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HARDNESS MEASUREMENTS VIA AN ELLIPSOID-SHAPED INDENTER

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In this theoretical study, we have chosen to use a body of an ellipsoidal geometric form as an indenter, where we determined the mathematical expression of the static hardness as function of the depth and the radii of the area of projected imprint. We used the general formula of the static hardness expressed by the ratio of a force applied perpendicular on the indenter to the resulting area of the imprint; also, we have established the real imprint (cap) of an indenter of revolution ellipsoid form. Finally, geometrical and mathematical approaches have been used to derive the formula of the static hardness expression.

Keywords: hardness measurement, mechanical characterization, bulk deformation, geometric modeling.

Notation

A, B, C	–	semi-axes of an ellipsoid
d	–	diameter of the projected imprint of a revolution ellipsoid indenter
H_e	–	static hardness of the ellipsoidal indenter
H_{ec}	–	hardness measured via a body of revolution, in case of a circular imprint
H_{er}	–	hardness measured via a body of revolution, in case of a true imprint
F	–	applied load
S	–	imprint surface
a^-, b^-	–	semi-axes of the projected elliptic surface
h	–	depth of the imprint
r	–	radius of the imprint projected surface

Introduction. Various dynamic and static hardness tests are applied to determine the hardness of uncoated and coated materials. They imply penetration of a harder indenter into a softer body [1].

The material hardness is a very important property in the material-related industries and technologies. It is defined as the mechanical resistance of the tested material to penetration of a harder material [2–10].

The hardness of coated and massive materials is an old problem, which has been a subject of many theoretical and experimental studies. There are numerous geometrical and mathematical models, which are used to determine the hardness of different materials. The static hardness H_e is expressed by the ratio of the applied load F to the imprint surface area S [2–10]. Its mathematical expression is given by the following formula:

$$H_e = \frac{F}{S}.$$

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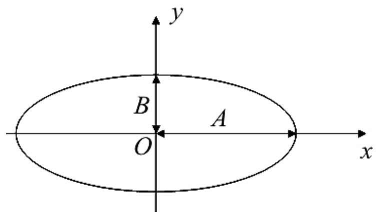

Fig. 1. The ellipse.

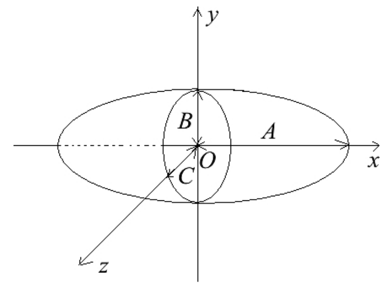

Fig. 2. The ellipsoid.

In the hardness tests, the indenter shape (pyramidal, conic, spheric, etc.) is a very significant factor because of the geometrical shape of the resulting imprint and the phenomena occurring during and after the tests (cracking, deformation, etc.).

In the present theoretical study, an indenter of an ellipsoidal geometrical shape is used to measure the static hardness of materials, which is calculated from the indenter semi-axes A, B, and C, semi-axes of the projected imprint a^-, b^-, and r, the applied load F, and the imprint depth h. Furthermore, the hardness is theoretically assessed by applying the geometrical approaches to the true imprint cap.

The paper is organized as follows: Firstly, general theoretical concepts related to the indenter theory are highlighted. Secondly, the area of the imprint is calculated as a function of its semi-axes and depth. Finally, the mathematical and geometrical assumptions are made to simplify the problem and yield the engineering formula for the material hardness.

1. Mathematical Concepts.

1.1. An Ellipse. An ellipse is formed by cutting a three-dimensional cone with a slanted plane. Its radius varies between A along the x axis and B along the y axis [11].

The standard equation of an ellipse (Fig. 1) with a center at the Cartesian system origin aligned with the axes is

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1. \quad (1)$$

Thus, the ellipse area (S) is

$$S = \pi AB. \quad (2)$$

1.2. An Ellipsoid. The standard characteristic equation of an ellipsoid centered at the origin of the Cartesian system and aligned with the axes is [11]:

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 + \left(\frac{z}{C}\right)^2 = 1. \quad (3)$$

For an ellipsoid of revolution of semi-axes (R, R, C), respectively, along axes Ox, Oy, and Oz (Fig. 2), its characteristic equation is

$$\left(\frac{x}{R}\right)^2 + \left(\frac{y}{R}\right)^2 + \left(\frac{z}{C}\right)^2 = 1. \quad (4)$$

We assume that

$$x^2 + y^2 = r^2. \quad (5)$$

By introducing Eq. (5) into Eq. (4), we get

$$\left(\frac{r}{R}\right)^2 + \left(\frac{z}{C}\right)^2 = 1. \quad (6)$$

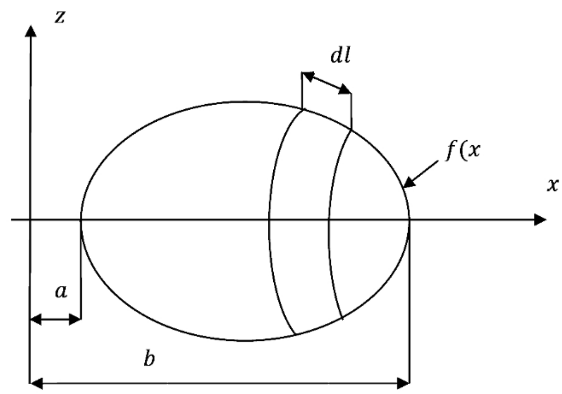

Fig. 3. A body of revolution.

1.3. Surface of a Body of Revolution. The side area of a body generated by a revolution of a curve of a characteristic equation $z = f(x)$, around axis Ox and ranging between points a and b (Fig. 3) is expressed by [11]:

$$S = \int_a^b 2\pi z dl, \tag{7}$$

where dl is the differential of the curve arc, which is given by the formula:

$$dl = \sqrt{(dz)^2 + (dx)^2}. \tag{8}$$

By introducing Eq. (8) into Eq. (7), we get

$$S = \int_a^b 2\pi z \sqrt{1 + \left(\frac{dz}{dx}\right)^2} dx \qquad \text{with} \qquad \frac{dz}{dx} = \frac{df(x)}{dx}. \tag{9}$$

Then, Eq. (7) may be written as

$$S = \int_a^b 2\pi f(x) \sqrt{1 + \left(\frac{df(x)}{dx}\right)^2} dx. \tag{10}$$

2. Hardness Measured by an Ellipsoidal Indenter.

2.1. Principle of Penetration. In case of hardness tests with an ellipsoidal indenter, we used the indenter with semi-axes A, B, and C subjected to the action of a known constant force applied perpendicular to the indenter and under defined conditions. We measure the dimensions of the imprint (transverse length and depth) and determine the hardness.

2.2. Static Hardness. The static hardness H_e is expressed by the ratio of the applied load F to the imprint projected surface S [12], as follows:

$$H_e = F/S. \tag{11}$$

For an elliptical indenter, the mathematical expression of the areas of the projected imprint with the semi-axes a^- and b^- is given by the following relation:

$$S = \pi a^- b^-. \tag{12}$$

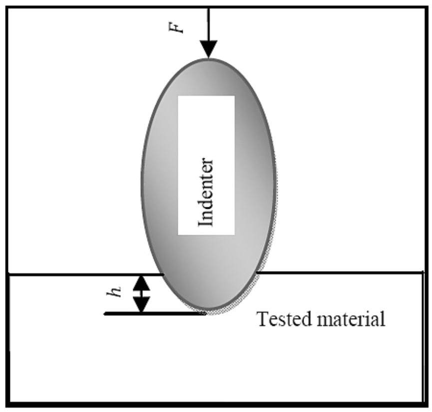

Fig. 4. Penetration of an ellipsoid.

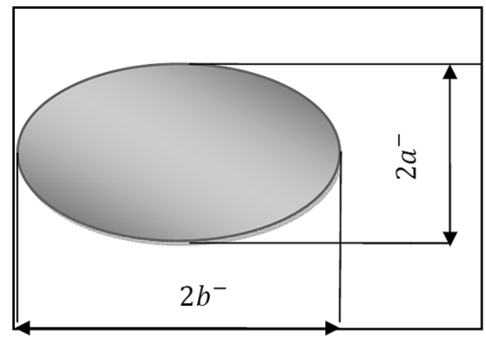

Fig. 5. Projected imprint area of an ellipsoidal indenter.

Introducing Eq. (12) into Eq. (11), the static hardness becomes:

$$H_e = \frac{F}{\pi a^- b^-}. \tag{13}$$

For an ellipsoidal indenter (Figs. 4 and 5), the semi-axes a^- and b^- of the imprint projected surface can be determined from the penetration depth h and the semi-axes of the indenter A, B, and C as follows:

$$a^- = A\sqrt{\frac{2h}{C} - \left(\frac{h}{C}\right)^2}, \tag{14}$$

$$b^- = B\sqrt{\frac{2h}{C} - \left(\frac{h}{C}\right)^2}. \tag{15}$$

Then the expression for hardness becomes

$$H_e = \frac{C^2 F}{\pi AB(2hC - (h)^2)}. \tag{16}$$

2.3. Hardness Measured via a Revolution Ellipsoidal Indenter.

2.3.1. A Circular Imprint. For an ellipsoidal indenter with a circular section ($C = R = B$), the projected surface of the imprint becomes a circular disk of a radius $r = d/2$ (Fig. 6). We can write the expression of the radius according of the semi-axes of the body of revolution A and R and the imprint depth h as follows:

$$\frac{d}{2} = r = R\sqrt{\frac{2h}{A} - \left(\frac{h}{A}\right)^2}. \tag{17}$$

Then, the hardness formula takes the following form:

$$H_{ec} = \frac{F}{\pi r^2} = \frac{4F}{\pi d^2} = \frac{A^2 F}{\pi R^2 (2hA - (h)^2)}. \tag{18}$$

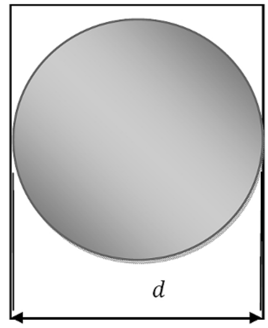

Fig. 6. Projected imprint area of a revolution ellipsoid indenter.

2.3.2. A True Imprint. For an ellipsoidal indenter (body of revolution) of a circular section ($C = B = R$), the imprint is a cap of a circular basis of diameter d and depth h (Fig. 4). Accordingly, the surface of this imprint is derived from the following formula:

$$S = \int_{A-h}^{A} 2\pi z \sqrt{1 + \left(\frac{dz}{dx}\right)^2}\, dx. \tag{19}$$

From the characteristic equation (1) we get:

$$z^2 = R^2\left(1 - \frac{x^2}{A^2}\right). \tag{20}$$

The derivative of Eq. (20) yields the following expression:

$$zz' = -\frac{R^2 x}{A^2}. \tag{21}$$

By introducing Eqs. (20) and (21) into Eq. (19), we get

$$S = \int_{A-h}^{A} 2\pi z \sqrt{1 + \left(\frac{dz}{dx}\right)^2}\, dx = \int_{A-h}^{A} 2\pi \sqrt{z^2 + \left(z\frac{dz}{dx}\right)^2}\, dx = 2\pi R \int_{A-h}^{A} \sqrt{1 - \frac{x^2}{A^2}\left(1 - \frac{R^2}{A^2}\right)}\, dx.$$

Let $t = \beta \frac{x}{A}$ with $\beta = \frac{A^2 - R^2}{A^2}$.

Then, the imprint area becomes

$$S = \frac{2\pi CA}{\beta} \int_{\frac{\beta(A-h)}{A}}^{\beta} \sqrt{1 - t^2}\, dt.$$

By integration, we get

$$S = \frac{\pi RA}{\beta}[t\sqrt{1 - t^2} + \arcsin t]_{\beta\frac{(A-h)}{A}}^{\beta}.$$

Thus,

$$S = \frac{\pi RA}{\beta}[\beta\sqrt{1 - \beta^2} + \arcsin\beta] - \frac{\pi RA}{\beta}\left[\left(\beta\frac{A-h}{A}\right)\sqrt{1 - \left(\beta\frac{A-h}{A}\right)^2} + \arcsin\left(\beta\frac{A-h}{A}\right)\right]$$

$$= \pi AR\sqrt{1-\beta^2} + \pi AR\frac{\arcsin\beta}{\beta} - \pi R(A-h)\sqrt{1-\left(\beta\frac{A-h}{A}\right)^2} - \frac{\pi(A-h)}{\beta\left(\frac{A-h}{A}\right)}\arcsin\left(\beta\frac{A-h}{A}\right). \quad (22)$$

For a low applied load, the imprint is very small ($A >> h$) so $\frac{A-h}{A} \approx 1$

$$S = \pi Rh\left(\sqrt{1-\beta^2} + \frac{\arcsin\beta}{\beta}\right). \quad (23)$$

Then, the hardness corresponding to the penetration depth h is controlled by the following equation:

$$H_{er} = \frac{\beta F}{\pi Rh(\beta\sqrt{1-\beta^2} + \arcsin\beta)}. \quad (24)$$

From Eq. (17), the penetration depth h is a function of both indenter dimensions (A, R) and the radius r of the projected imprint,

$$h = \frac{AR - A\sqrt{R^2 - r^2}}{R} \quad \text{(accepted),} \qquad \text{and} \qquad h = \frac{AR + A\sqrt{R^2 - r^2}}{R} > A \quad \text{(rejected).}$$

Thus,

$$h = A\left(1 - \sqrt{1-\left(\frac{r}{R}\right)^2}\right). \quad (25)$$

For a small imprint (in cases of microhardness and nanohardness), the ratio $\left(\frac{r}{R}\right)$ is negligible [12]:

$$\left(\sqrt{1-\left(\frac{r}{R}\right)^2}\right) = 1 - \frac{1}{2}\left(\frac{r}{R}\right)^2. \quad (26)$$

By introducing Eq. (26) into Eq. (25), we get

$$h = \frac{A}{2}\left(\frac{r}{R}\right)^2. \quad (27)$$

Then, the expression for the imprint surface is reduced to

$$S = \pi\frac{Ar^2}{2R}\left(\sqrt{1-\beta^2} + \frac{\arcsin\beta}{\beta}\right). \quad (28)$$

Then, the hardness can be described by the following formula:

$$H_{er} = \frac{2R\beta F}{\pi Ar^2(\beta\sqrt{1-\beta^2} + \arcsin\beta)} = G\frac{F}{r^2} \quad (29)$$

with the constant

$$G = \frac{2R\beta}{\pi A(\beta\sqrt{1-\beta^2} + \arcsin\beta)}. \quad (30)$$

CONCLUSIONS

1. In the present paper, the most important results of using an ellipsoid-shaped indenter for measuring the material hardness are the derived mathematical expressions for the imprints of different geometric shapes.

2. In the derivation of the hardness expression for the case of an ellipsoid-shaped indenter, the considered surface of the resulting imprint is, in the first case, taken as an elliptical section (projection of the true imprint). The related calculations are simple, but in the second case, where the surface of the imprint is treated as an ellipsoidal cap, the hardness expression is more complex. Thus, a geometrical approach is used to simplify the mathematical expressions for the resulting imprint configuration and hardness value.

3. The main conclusion is that application of an ellipsoid-shaped indenter yields new theoretical and experimental findings, which widen the scope of applications of various hardness test methods.

4. In this theoretical study, the hardness expression of a spherical indenter is derived. Also, the differences between an ellipsoid and spherical indenters are demonstrated.

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