware and ArduCopter software project. A test-bench for attitude control was assembled using holder with rotation ball joint for security purposes. The obtained experimental results show the effectiveness and performances of the proposed solutions.

General Conclusion

This thesis gaves some contributions on the theoretical background of the SAC and on the problem of the Synergetic Controller design when the system to be controlled is unknown. There are especially three majors contributions presented in this dissertation :

The first one was given in chapter 1, where an extension in decentralized simple adaptive control with the new relaxed W-Passivity conditions is presented. A formal proof of stability of the DSAC is established using the new WASP theorem for square nonlinear systems with not necessarily symmetric matrix CB.

The second contribution of this dissertation was presented in Chapter 2, in which a novel fractional order decentralized simple adaptive control law were proposed. Firstly, the ASPR conditions for the design of a stable fractional adaptive system were established. A fractional Order parallel feedforward compensator (FO-PFC) to realize an augmented ASPR system were then provided and a new ASPR-based FODSAC controller were proposed. The stability analysis of the proposed control scheme were presented and simulation example comparing the standard DSAC with the new FODSAC were provided, showing the performance of the proposed method.

The third contribution was presented in Chapter 3, where a new Adaptive Synergetic Controller was proposed as a solution for the problem of the design of the standard Synergetic Control law when the system parameters and dynamics are unknown. Thanks to the structure of the ASC controller with an adequate choice of the lyapunov function, we were able to eliminate the residual term in the derivative of the lyapunov function of the standard SAC, that affect negativity of such function. For a clear discussion of the system stability, and with a mathematical sense we substitute the residual term by its Jacobian linear approximation and with an appropriate choice of the macro-variable we were able to achieve the asymptotic convergence of the system to a desired manifold. The performances and effectiveness of the proposed solution were illustrated via simulation results and compared with existing work. Experimental application to a quadrotor system was given to validate the proposed ASC scheme.

Throughout the completion of the quadrotor project, several future challenges were identified. The first one related to the proposed ASC controller, in which the computation time of its algorithm could be decreased by taking only the diagonal elements, as in the standard DSAC presented in chapter 1. The second challenge is the implementation of the proposed controllers (ASC and the FODSAC) into a "Quadrotor manipulator". The Quadrotor manipulator project is still under developments, and the developed initial platform of the project can be seen in appendix B, wich it is a quadrotor equipped with a 7 degree of freedom robotic arm. The motion control of such vehicle is a challenging issue since the vehicle is characterized by an unstable dynamics and the presence of the object causes non trivial coupling effects. The proposed adaptive controllers presented in this thesis will be then tested in this vehicle.

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Appendix A

Proof of lemma 2.4

Using matrix trace properties and definitions we have

$$tr\left\{AWA^T\right\} = tr\left\{WA^TA\right\} = tr\left\{W(A^TA)\right\} \tag{4.1}$$

Since W is diagonal, then one can write:

$$\begin{aligned} tr\left\{W(A^TA)\right\} &= \sum_{i=1}^{n}\left[W\left(A^TA\right)\right]_{ii} = \sum_{i=1}^{n} W_{ii}\left(A^TA\right)_{ii} \\ &= \sum_{i=1}^{n} W_{ii}\left(A^TA\right)_{ii} = \sum_{i=1}^{n} W_{ii}\sum_{j=1}^{m} A_{ij}^T A_{ji} \\ &= \sum_{i=1}^{n}\sum_{j=1}^{m} W_{ii}A_{ij}^T A_{ji} = tr\left\{AWA^T\right\} \end{aligned} \tag{4.2}$$

since $A_{ij}^T = A_{ji}$, then we can write

$$tr\left\{AWA^T\right\} = \sum_{i=1}^{n}\sum_{j=1}^{m} W_{ii}A_{ji}^2 \tag{4.3}$$

for the case of the time varying matrix $A(t)$, expression (4.3) can be written as

$$tr\left\{A(t)WA^T(t)\right\} = \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ii}a_{ji}^2(t) \tag{4.4}$$

where w_{ii} are diagonal elements of W, and a_{ji} are the elements of the matrix $A(t)$. Applying the caputo fractional derivative to (4.4) we get, $\forall\alpha \in (0,1)$, $\forall t \geqslant t_0$

$${}_{t_0}^{C}D_t^{\alpha}\left[tr\left\{A(t)WA^T(t)\right\}\right] = {}_{t_0}^{C}D_t^{\alpha}\left[\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ii}a_{ji}^2(t)\right] \tag{4.5}$$

Using the caputo fractional derivative property of linearity, we get

$$ {}_{t_0}^{C}D_t^{\alpha}\left[tr\left\{A(t)WA^T(t)\right\}\right] = \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ii}\,{}_{t_0}^{C}D_t^{\alpha}\left[a_{ji}^2(t)\right] \tag{4.6} $$

Applying lemma 2.1 we get

$$ {}_{t_0}^{C}D_t^{\alpha}\left[tr\left\{A(t)WA^T(t)\right\}\right] \leqslant 2\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ii}a_{ji}(t){}_{t_0}^{C}D_t^{\alpha}\left[a_{ji}(t)\right] \tag{4.7} $$

since w_{ii} are diagonal elements of W, then the following relationship holds

$$ \sum_{i=1}^{n}\sum_{j=1}^{m} w_{ii}a_{ji}(t){}_{t_0}^{C}D_t^{\alpha}\left[a_{ji}(t)\right] = tr\left\{A(t)W\,{}_{t_0}^{C}D_t^{\alpha}A^T(t)\right\} \tag{4.8} $$

substituting (4.8) in (4.7), relation (4.7) can be written as

$$ {}_{t_0}^{C}D_t^{\alpha}\left[tr\left\{A(t)WA^T(t)\right\}\right] \leqslant 2tr\left\{A(t)W\,{}_{t_0}^{C}D_t^{\alpha}A^T(t)\right\} \tag{4.9} $$

this complete the proof for the case when $\alpha \in (0,1)$. When $\alpha = 1$ the result coincide with the standard integer order derivatives, where $\frac{d}{dt}[tr\{A(t)WA^T(t)\}] = 2tr\{A(t)W\frac{d}{dt}A^T(t)\}$. Finally, this ends the proof.

Appendix B

The developed platform of the Quadrotor manipulator

Figure B.1: Initial developed platform of the Quadrotor manipulator: The Quadrotor is equipped with a 7 degree of freedom robotic arm.

ملخص: قدمت بعض تطبيقات تقنية التحكم التكيفي البسيط مشكلة تعقيد التصميم الناشئة عن عدد كبير من المعاملات التي يتم اختيارها. تم تقليل هذه المشكلة مؤخرًا من خلال فكرة وحدة التحكم التكيفي البسيط اللامركزية، والتي تأخذ بعين الاعتبار فقط قطر المصفوفات المتغيرة مع الزمن. لذلك، سهلت المتطلبات الحسابية المنخفضة لـوحدة التحكم التكيفي البسيط اللامركزية التنفيذ في الوقت الفعلي. في الجزء الأول من هذه الأطروحة، قمنا بزيادة تحسين مرونة وحدة التحكم التكيفي البسيط اللامركزية من خلال النظر في المشتق الكسري للمكاسب التكيفية. أولاً، يتم وضع شروط تقريبا حقيقي إيجابي بدقة لتصميم نظام تكيف كسري مستقر. بعد ذلك يتم توفير معوض تغذية كسري حيث يتم تركيبه موازيا للنظام الأصلي بغرض تحقيق نظام تقريبا حقيقي إيجابي بدقة معزز. تم اثبات استقرار وحدة التحكم المقترحة، مع توفير مثال محاكاة مقارنة وحدة التحكم التكيفي البسيط اللامركزية مع وحدة التحكم التكيفي البسيط اللامركزية الكسرية الجديدة. في الجزء الثاني من هذه الأطروحة، تم اقتراح وحدة تحكم تآليل تكيفية جديدة كحل لمشكلة تصميم قانون التحكم التآزري القياسي عندما تكون معلمات النظام ودينامياته غير معروفة. تم اثبات استقرار وحدة التحكم التكيفية المقترحة رسميًا. تم تجريبيا تطبيق وحدة التحكم المقترحة على نظام طائرة بدون طيار للتحقق من صحة النتائج النظرية.

كلمات مفتاحية: التحكم التكيفي البسيط، التحكم التآزري التكيفي، الاستقرار، طائرة بدون طيار، نظام كسري

Résumé : Certaines applications de la technique SAC ont présenté un problème de complexité de conception découlant d'un grand nombre de paramètres et coefficients sélectionné. Ce problème a été récemment résolu par l'idée du contrôleur adaptatif simple décentralisé (DSAC), qui ne considère que la diagonale des matrices de gain variant dans le temps. Par conséquent, la diminution des exigences de calcul du DSAC a facilité la mise en œuvre en temps réel. Dans la première partie de cette thèse, nous améliorons encore la flexibilité du DSAC en considérant la dérivée fractionnelle des gains adaptatifs. Le nouveau contrôleur proposé est appelé DSAC d'ordre fractionnaire (FODSAC). Premièrement, les conditions ASPR pour la conception d'un système adaptatif fractionnaire stable sont établies. Un compensateur à action directe parallèle d'ordre fractionnaire (FOPFC) pour réaliser un système ASPR augmenté est alors fourni et un nouveau contrôleur FODSAC basé sur ASPR est proposé. L'analyse de stabilité du schéma de commande proposé est présentée et un exemple de simulation comparant le DSAC standard au nouveau FODSAC est fourni, montrant les performances de la méthode proposée. Dans la seconde partie de cette thèse, un nouveau contrôleur adaptatif synergétique (ASC) est proposé comme solution au problème de la conception de la loi de commande synergétique standard lorsque les paramètres et la dynamique du système sont inconnus. La stabilité du contrôleur adaptatif proposé est formellement prouvée via le Lyapunov et le principe d'invariance de Lasalle. Une application expérimentale sur un quadrirotor est donnée pour valider les résultats théoriques.

Mots-clés: Commande adaptatif simple, Commande synergétique adaptative, stabilité, quadrirotor, systéme d'ordre fractionnaire

Abstract: Some applications of the Simple Adaptive Control (SAC) technique presented a design complexity issue arising from a large number of parameters and coefficients to select. This issue was recently decreased by the idea of the Decentralized Simple Adaptive Controller (DSAC), which considers only the diagonal of the time-varying gain matrices. Therefore, the decreased computational requirements of the DSAC facilitated real-time implementation. In the first part of this thesis, we further improve the flexibility of the DSAC by considering the fractional derivative of the adaptive gains. The proposed new controller is called Fractional Order DSAC (FO-DSAC). Firstly, the ASPR conditions for the design of a stable fractional adaptive system are established. A fractional Order parallel feedforward compensator (FO-PFC) to realize an augmented ASPR system is then provided and a new ASPR-based FODSAC controller is proposed. The stability analysis of the proposed control scheme is presented and simulation example comparing the standard DSAC with the new FODSAC is provided, showing the performance of the proposed method. In the second part of this thesis, a new Adaptive Synergetic Controller (ASC) is proposed as a solution for the problem of the design of the standard Synergetic Control law when the system parameters and dynamics are unknown. The stability of the proposed adaptive controller is formally proven via the Lyapunov approach. Experimental application to a quadrotor system is given to validate the theoretical results.

Keywords: Simple Adaptive Control, Adaptive synergetic control, stability, quadrotor, fractional order system