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A New Operational Matrix of Orthonormal Bernstein Polynomials and Its Applications

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Abstract

In this work, we introduce a new general procedure of finding operational matrices of integration P, differentiation D, and product $\hat{C}$ for the orthonormal Bernstein polynomials (OBPs) in the same way as in [12], S. A. Yousefi, M. Behroozifar, *Operational matrices of Bernstein polynomials and their applications,* Int. J. Syst. Sci.41, 709-716, 2010. As an application, these matrices can be used to solve integral and integro-differential equations, differential equations, and in some problems in the calculus of variations and optimal control. The efficiency of this approach is shown by applying this procedure on some illustrative examples.

Keywords: Orthonormal Bernstein polynomials(OBPs), Operational Matrix, Integro-differential equation, Differential equation.
2010MSC: 34K28, 45G10, 45D05.

1. Introduction

In the last three decades, approximations by orthonormal family of functions have played a vital role in the development of physical sciences, engineering and technology in general and mathematical analysis. They have been playing an important part in the evaluation of new techniques to solve problems such as identification, analysis and optimal control. The aim of these techniques is to obtain effective algorithms that are suitable for the digital computers. The motivation and philosophy behind this approach is that it transforms the underlying differential equation of the problem to an algebraic equation, thus simplifying the solution process of the problem to a great extent. The Bernstein polynomials are not orthonormal so their use in the least square approximation are limited. Also, to the best of our knowledge, the operational matrix for orthonormal Bernstein polynomials (OBPs for short) was not investigated. To overcome this difficulty, Gram-Schmidt orthonormalization process can be used to construct the orthonormal Bernstein polynomials. In this paper, a general procedure of forming the operational matrices of integration P. differentiation D, and product $\hat{C}$ for the orthonormal Bernstein polynomials are given

$$\int_0^x OB(t)dt \approx P \times OB(x)$$

$$\frac{d}{dx} OB(x) \approx D \times OB(x)$$

$$c^T OB(x) OB(x)^T \approx OB(x) \times \hat{C}$$

where $OB(x) = [OB_0(x), OB_1(x), .., OB_m(x)]^T$ and c is an arbitrary vector and the matrices P, D, and $\hat{C}$ are of order $(m+1)\times(m+1)$.

Special attention has been given to applications of Walsh functions [6], block-pulse functions (see [2], [16], and [3]), Laguerre polynomials [5], Legendre polynomials [4],[9], Chebyshev polynomials [10], [14], Taylor series [13], and Fourier series [7], [11]. This paper is structured as follows. In Sections 2 and 3, we describe the basic formulation of orthonormal Bernstein polynomials and expansion of OBPs in terms of Taylor basis. In Sections 4 to 6, we explain general procedure of operational matrices of integration, differentiation and product, respectively. In Section 7, we demonstrate the validity, the accuracy, and the applicability of these operational matrices by considering some numerical examples. In Section 8, we conclude.

2. Orthonormal Bernstein polynomials (OBPs)

The explicit representation of the orthonormal Bernstein polynomials of mth degree are defined on the interval [0, 1] in [8] by

$$OB_{j,m}(x) = \sqrt{2(m-1)+1}(1+x)^{m-j} \sum_{k=0}^{j} (-1)^k \binom{2m+1-k}{j-k} \binom{j}{k} x^{j+k}, \, j = 0, ..., m \tag{1}$$

In addition, (1) can be written in a simpler form in terms of original non-orthonormal Bernstein basis functions as in [8]

$$OB_{j,m}(x) = \sqrt{2(m-1)+1}(1+x)^{m-j} \sum_{k=0}^{j} (-1)^k \frac{\binom{2m+1-k}{j-k}\binom{j}{k}}{\binom{m-k}{j-k}} B_{j-k,m-k}(x), \, j = 0, ..., m \tag{2}$$

These polynomials satisfy the following orthogonality relation

$$\int_0^1 OB_{i,m}(t)OB_{j,m}(t)dt = \delta_{i,j}, \qquad i,j = 0,\dots,m \tag{3}$$

where $\delta_{i,j}$ is the Kronecker delta function. For example with $m=5$ we have

$$OB_{0,5}(x) = \sqrt{11}(1-x)^5$$
$$OB_{1,5}(x) = 3(11x-1)(1-x)^4$$
$$OB_{2,5}(x) = \sqrt{7}(55x^2-20x+1)(1-x)^3$$
$$OB_{3,5}(x) = \sqrt{5}(165x^3-135x^2+27x-1)(1-x)^2$$
$$OB_{4,5}(x) = \sqrt{3}(330x^4-480x^3+216x^2-32x+1)(1-x)$$
$$OB_{5,5}(x) = 462x^5-1050x^4+840x^3-280x^2+35x-1$$

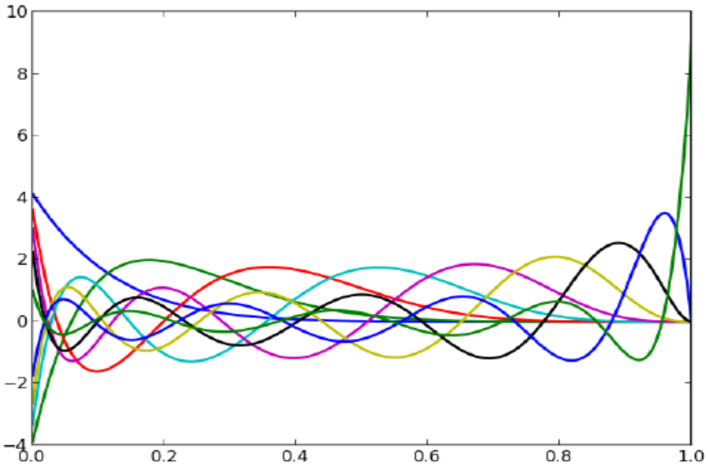

Figure 1 : Orthonormal Bernstein polynomials with n=8

A function $y(x)$ belongs to the space $L^2[0,1]$ may be expanded by Bernstein orthonormal polynomials as follows [1], [16]

$$y(x) = \sum_{j=0}^{\infty} c_{j,m} OB_{j,m}(x) \tag{4}$$

here if $<:>$ be the standard inner product on $L^2[0,1]$ then

$$c_{j,m} = \langle y(x), OB_{j,m}(x) \rangle \tag{5}$$

By truncating the series (4) up to (m + 1)th term we can obtain an approximation for $y(x)$ as follows

$$y(x) = \sum_{j=0}^{\infty} c_{j,m} OB_{j,m}(x) = c^T OB(x) \tag{6}$$

Where $c = [c_{0,m}, c_{1,m}, .., c_{m,m}]^T$ and $OB(x) = [OB_{0,m}(x), OB_{1,m}(x), .., OB_{m,m}(x)]^T$. It can be easily seen that the elements of $OB(x)$ in the $[0. 1]$ are orthogonal.

3. Expansion of OBPs in terms of Taylor basis

By using (1) and (2) we have :

$$OB_{j,m}(x) = \sqrt{2(m-1)+1}\left(\sum_{r=0}^{m-i}\alpha_{i,r}x^r\right)\left(\sum_{j=0}^{i}\beta_{i,j}x^j\right)$$

$$OB_{j,m}(x) = \sqrt{2(m-1)+1}\sum_{j=0}^{m}\left(\sum_{k=\max\{0,j-m+i\}}^{\min\{i,j\}}\alpha_{i,j-k}\beta_{i,k}\right)x^j \tag{7}$$

$$x \in [0,1], i = 0,...,m.$$

where

$$\alpha_{i,r} = (-1)^r \binom{m-i}{r}, \qquad r = 0,...,m \tag{8}$$

$$\beta_{i,j} = (-1)^{i-j}\binom{2m+1-i+j}{j}\binom{i}{i-j}, \qquad j = 0,...,i \tag{9}$$

Equation (7) can be displayed in the following matrix form

$$OB(x) = MT_m(x), x \in [0,1] \tag{10}$$

where

$$M_{i,j} = \sqrt{2(m-1)+1}\sum_{k=\max\{0,j-m+i\}}^{\min\{i,j\}}\alpha_{i,j-k}\beta_{i,k}, \qquad i,j = 0,...,m \tag{11}$$

and $T_m(x) = [1, x, x^2, ..., x^m]$. For example with m = 5 we have

$$M_5 = \begin{bmatrix} \sqrt{11} & -5\sqrt{11} & 10\sqrt{11} & -10\sqrt{11} & 5\sqrt{11} & -\sqrt{11} \\ -3 & 45 & -150 & 210 & -135 & 33 \\ \sqrt{7} & -23\sqrt{7} & 118\sqrt{7} & -226\sqrt{7} & 185\sqrt{7} & -55\sqrt{7} \\ -\sqrt{5} & 29\sqrt{5} & -190\sqrt{5} & 462\sqrt{5} & -465\sqrt{5} & 165\sqrt{5} \\ \sqrt{3} & -33\sqrt{3} & 248\sqrt{3} & -696\sqrt{3} & 810\sqrt{3} & -330\sqrt{3} \\ -1 & 35 & -280 & 840 & -1050 & 462 \end{bmatrix}$$

4. OBPs operational matrix of integration

Let P be an $(m+1)\times(m+1)$ operational matrix of integration, then

$$\int_0^x OB(t)dt \approx P\times OB(x), \qquad x\in[0,1] \tag{12}$$

By (10) we have

$$\int_0^x OB(t)dt = M\begin{bmatrix}1 & 0 & 0 & \dots & 0\\ 0 & 1/2 & 0 & \dots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & \dots & 1/m+1\end{bmatrix}\begin{bmatrix}x\\ x^2\\ \vdots\\ x^{m+1}\end{bmatrix} = M\Lambda X$$

where Λ is $(m+1)\times(m+1)$ matrix: $\Lambda = \begin{vmatrix}1 & 0 & 0 & \dots & 0\\ 0 & 1/2 & 0 & \dots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & \dots & 1/m+1\end{vmatrix}$ and $X = \begin{bmatrix}x\\ x^2\\ \vdots\\ x^{m+1}\end{bmatrix}$

Now, we approximate the elements of vector X in terms of $\{OB_{j,m}(x)\}_{j=0}^{m}$ by (10), we have $T_m(x) = M^{-1}OB(x)$ then for $k = 0,\dots,m.$

$$x^k = M_{k+1}^{-1}OB(x) \tag{13}$$

Where M_{k+1}^{-1} is (k + 1)th row of M^{-1} for $k = 0,\dots,m,$ that is $M^{-1} = \begin{bmatrix}M_1^{-1}\\ M_2^{-1}\\ \vdots\\ M_{k+1}^{-1}\end{bmatrix}$

So, we just need to approximate x^{m+1} by using (5), we have $x^{m+1} = c_{m+1}^T OB(x)$
Where

$$c_{m+1} = \int_0^1 t^{m+1}OB(t)dt \tag{14}$$

and then, $X=\begin{bmatrix} M_2^{-1} \\ M_3^{-1} \\ \vdots \\ M_{k+1}^{-1} \\ c_{m+1}^T \end{bmatrix} OB(x)$. Let $B=\begin{bmatrix} M_2^{-1} \\ M_3^{-1} \\ \vdots \\ M_{k+1}^{-1} \\ c_{m+1}^T \end{bmatrix}$, we have

$$\int_0^x OB(t)dt \approx M\Lambda B \times OB(x), \qquad x \in [0,1] \tag{15}$$

and therefore we have the operational matrix of integration as $P \approx M\Lambda B$

For m = 5, the matrix P_5 is denoted by P and is given as follows:

$$P_5=\begin{bmatrix} \frac{11}{72} & \frac{23}{264}\sqrt{11} & \frac{109}{3960}\sqrt{77} & \frac{331}{11880}\sqrt{55} & \frac{659}{23760}\sqrt{33} & \frac{925}{33264}\sqrt{11} \\ -\frac{1}{264}\sqrt{11} & \frac{1}{8} & \frac{11}{120}\sqrt{7} & \frac{29}{360}\sqrt{5} & \frac{61}{720}\sqrt{3} & \frac{83}{1008} \\ \frac{1}{3960}\sqrt{77} & -\frac{1}{120}\sqrt{7} & \frac{7}{72} & \frac{7}{216}\sqrt{35} & \frac{11}{432}\sqrt{21} & \frac{89}{3024}\sqrt{7} \\ -\frac{1}{11880}\sqrt{55} & \frac{1}{360}\sqrt{5} & -\frac{1}{216}\sqrt{35} & \frac{5}{72} & \frac{5}{144}\sqrt{15} & \frac{23}{1008}\sqrt{5} \\ \frac{1}{23760}\sqrt{33} & -\frac{1}{720}\sqrt{3} & \frac{1}{432}\sqrt{21} & -\frac{1}{144}\sqrt{15} & \frac{1}{24} & \frac{19}{504}\sqrt{3} \\ -\frac{1}{33264}\sqrt{11} & \frac{1}{1008} & -\frac{5}{3024}\sqrt{7} & \frac{5}{1008}\sqrt{5} & -\frac{5}{504}\sqrt{3} & \frac{1}{72} \end{bmatrix}$$

5. OBPs operational matrix of Derivative

In this section, we want to derive an explicit formula for orthonormal Bernstein polynomials of m-th-degree operational matrix of differentiation. Suppose that D is an $(m+1)\times(m+1)$ operational matrix of differentiation, then

$$\frac{d}{dx}OB(x) \approx D \times OB(x) \text{ , where } x \in [0,1] \tag{16}$$

From (10) we have, $OB(x)=MT_m(x)$, and then

$$\frac{d}{dx}OB(x)=M\begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & m \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ x \\ \vdots \\ x^{m-1} \end{bmatrix}=M\Lambda' X'$$

where Λ' is $(m+1)\times m$ matrix $\Lambda' = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & m \end{bmatrix}$ and $X' = \begin{bmatrix} 0 \\ 1 \\ x \\ \vdots \\ x^{m-1} \end{bmatrix}$

Now, we expand vector X' in terms of $\{OB_{j,m}(x)\}_{j=0}^{m}$

By using (13), we can write $X' = B'OB(x)$, where $B' = \begin{bmatrix} M_1^{-1} \\ M_3^{-1} \\ \vdots \\ M_m^{-1} \end{bmatrix}$

Thus

$$\frac{d}{dx}OB(x) \approx M\Lambda'X' \times OB(x) \tag{17}$$

and therefore we have the operational matrix of differentiation as $D = M\Lambda'X'$
For example with $m = 5$ we have

$$D_5 = \begin{bmatrix} -\frac{11}{2} & -\frac{1}{6}\sqrt{11} & 0 & 0 & 0 & 0 \\ \frac{10}{6}\sqrt{11} & -\frac{9}{2} & -\frac{10}{21}\sqrt{7} & 0 & 0 & 0 \\ -\sqrt{7}\sqrt{11} & \frac{73}{21}\sqrt{7} & -\frac{7}{2} & -\frac{27}{70}\sqrt{35} & 0 & 0 \\ \sqrt{5}\sqrt{11} & -3\sqrt{5} & \frac{97}{70}\sqrt{35} & -\frac{5}{2} & -\frac{16}{15}\sqrt{15} & 0 \\ -\sqrt{3}\sqrt{11} & 3\sqrt{3} & -\sqrt{21} & \frac{31}{15}\sqrt{15} & -\frac{3}{2} & -\frac{35}{6}\sqrt{3} \\ \sqrt{11} & -3 & \sqrt{7} & -\sqrt{5} & \frac{41}{6}\sqrt{3} & \frac{35}{2} \end{bmatrix}$$

6. OBPs operational matrix of product

In this section, we want to derive an explicit formula for orthonormal Bernstein polynomials of m[th] degree operational matrix of product. Suppose that c is an arbitrary $(m+1)\times 1$ matrix, then $\hat{C}$ is an $(m+1)\times(m+1)$ operational matrix of product whenever

$$c^T OB(x) OB(x)^T \approx OB(x) \times \hat{C} \tag{18}$$

By (10) and since $c^T OB(x) = \sum_{j=0}^{m} c_j OB_{j,m}(x)$, we have

$$c^T OB(x) OB(x)^T \approx OB(x) \times T_m(x)^T M^T$$
$$= \left[c^T OB(x), x\left(c^T OB(x)\right), x^2\left(c^T OB(x)\right), ..., x^m\left(c^T OB(x)\right)\right] M^T$$
$$= \left[\sum_{k=0}^{m} c_k OB_{j,m}(x), \sum_{k=0}^{m} c_k x OB_{k,m}(x), \sum_{k=0}^{m} c_k x^2 OB_{k,m}(x), ..., \sum_{k=0}^{m} c_k x^m OB_{k,m}(x)\right] M^T \quad (19)$$

Now, we approximate all functions $x^j OB_{k,m}(x), j = 0,...,m,$ in terms of $\{OB_{j,m}(x)\}_{j=0}^{m}$.

Let

$$e_{j,k}^T = \left[e_0^{j,k}, e_1^{j,k}; ..., e_m^{j,k}\right]$$

by (5), we have

$$x^j OB_{k,m}(x) = e_{j.k}^T OB(x), j,k = 0,...,m$$

Thus, we obtain

$$\sum_{k=0}^{m} c_k x^j OB_{k,m}(x) = \sum_{k=0}^{m} c_k \left(\sum_{i=0}^{m} e_{i.k}^T OB_{i,m}(x)\right)$$
$$= \sum_{i=0}^{m} OB_{i,m}(x) \left(\sum_{k=0}^{m} c_k e_{i.k}^T\right)$$
$$= OB(x)^T \left[e_{j,0}, e_{j,1}, ..., e_{j,m}\right] c$$
$$= OB(x)^T \hat{E}_{j+i} \quad (20)$$

where $\hat{E}_{j+i} = \left[e_{j,0}, e_{j,1}, ..., e_{j,m}\right] c$

By defining matrix $\hat{E} = \left[\hat{E}\right]_{(m+1)\times(m+1)} = \left[\hat{E}_1, \hat{E}_2, .., \hat{E}_m\right]$ and inserting (19) into (20) we have

$$c^T OB(x) OB(x)^T \approx OB(x) \times \hat{E} M^t$$

So

$$\hat{C} = \hat{E} M^t$$

7. Applications of the operational matrix of OBPs

7.1 Application to the Emden-Fowler equation

Consider the Emden-Fowler equation given in Wazwaz, [15] by

$$y''(x)+\frac{k}{x}y'(x)+x^r y^n(x)=0, 0<x\le 1, n\in \mathbb{N}\cup\{0\} \tag{21}$$

with initial conditions

$$y(0)=1, \qquad y'(0)=0 \tag{22}$$

We assume that the unknown function $y''(x)$ is approximated by

$$y''(x)=c^T OB(x) \tag{23}$$

Using (15) and the initial conditions (22) we have

$$y'(x)=c^T P\times OB(x), y(x)=c^T P^2\times OB(x)+d^T OB(x)=h^T OB(x) \tag{24}$$

Where $d^T OB(x)=1$ and $h=(P^T)^2 c+d$, by (24) and (18) we have

$$y^2(x)=h^T OB(x)OB(x)^T h=OB(x)\hat{h}h$$

and

$$y^3(x)=h^T OB(x)OB(x)^T \hat{h}h=OB(x)\hat{h}^2 h$$

So, by induction we

$$y^n(x)=OB(x)^T \hat{h}^{n-1}h, n\in \mathbb{N} \tag{25}$$

where $\hat{h}$ is operational matrix of product. We can express the functions x and x^{r+1} as

$$x=e^T OB(x), x^{r+1}=k^T OB(x) \tag{26}$$

Substituting (23) and (26) in (21) we obtain

$$e^T OB(x)OB(x)^T c+2c^T P\times OB(x)+k^T OB(x)OB(x)^T \hat{h}^{n-1}h=0 \tag{27}$$

Using (18) we have

$$e^T OB(x)OB(x)^T=OB(x)^T \hat{E} \tag{28}$$

and

$$k^T OB(x)OB(x)^T=OB(x)^T \hat{K} \tag{29}$$

Substituting (29) and (28) in (27) we get

$$OB(x)^T \hat{E}c+2OB(x)^T P^T c\times OB(x)+OB(x)^T \hat{K}\hat{h}^{n-1}h=0$$

or

$$\hat{E}c+2P^T c+\hat{K}\hat{h}^{n-1}h=0 \tag{30}$$

Equation (30) is a set of algebraic equations which can be solved for c.
Now, we apply the above presented method with $m=3$ and $m=5$ for solving Equation (21) with $r=0$ and $n=1$ which has the exact solution $y(x)=\frac{\sin(x)}{x}$.

In Table 1, a comparison is made between the approximate values using the present approach together with the exact solution.

We found the approximated solution for $m = 3$ and $m = 5$ as follows:

$$y_3(x) = 1.5628\times10^{-2}x^3 - 0.17615x^2 + 1.9428\times10^{-3}x + 0.99996$$

$$y_3(x) = -2.7212\times10^{-3}x^5 + 1.4148\times10^{-2}x^4 - 5.3174\times10^{-3}x^3$$

$$- 0.16421x^2 - 4.6180\times10^{-4}x + 1.0000$$

Table 1. Estimated and exact values

x	Present method with		Exact solution
	m=3	*m=5*	
0.1	0.99837	0.998 31	0:9983341664682815
0.2	0.99340	0.993 32	0:9933466539753061
0.3	0.98509	0.985 05	0:9850673555377986
0.4	0.97352	0.973 54	0:9735458557716263
0.5	0.95881	0.958 85	0:958851077208406
0.6	0.94104	0.941 08	0:9410707889917257
0.7	0.92031	0.920 33	0:9203109817681301
0.8	0.89672	0.896 72	0:8966951136244035
0.9	0.87037	0.870 37	0:8703632329194261
1.0	0.84136	0.841 44	0:8414709848078965

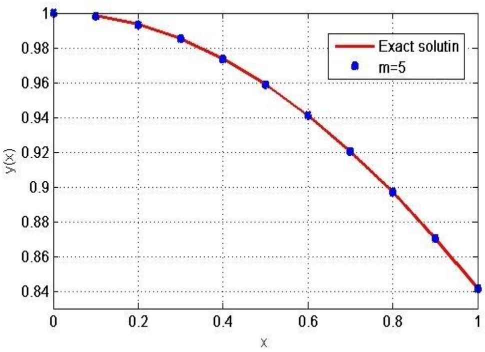

Figure 2: Graph of exact solution and approximate solution at m=5

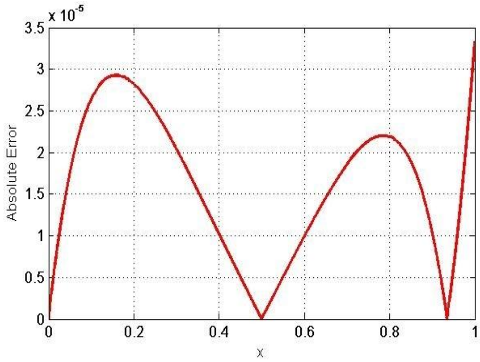

Figure 3: Graph of absolute errors for m=5

7.2 Application to the Linear Fredholm Integro-Differential Equation

We consider the following linear Fredholm integro-differential equation

$$\begin{cases} y'(x) = f(x) + y(x) + \int_0^1 k(x,s)y(s)ds, & 0 \le x \le 1 \\ y(0) = y_0 \end{cases} \tag{31}$$

where the function $f(x) \in L^2[0,1]$, the kernel $k(x,s) \in L^2([0,1]\times[0,1])$

$$y'(x) = Y'^T OB(x) \qquad y(0) = Y_0^T OB(x) \quad , f(x) = F^T OB(x)$$

then $k(x,s) = OB(x)^T KOB(s),$ where $k_{i,j} =< OB_{i,m}(x)^T, < k(x,s), OB_{j,m}(s) >>$

Then

$$y(x) + = \int_0^1 y'(s)ds + y(0) = \int_0^1 Y'^T OB(s)ds + Y_0{}^T OB(x)$$
$$= \left(Y'^T P + Y_0{}^T\right) OB(x)$$

Substituting into (31) we have

$$OB(x)^T Y' = OB(x)^T F + OB(x)^T \left(P^T Y' + Y_0\right)$$
$$+ \int_0^x OB(x)^T KOB(s)OB(x)^T \left(P^T Y' + Y_0\right) ds$$
$$= OB(x)^T F + OB(x)^T \left(P^T Y' + Y_0\right) + OB(x)^T K\left(P^T Y' + Y_0\right)$$
$$Y' = F + \left(P^T Y' + Y_0\right) + K\left(P^T Y' + Y_0\right)$$
$$\Rightarrow Y' = \left(I - KP^T - P^T\right)^{-1}\left(KY_0 + Y_0 + F\right)$$

By solving the above linear system we can find the vector Y', so

$$Y^T = Y'^T P + Y_0^T \text{ or } y(x) = Y^T OB(x)$$

Now, We consider the following linear Fredholm integro-differential equation

$$\begin{cases} y'(x) = xe^x + e^x - x + \int_0^1 xy(s)ds \\ y(0) = 0 \end{cases}$$

with exact solution $y(x) = xe^x$

We found the approximated solution for m = 3 and m = 5 as follows

$$y_3(x) = 0.980\,90x^3 + 0.683\,26x^2 + 1.065\,1x - 2.825\,8\times10^{-3}$$
$$y_3(x) = 6.342\,3\times10^{-3}x^5 + 0.155\,68x^4 + 0.495\,34x^3$$
$$+ 1.004\,8x^2 + 0.998\,97x + 5.587\,6\times10^{-5}$$

Table 2. Approximate and exact solutions for example 2.

x	Present method with		Exact solution
	m=3	*m=5*	
0.1	0.11150	0.11051	0.110517091807565
0.2	0.24537	0.24427	0.244280551632034

0.3	0.40468	0.40497	0.404957642272801
0.4	0.59531	0.59675	0.596739879056508
0.5	0.82315	0.82437	0.824360635350064
0.6	1. 0941	1. 0933	1. 093271280234305
0.7	1. 4140	1. 4096	1. 409626895229334
0.8	1. 7888	1. 7805	1. 780432742793974
0.9	2. 2243	2. 2137	2. 213642800041255
1.0	2. 7264	2. 7183	2.718281828459046

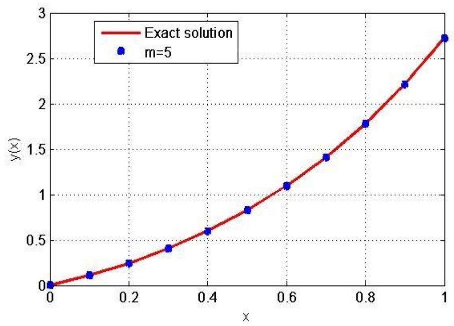

Figure 4: Graph of exact solution and approximate solution at *m=5*

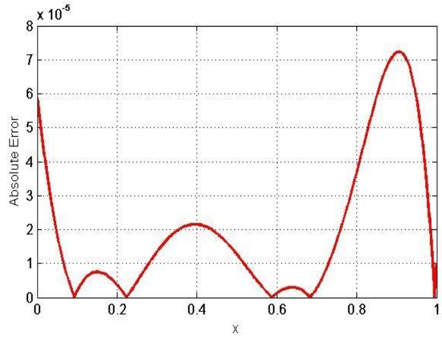

Figure 5: Graph of absolute errors for *m=5*

Conclusion

n this paper, we have first constructed orthonrmal polynomials OB(x) of degree n by applying Gram-Schmidt orthonormalization process on the Bernstein polynomials B(x). Then, we have used another different numerical procedure to derive the OBPs operational matrices of integration P, differentiation D and product $\hat{C}$. A general procedure of forming these matrices is given. These matrices can be used to solve problems such as the calculus of variations, integro-differential equation, differential equations, optimal control and integral equations, like that of other basis. The method is general, easy to implement, and yields very accurate results. Moreover, only a small number of bases are needed to obtain a satisfactory result. Numerical treatment is included to demonstrate the validity and applicability of these operational matrices. What is new in this work, besides its difficulty compared to Bernstein polynomials B(x), is that we can write OB(x) in terms of Taylor basis. Here, the advantage in the form of OB(x) is the easier computation of the coefficients c compared to that of B(x). We have estabilished the general form of P, and computed the operational matrix of derivation D, and we have given the general form of the matrix derived from the product. The efficiency of this method is shown on examples such as the Lane-Fowler equations and on some integro-differential equations. The first example shows the efficient use of theses matrices by transforming the problem to a set of algebraic equations which are easy to solve. The results obtained are very precise compared to those obtained in [12]. Example 2 shows how it is easier to solve integral and integro-differential equations.

References

[1] M. I. Bhatti, P. Bracken, Solutions of differential equations in a Bernstein polynomial basis, J. Comput. Appl. Math. 205, 272-280.2007.

[2] A. Deb, A. Dasgupta, G. Sarkar, A new set of orthogonal functions and its application to the analysis of dynamic systems, J. Frank. Inst. 343, 1-26, 2006.

[3] Z. H. Jiang, W. Schaufelberger, Block Pulse Functions and Their Applications in Control Systems, Springer-Verlag, 1992.

[4] F. Khellat, S. A. Yousefi, The linear Legendre wavelets operational matrix of integration and its application, J. Frank. Inst. 343, 181-190, 2006.

[5] F. C.Kung, and H. Lee, Solution and parameter estimation of linear time invariant delay systems using Laguerre polynomial expansion, Journal on Dynamic Systems, Measurement, and Control, 297-301, 1983.

[6] I. Lazaro , J. Anzurez , M. Roman M, Parameter estimation of linear systems based on walsh series, The Electronics, Robotics and Automotive Mechanics Conference, 355-360, 2009.

[7] M. H. Farahi, M. Dadkhah, Solving Nonlinear Time Delay Control Systems by Fourier series, Int. Journal of Engineering Research and Applications, Vol. 4, Issue 6 , 217-226, 2014.

[8] S. Javadi, E. Babolian, Z. Taheri, Solving generalized pantograph equations by shifted orthonormal Bernstein polynomials, J. Comput. Appl. Math, 303 (2016)1-14

[9] F. Marcellan, and W.V. Assche, Orthogonal Polynomials and Special Functions (a Computation and Applications), Springer-Verlag Berlin Heidelberg, 2006.

[10] H. R. Marzban and M. Shahsiah, Solution of piecewise constant delay systems using hybrid of block- pulse and Chebyshev polynomials, Optim. Contr. Appl. Met., Vol. 32, pp.647-659, 2011.

[11] A. Richard . A. Bernatz. Fourier series and numerical methods for partial differential equations. John Wiley & Sons, Inc., New York, 2010.

[12] S. A. Yousefi, M. Behroozifar, Operational matrices of Bernstein polynomials and their applications, Int. J. Syst. Sci. 41, 709-716, 2010.

[13] M. Sezer, and A. A. Dascioglu, Taylor polynomial solutions of general linear differential-difference equations with variable coefficients, Appl. Math.Comput.174, 1526-1538, 2006.

[14] M. Shaban and S. Kazem and J. A. Rad, A modification of the homotopy analysis method based on Chebyshev operational matrices, Math. Comput. Model, in press. 2013.

[15] A. M. Wazwaz, Adomian decomposition method for a reliable treatment of the Emden-Fowler equation, Appl.Math.Comput.161 543-560.2005.

[16] X. T. Wang, Numerical solutions of optimal control for time delay systems by hybrid of block-pulse functions and Legendre polynomials Applied Mathematics and Computation, 184, 849-856, 2007.