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Robust backstepping control for uncertain chaotic multi-inputs multi-outputs systems using type 2 fuzzy systems

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Abstract

In this paper, a new backstepping type 2 fuzzy control is developed for a class of uncertain multi-inputs multi-outputs (MIMO) chaotic systems, with a complicate of uncertainties and external disturbances. The system consists of interconnected subsystems that are duffing equation. In the control design, type 2 fuzzy logic systems are used to approximate the unknown functions. Hybrid adaptive robust tracking control schemes that are based upon a combination of bounds of type 2 fuzzy approximation parameters and the backstepping design are developed such that all the states and signals are bounded and the proposed approach alleviate the online computation burden and improves the robustness to dynamic uncertainties and external disturbances. Finally, the control of chaotic duffing systems with unknown parameters is given as example to verify the effectiveness of the proposed approach.

Keywords

Adaptive control, type 2 fuzzy control, backstepping technique, robust control, MIMO chaotic systems.

Introduction

A chaotic system has complex dynamical behaviors that possess some special features, such as excessive sensitivity to initial conditions; a small change in the initial conditions can drastically change the long-term behavior of a system (Branislav, 2011). Chaotic phenomena can be found in many engineering systems such as biological systems, circuit systems, power converters, chemical reactions and physical systems (Chen, 1999). In the past decades, there has been a rapid growth of research efforts aiming at the development of systematic design methods for control of chaotic dynamical systems. Successful methods and techniques have been reviewed in Wang and Ip (2005), Hugues-Salas et al. (2008), Zribi et al. (2009) and Wong and Kakmeni (2004). In such schemes, it is assumed that an accurate model of the plant is available, and the unknown parameters are assumed to appear linearly with respect to known nonlinear functions. This assumption is not sufficient for many practical situations because it is difficult to describe a nonlinear plant by known nonlinear functions precisely. Fuzzy systems are usually used to controlling uncertain nonlinear functions owing to their inherent capabilities in function approximation (Lee and Chung, 2012; Ozbek, 2015; Yang and Zhou, 2005; Yoshimura, 2012). Several approximation based adaptive neural /fuzzy control approaches have been successfully applied for the control of uncertain chaotic systems (for synchronization, tracking, or stabilization purposes), and lots of significant results have been reported, such that the synchronization of uncertain chaotic systems with random-varying parameters (Lin and Wang, 2011), fuzzy logic controller is designed for controlling non-linear behaviors in a rod-type plasma torch system (Khari et al., 2015); nonlinear and chaos control of a micro-electro-mechanical system by using second order fast terminal sliding mode control (Zhankui and Sun, 2013); second order terminal sliding mode control for a class of chaotic systems with unmatched uncertainties (Xiang and Huangpu, 2010); chaos synchronization of nonlinear gyros using self-learning PID control approach (Hsu et al., 2012); robust ISS-satisfying variable universe indirect fuzzy control for chaotic systems (Wang et al., 2009); a fuzzy adaptive variable structure control scheme for uncertain chaotic multi-inputs multi-outputs (MIMO) systems with sector nonlinearities and dead zones applied for duffing chaotic systems (Boulkroune and M'Saad, 2011); and observer-based decentralized fuzzy neural sliding mode control for interconnected unknown MIMO chaotic systems via network structure adaptation (Lin and Wang, 2010). In the systems that have spatial form (strict feedback form), the difficulties encountered in handing chaotic systems have posed a real need for using some kind of intelligent approach. To overcome these drawbacks, the backstepping

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design method for the chaotic dynamic systems has been designed (Lin and Li, 2013; Peng and Hsu, 2009; Lin et al., 2010). In these schemes, the adaptive laws were obtained by adjusting the estimation vectors of the optimal parameters vectors. These proposed schemes suffered the well-known "curse of dimensionality", that is, to achieve a better approximation result, the number of parameters to be adjusted online is very large, in particular for high dimensional systems, and the learning time tends to become unacceptably long during the implementation. This problem was solved in Liu et al. (2010), Li et al. (2010) and Liu and Wang (2007). In those approaches, fuzzy systems are used to approximate the unknown functions and it is assumed that the norms of optimal vectors and the fuzzy approximation errors are bounded by unknown bounds. By only adjusting estimations of unknown bounds, this problem is solved.

The type 1 fuzzy logic systems (T1FLSs) employed in Doudou and Khaber (2012), Khaber et al. 2006, Pourkargar and Shahrokhi (2011) and Yin et al. (2016) are developed using type 1 membership functions that are unable to handle directly the uncertainties. To take care of these uncertainties, type 2 fuzzy logic systems (T2FLS) have been used. Many research show that the T2FLS are better able to handle uncertainties than their T1FLS (Biglarbegian et al., 2010; Lam and Seneviratne, 2008; Shahnazi, 2016; Manceur et al., 2013).

Type 2 fuzzy adaptive backstepping control schemes can provide a systematic framework for the design of synchronization or stabilization of chaotic systems, in which the FLS are used to approximate the unknown nonlinear functions, and an adaptive fuzzy controller is constructed recursively (Lin et al., 2011).

In this paper, motivated by above-mentioned works in literature, a novel adaptive fuzzy control approach based on type 2 fuzzy systems is proposed and developed for MIMO chaotic systems with modeling complicated uncertainties and external disturbances. The chaotic MIMO systems are composed of interconnected subsystems, where the system state interconnections appear in every equation of each subsystem. The Lyapunov analysis method is employed to construct the control inputs and the adaptation laws without the requirement of integral type Lyapunov functions. The use of type 2 fuzzy systems can fully handle the uncertainties, and achieve higher performances. In the controller design, it is assumed that the norms of the optimal type 2 fuzzy basic functions and the fuzzy approximation errors vectors are bounded by unknown bounds. By only adjusting estimations of unknown bounds, the developed control approach alleviates the online computation burden and improves the robustness to dynamic uncertainties and external disturbances. The proposed backstepping type 2 fuzzy method can solve the control problem of duffing chaotic system under appropriate assumptions and assures the stability of the resulting closed loop and the tracking errors converge to a small neighborhood around zero.

The paper is organized as follows. First, a model description and the mathematics lemmas of the studied class systems are presented in section 2. The backstepping controller design using interval type 2 fuzzy systems is designed in section 3. In section 4, simulation example under matlab environment is given. Finally, the obtained results are summarized in section 5.

Models descriptions and mathematics lemmas

Consider a large-scale system Q that is composed of m interconnected chaotic subsystems q_j. Each chaotic subsystem q_j ($j = 1, 2, ..., m$), with modeling uncertainties and external disturbances are described by

$$q_j \begin{cases} \dot{x}_{j,i_j} = z_{j,i_j}(X) + \bar{x}_{j,i_j+1} + \Delta_{j,i_j} \quad i_j = 1, \ldots, \rho_j - 1 \\ \dot{x}_{j,\rho_j} = z_{j,\rho_j}(X) + f_j + u_j + \Delta_{j,\rho_j} \\ y_j = x_{j,1} \end{cases} \tag{1}$$

where $x_{j,i_j}, i_j = 1, \ldots, \rho_j$ are the states of the j^{th} subsystem with ρ_j denoting the order of the j^{th} subsystem. Let $\rho_1 + \rho_2 + \ldots \rho_m = n$, u_j, y_j are the input and the output of the j^{th} subsystem, respectively. z_{j,i_j}, represents the strength of interconnections from other chaotic subsystems that are considered as unknown smooth continuous functions, Δ_{j,i_j} denotes bounded external disturbances. $\bar{x}_{1,i_j} = [x_{j,1}, \ldots, x_{j,i_j}]^T \in R^{i_j}$ is the vector of partial state variables in the j^{th} subsystem. $X = [\bar{x}_{1,\rho_1}, \ldots \bar{x}_{m,\rho_m}]^T$ is the state variable of the overall system, j, i_j, ρ_j,m and n are positive integers. For simplicity, throughout this paper, the following notations are used: z_{j,i_j} and z_{j,ρ_j} will be used to denote $z_{j,i_j}(X)$ and $z_{j,\rho_j}(\bar{x})$, respectively.

The control objective is to construct a robust adaptive fuzzy controller for the system (1) such that all the signals of the resulting closed-loop system are uniformly bounded and the tracking error $e_{j,1} = y_j - y_{jd}$ can be rendered small. To achieve the control objective, we need to make the following assumptions:

Assumption 1: (Hsu et al., 2012; Wang et al., 2009): external disturbances Δ_{j,i_j} is bounded, that is, there exist unknown constants $d^*_{j,ij} > 0$ such that $\Delta_{j,i_j} < d^*_{j,ij}$ with $i_j = 1, \ldots, \rho_j$, $j = 1, 2, \ldots, m$.

Assumption 2: (Hsu et al., 2012; Pourkargar and Armaou, 2015; Wang et al., 2009): The reference signal $y_{j,d}$ is a sufficiently smooth function and k^{th}-order derivative $y^{(k)}_{j,d}$ $k = 1, ..., \rho_j$, $j = 1, 2, \ldots, m$ are assumed to be bounded and continuous.

Remark 1: The problems of "explosion of complexity", which is inherent in the backstepping technique, caused by the repeated differentiating of virtual control in the backstepping design, in particular, the complexity of controller grows hugely as the order of the system increases. The problem is solved in Yu et al. (2017), Li et al. (2010), Swaroop et al. (2000), Li et al. (2015), Khebbache et al. (2016), by introducing a new state variable and let the virtual control pass through a first-order filter so called dynamic surface control. In our approach, the uncertain MIMO nonlinear systems are composed of m interconnected subsystems where both the interconnection terms and the external disturbances appear in every equation of each subsystem. Owing to the special structure of the considered systems, our research situation is difficult and complicated. The problem is solved by the use of T2FLS for approximating the unknown functions and the derived of virtual control. The computation burden is owing to the large number of vectors adaptation parameters. In our approach, this problem is handled by the adaptation only of the bounds of the vector parameters and not all these elements motivated by Liu et al. (2010).

Backstepping controller design using interval type 2 fuzzy system

Interval type 2 fuzzy system description

Consider a T2FLS having p inputs $x_1 \in X_1, \cdots, x_p \in X_p$ and one output $y \in Y$. The type 2 fuzzy rule base consists of a collection of IF-THEN rules. We assume that there are M rules and the rule of a type 2 relation between the input space $X_1 \times X_2 \times \cdots \times X_p$ and the output space Y can be expressed as:

$$R^i : IF\ x_1\ is\ F_1^i\ and\ x_2\ is\ F_2^i\ and\ \ldots\ and\ x_p\ is\ F_p^i\ THEN\ \ y\ \ is\ \ G^i \quad i = 1, 2, \ldots, m \tag{2}$$

The inference engine combines rules and gives a mapping from T2FSs input to T2FSs output. To achieve this process, we have to compute unions and intersections of type 2 sets, as well as compositions of type 2 relations. The output inference engine block is a type 2 set. By using the extension principle of type 1 defuzzification method, type reduction process takes us from type 2 output sets to a type 1 set called the "type reduced set". This set may then be defuzzified to obtain a single crisp value.

In Figure 1, we only consider singleton input fuzzification throughout this paper. Similar to T1FLS, the firing set in (2) can be obtained by the following inference process:

$$F^i = \coprod_{x \in X} \left[\prod_{k=1}^{p} \mu_{\tilde{F}_k^i}(x_k) \right] \tag{3}$$

where Π denote the MEET operation and $\coprod$ the JOIN one.

The result of the JOIN operation is an interval type 1 set defined by $F^i = \left[\underline{f}^i\ \bar{f}^i \right]^T$ (Lin et al., 2011), where

$$\begin{cases} \underline{f}^i = \underline{\mu}_{F_1^i}(x_1) * \cdots * \underline{\mu}_{F_p^i}(x_p) \\ \bar{f}^i = \bar{\mu}_{F_1^i}(x_1) * \cdots * \bar{\mu}_{F_p^i}(x_p) \end{cases} \tag{4}$$

There are many kinds of type reduction method, such as centroid, height, modified weight and center of sets. The center-of-sets type reduction will be used in this paper and can be expressed as

$$\begin{aligned} Y_{\cos}(x) &= [y_l, y_r] \\ &= \int_{y^1 \in [y_l^1, y_r^1]} \cdots \int_{y^M \in [y_l^M, y_r^M]} \\ &\quad \times \int_{f^1 \in [\underline{f}^1, \bar{f}^1]} \cdots \int_{f^M \in [\underline{f}^M, \bar{f}^M]} \Big/ \frac{\sum_{i=1}^M f^i y^i}{\sum_{i=1}^M f^i} \end{aligned} \tag{5}$$

Note that y_l and y_r depend only on mixture of $\underline{f}^i$ or $.\bar{f}^i$ values. Hence, left-most point y_l and right-most point y_r can be expressed as

$$y_l \frac{\sum_{i=1}^M f_l^i y_l^l}{\sum_{i=1}^M f_l^l}, \tag{6}$$

$$y_r \frac{\sum_{i=1}^M f_r^i y_r^l}{\sum_{i=1}^M f_r^l} \tag{7}$$

For illustrative purposes, we briefly provide the computation procedure for y_r. Without loss of generality, assume the y_r^i are arranged in ascending order, that is, $y_r^1 \le y_r^2 \le \cdots \le y_r^M$

Step 1: Compute y_r in (6) by initially using $f_r^r = \left(\bar{f}^i + \underline{f}^i\right)/2$ for $j = 1, 2, \ldots, m$, where $\bar{f}^i$ and $\underline{f}^i$ are precomputed by (4); and let $y_r' = y_r$

Step 2: Find $R(1 \le R \le M-1)$ such that $y_r^R \le y_r' \le y_r^{R+1}$.

Step 3: Compute y_r in (7) with $f_r^i = \underline{f}^i$ for $i \le R$ and $f_r^i = \bar{f}^i$ for i > R and let $y_r'' = y_r$.

Step 4: If $y_r'' \ne y_r'$ then go to Step 5. If $y_r'' = y_r'$ then stop and set $y_r'' = y_r$

Step 5: Set y_r' equal to y_r'' and return to Step 2.

The y_r in (7) can be reexpressed as

$$\begin{aligned} y_r &= y_r\left(\underline{f}^1, \cdots, \underline{f}^R, \bar{f}^{R+1}, \cdots, \bar{f}^M, y_r^1, \cdots, y_r^M\right) \\ &= \frac{\sum_{i=1}^R \underline{f}^i y_r^i + \sum_{i=R+1}^M \bar{f}^i y_r^i}{\sum_{i=1}^R \underline{f}^i + \sum_{i=R+1}^M \bar{f}^i} \end{aligned} \tag{8}$$

The same procedure is used to compute y_l. In Step 2, it only needs to find L $(1 \le L \le M-1)$, such that

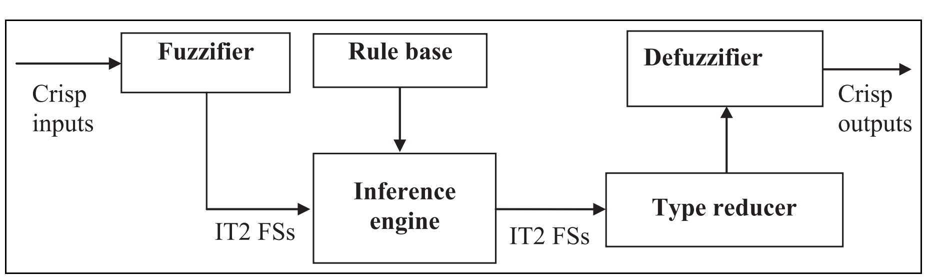

Figure 1. Structure of T2FLS.

$y_l^L \le y_l' \le y_l^{L+1}$. In Step 3, let $f_l^i = \bar{f}^i$ for $i \le L$, and $f_l^i = \underline{f}^i$ for $i > L$. The y_l in (6) can be also rewritten as

$$\begin{aligned} y_l &= y_l\left(\bar{f}^1, \cdots, \underline{f}^L, \underline{f}^{L+1}, \cdots, \underline{f}^M, y_l^1, \cdots, y_l^M\right) \\ &= \frac{\sum_{i=1}^{L} \bar{f}^i y_l^i + \sum_{i=L+1}^{M} \underline{f}^i \bar{f}^i y_l^i}{\sum_{i=1}^{L} \bar{f}^i + \sum_{i=L+1}^{M} \underline{f}^i} \end{aligned} \tag{9}$$

The defuzzified crisp output from T2FLS is the average of y_l and y_r

$$y(x) = (y_l + y_r)/2 \tag{10}$$

Adaptive backstepping type 2 fuzzy controller design

The backstepping procedure is an effective design approach for strict feedback nonlinear systems. For the j^{th} subsystem of the systems (1), the design procedure contains ρ_j steps. From step 1 to step $\rho_j - 1$, the virtual control inputs α_{j,i_j}, $i_j = 1, \cdots, \rho_j - 1$ are designed to make the error variables $|e_{j,i_j}|$ as small as possible.

Step 1: Define the tracking error variable $e_{j,1} = x_{j,1} - y_{d,j}$

$$\dot{e}_{j,1} = z_{j,1}(\bar{x}_{j,1,i_j}) + \bar{x}_{j,2} + \Delta_{j,1} - \dot{y}_{d,j} \tag{11}$$

Define the unknown function as follows

$$M_{j,1}(E_{j,1}) = \left(z_{j,1} - \dot{y}_{d,j}\right) \tag{12}$$

We employ the approximation property of T2FLS to approximate $M_{j,1}(E_{j,1})$. Then (11) can be rewritten as

$$M_{j,1}(Z_{j,1}) = \frac{1}{2}\theta_{lj,1}^{*T}\xi_{lj,1}(E_{j,1}) + \frac{1}{2}\theta_{rj,1}^{*T}\xi_{rj,1}(E_{j,1}) + \delta_{j,1}(E_{j,1}) \tag{13}$$

where $E_{j,1} = [\bar{x}_{1,1}^T, \ldots, \bar{x}_{m,i_j}^T]^T \in \Omega_{j,1}$ is selected as the input vector of the T2FLS, $\theta_{lj,1}^{*T}$ is the optimal parameters vector of left-most point and $\theta_{rj,1}^{*T}$ is the optimal parameters vector of right-most point, $\xi_{lj,1}(E_{j,1})$ and $\xi_{rj,1}(E_{j,1})$ are the fuzzy basis function vectors of left-most point and right-most point, respectively, and $\delta_{j,1}(E_{j,1})$ denotes the fuzzy approximation error. Throughout this paper, we will make the following assumption:

Assumption 3: (Hsu et al., 2012; Wang et al., 2009): on the compact set Ω_{j,i_j} there exist bounded constants a_{lj,i_j}, a_{rj,i_j} and δ_{j,i_j}^* such that $\left\|\theta_{lj,i_j}^*\right\| \le a_{lj,i_j}$, $\left\|\theta_{rj,i_j}^*\right\| \le a_{rj,i_j}$, $|\delta_{j,i_j}(E_{j,i_j})| \le \delta_{j,i_j}^*$, $a_{lj,i_j} > 0$, $a_{rj,i_j} > 0$ and $\delta_{j,i_j}^* > 0$ are unknown .

Let $q_{j,i_j} = \delta_{j,i_j}^* + d_{j,i_j}^*$ q_{j,i_j} is an unknown and positive constant. Define $\tilde{a}_{lj,i_j} = \hat{a}_{lj,i_j} - a_{lj,i_j}$, $\tilde{a}_{rj,i_j} = \hat{a}_{rj,i_j} - a_{rj,i_j}$ and $\tilde{q}_{j,i_j} = \hat{q}_{j,i_j} - q_{j,i_j}$ as estimations errors. $\hat{a}_{lj,i_j} > 0$, $\hat{a}_{rj,i_j} > 0$ and $\hat{q}_{j,i_j} > 0$ will be utilized to denote the estimations of a_{lj,i_j}, a_{rj,i_j} and q_{j,i_j}, respectively.

By substituting (12) and (13) into (11) and introducing the error variable $e_{j,2} = x_{j,2} - \alpha_{j,1}$, we have

$$\begin{aligned} \dot{e}_{j,1} = {} & \frac{1}{2}\theta_{lj,1}^{*T}\xi_{lj,1}(E_{j,1}) + \frac{1}{2}\theta_{rj,1}^{*T}\xi_{rj,1}(E_{j,1}) \\ & + \delta_{j,1}(E_{j,1}) + e_{j,2} + \alpha_{j,1} + \Delta_{j,1} \end{aligned} \tag{14}$$

where

$$\begin{aligned} \alpha_{j,1} = {} & -k_{j,1}e_{j,1} - \frac{\hat{a}_{lj,1}^2 e_{j,1}\|\xi_{lj,1}(E_{j,1})\|^2}{2(\hat{a}_{lj,1}|e_{j,1}|\|\xi_{lj,1}(E_{j,1})\| + \varepsilon_{lj,1})} \\ & - \frac{\hat{a}_{rj,1}^2 e_{j,1}\|\xi_{rj,1}(E_{j,1})\|^2}{2(\hat{a}_{rj,1}|e_{j,1}|\|\xi_{rj,1}(E_{j,1})\| + \varepsilon_{rj,1})} - \frac{\hat{q}_{j,1}^2 e_{j,1}}{\hat{q}_{j,1}|e_{j,1}| + \omega_{j,1}} \end{aligned} \tag{15}$$

is chosen as the virtual control input of the subsystem (11), with design parameters $\varepsilon_{lj,1}$, $\varepsilon_{rj,1}$ and $\omega_{j,1}$.

Consider the Lyapunov function candidate

$$V_{j,1} = \frac{1}{2}e_{j,1}^2 + \frac{1}{2\gamma_{lj,1}}\tilde{a}_{lj,1}^2 + \frac{1}{2\gamma_{rj,1}}\tilde{a}_{rj,1}^2 + \frac{1}{2\lambda_{j,1}}\tilde{q}_{j,1}^2 \tag{16}$$

where $\gamma_{lj,1} > 0$, $\gamma_{rj,1} > 0$ and $\lambda_{j,1} > 0$ are the design constants.

The time derivative of $V_{j,1}$ is

$$\dot{V}_{j,1} = \frac{1}{2}e_{j,1}\dot{e}_{j,1} + \frac{1}{\gamma_{lj,1}}\tilde{a}_{lj,1}\dot{\hat{a}}_{lj,1} + \frac{1}{\gamma_{rj,1}}\tilde{a}_{rj,1}\dot{\hat{a}}_{rj,1} + \frac{1}{2\lambda_{j,1}}\tilde{q}_{j,1}\dot{\hat{q}}_{j,1} \tag{17}$$

By substituting (14) and (15) into $\dot{V}_{j,1}$ and select the following adaptation laws

$$\dot{\hat{a}}_{lj,1} = -0.5\sigma_{lj,1}\hat{a}_{lj,1} + 0.5\gamma_{lj,1}|e_{lj,1}|\|\xi_{lj,1}(E_{j,1})\| \tag{18}$$

$$\dot{\hat{a}}_{rj,1} = -0.5\sigma_{rj,1}\hat{a}_{rj,1} + 0.5\gamma_{rj,1}|e_{rj,1}|\|\xi_{rj,1}(E_{j,1})\| \tag{19}$$

$$\dot{\hat{q}}_{j,1} = -\eta_{j,1}\hat{q}_{j,1} + \lambda_{j,1}|e_{j,1}| \tag{20}$$

$\dot{V}_{j,1}$ can be obtained like this

$$\begin{aligned} \dot{V}_{j,1} \le {} & e_{j,1}e_{j,2} - k_{j,1}^* e_{j,1}^2 - \frac{\sigma_{lj,1}}{2\gamma_{lj,1}}\tilde{a}_{lj,1}^2 \\ & - \frac{\sigma_{rj,1}}{2\gamma_{rj,1}}\tilde{a}_{rj,1}^2 - \frac{\eta_{j,1}}{2\lambda_{j,1}}\tilde{q}_{j,1}^2 + \frac{\sigma_{lj,1}}{2\gamma_{lj,1}}a_{lj,1}^2 \\ & + \frac{\sigma_{rj,1}}{2\gamma_{rj,1}}a_{rj,1}^2 + \frac{\eta_{j,1}}{2\lambda_{j,1}}q_{j,1}^2 + \varepsilon_{j,1} + \omega_{j,1} \end{aligned} \tag{21}$$

where the coupling term $e_{j,1}e_{j,2}$ will be cancelled in the next step.

Step i: $(i = 2, \ldots, \rho_j - 1)$ Similar to Step 1, define the error variable $e_{j,i_j} = x_{j,i_j} - \alpha_{j,i_j-1}$, by similar manipulations in Step 1, we chose as the virtual control input

$$\alpha_{j,i_j} = -e_{j,i_j-1} - k_{j,i_j}e_{j,i_j} - \frac{\hat{a}_{lj,1}^2 e_{j,i_j}\left\|\xi_{lj,i_j}\left(E_{lj,i_j}\right)\right\|^2}{2\left(\hat{a}_{lj,i_j}\left|e_{j,i_j}\right|\left\|\xi_{lj,i_j}\left(E_{lj,i_j}\right)\right\| + \varepsilon_{lj,i_j}\right)} - \frac{\hat{a}_{rj,1}^2 e_{j,i_j}\left\|\xi_{rj,i_j}\left(E_{j,i_j}\right)\right\|^2}{2\left(\hat{a}_{rj,i_j}\left|e_{j,i_j}\right|\left\|\xi_{rj,i_j}\left(E_{j,i_j}\right)\right\| + \varepsilon_{rj,i_j}\right)} - \frac{\hat{q}_{j,i_j}^2 e_{j,i_j}}{\hat{q}_{j,i_j}\left|e_{j,i_j}\right| + \omega_{j,i_j}} \tag{22}$$

of the subsystems with small design parameters $\omega_{j,i_j} > 0$, $\varepsilon_{lj,i_j} > 0$ and $\varepsilon_{rj,i_j} > 0$.

Consider the Lyapunov function candidate

$$V_{j,i_j} = V_{j,i_j-1} + \frac{1}{2}e_{j,i_j}^2 + \frac{1}{2\gamma_{lj,i_j}}\tilde{a}_{lj,i_j}^2 + \frac{1}{2\gamma_{rj,i_j}}\tilde{a}_{rj,i_j}^2 + \frac{1}{2\lambda_{j,i_j}}\tilde{q}_{j,i_j}^2 \tag{23}$$

where $\gamma_{lj,i_j} > 0$, $\gamma_{rj,i_j} > 0$ and $\lambda_{j,i_j} > 0$ are the design constants. The time derivative of V_{j,i_j} is

$$\dot{V}_{j,i_j} = \dot{V}_{j,i_j-1} + e_{j,i_j}\dot{e}_{j,i_j} + \frac{1}{\gamma_{lj,i_j}}a_{lj,i_j}\dot{\hat{a}}_{lj,i_j} + \frac{1}{\gamma_{rj,i_j}}a_{rj,i_j}\dot{\hat{a}}_{rj,i_j} + \frac{1}{\lambda_{j,i_j}}q_{j,i_j}\dot{\hat{q}}_{j,i_j} \tag{24}$$

Select the following adaptation laws

$$\dot{\hat{a}}_{lj,i_j} = -0.5\sigma_{lj,i_j}\hat{a}_{lj,i_j} + 0.5\gamma_{lj,i_j}\left|e_{j,i_j}\right|\left\|\xi_{lj,i_j}\left(E_{j,i_j}\right)\right\| \tag{25}$$

$$\dot{\hat{a}}_{rj,i_j} = -0.5\sigma_{rj,i_j}\hat{a}_{rj,i_j} + 0.5\gamma_{rj,i_j}\left|e_{rj,i_j}\right|\left\|\xi_{rj,i_j}\left(E_{j,i_j}\right)\right\| \tag{26}$$

$$\dot{\hat{q}}_{j,i_j} = -\eta_{j,i_j}\hat{q}_{j,i_j} + \lambda_{j,i_j}\left|e_{j,i_j}\right| \tag{27}$$

where $\sigma_{lj,i_j} > 0$, $\sigma_{rj,i_j} > 0$ and $\eta_{j,i_j} > 0$ are the design constants. After some manipulations we obtain

$$\dot{V}_{j,i_j} \le e_{j,i_j}e_{j,i_j+1} - \sum_{p=1}^{i_j-1}k_{j,p}e_{j,p}^2 - \sum_{p=1}^{i_j-1}\left(\frac{1}{2}\left(\frac{\sigma_{lj,p}}{2\gamma_{lj,p}}\tilde{a}_{lj,p}^2 + \frac{\sigma_{rj,p}}{2\gamma_{rj,p}}\tilde{a}_{rj,p}^2\right) + \frac{\eta_{j,p}}{2\lambda_{j,p}}\tilde{q}_{j,p}^2\right) + \sum_{p=1}^{i_j-1}\left(\frac{1}{2}\left(\frac{\sigma_{lj,p}}{2\gamma_{lj,p}}a_{lj,p}^2 + \frac{\sigma_{rj,p}}{2\gamma_{rj,p}}a_{rj,p}^2 + \varepsilon_{lj,p} + \varepsilon_{rj,p}\right) + \frac{\eta_{j,p}}{2\lambda_{j,p}}q_{j,p}^2 + \omega_{j,p}\right) \tag{28}$$

where the coupling term $e_{j,i_j}e_{j,i_j+1}$ will be cancelled in the next step.

Step ρ_j : Define the variable error $e_{j,\rho_j} = x_{j,\rho_j} - \alpha_{j,\rho_j-1}$. In this step, the practical control input u_j will be constructed. In step $\rho_j - 1$ we have obtained the virtual control input α_{j,ρ_j-1}, defined by

$$\dot{\alpha}_{j,\rho_j-1} = \sum_{l=1}^{m}\sum_{k=1}^{\rho_j-1}\frac{\partial\alpha_{j,\rho_j-1}}{\partial x_{l,k}}\left(x_{l,k+1} + z_{l,k} + \Delta_{l,k}\right) + \psi_{j,\rho_j-1} \tag{29}$$

where $\psi_{j,\rho_j-1} = \sum_{p=1}^{\rho_j}\left(\partial\alpha_{j,\rho_j-1}/\partial y_{d,j}^{p-1}\right)y_{r,j}^{p} + \sum_{p=1}^{\rho_j-1}\left(\partial\alpha_{j,\rho_j-1}/\partial\hat{a}_{lj,p}\right)\dot{\hat{a}}_{lj,p} + \left(\partial\alpha_{rj,\rho_j-1}/\partial\hat{a}_{rj,p}\right)\dot{\hat{a}}_{rj,p} + \sum_{p=1}^{\rho_j-1}\left(\partial\alpha_{j,\rho_j-1}/\partial\hat{q}_{j,p}\right)\dot{\hat{q}}_{j,p}$

the derivative of e_{j,ρ_j} is

$$\dot{e}_{j,\rho_j} = \dot{x}_{j,\rho_j} - \dot{\alpha}_{j,\rho_j} = z_{j,\rho_j} + u_j + f_j + \Delta_{j,\rho_j} - \dot{\alpha}_{j,\rho_j} \tag{30}$$

Define the unknown function $M_{j,\rho_j}\left(E_{j,\rho_j}\right)$ as follows

$$M_{j,\rho_j}\left(E_{j,\rho_j}\right) = z_{j,\rho_j} - \dot{\alpha}_{j,\rho_j} \tag{31}$$

where $E_{j,\rho_j} = X \in \Omega_{j,\rho_j}$. We use the approximation property of fuzzy systems to approximate $M_{j,\rho_j}\left(E_{j,\rho_j}\right)$, then, (30) can be rewritten as

$$M_{j,\rho_j}\left(E_{j,\rho_j}\right) = \frac{1}{2}\theta_{lj,\rho_j}^{*T}\xi_{lj,\rho_j}\left(E_{j,\rho_j}\right) + \frac{1}{2}\theta_{rj,\rho_j}^{*T}\xi_{rj,\rho_j}\left(E_{rj,\rho_j}\right) + \delta_{j,\rho_j}\left(E_{j,\rho_j}\right) \tag{32}$$

where θ_{lj,ρ_j}^{*T} and θ_{rj,ρ_j}^{*T} denote the optimal parameters vectors and δ_{j,ρ_j} is the fuzzy approximation error. By substituting (29) and (32) into (30), it follows that

$$\dot{e}_{j,\rho_j} = g_{j,\rho_j}\left[\frac{1}{2}\theta_{lj,\rho_j}^{*T}\xi_{lj,\rho_j}\left(E_{j,\rho_j}\right) + \frac{1}{2}\theta_{rj,\rho_j}^{*T}\xi_{rj,\rho_j}\left(E_{j,\rho_j}\right) + \delta_{j,\rho_j}\left(E_{j,\rho_j}\right) + u_j + f_j + \Delta_{j,\rho_j}\right] \tag{33}$$

In similar way, we define the control input u_j as follows

$$u_j = -e_{j,\rho_j-1} - k_{j,\rho_j}e_{j,\rho_j} - f_j - \frac{\hat{a}_{lj,\rho_j}^2 e_{j,\rho_j}\left\|\xi_{lj,\rho_j}\left(E_{j,\rho_j}\right)\right\|^2}{2\left(\hat{a}_{lj,\rho_j}\left|e_{j,\rho_j}\right|\left\|\xi_{lj,\rho_j}\left(E_{j,\rho_j}\right)\right\| + \varepsilon_{lj,\rho_j}\right)} - \frac{\hat{a}_{rj,\rho_j}^2 e_{j,\rho_j}\left\|\xi_{rj,\rho_j}\left(E_{j,\rho_j}\right)\right\|^2}{2\left(\hat{a}_{rj,\rho_j}\left|e_{j,\rho_j}\right|\left\|\xi_{rj,\rho_j}\left(E_{j,\rho_j}\right)\right\| + \varepsilon_{rj,\rho_j}\right)} - \frac{\hat{q}_{j,\rho_j}^2 e_{j,\rho_j}}{\hat{q}_{j,\rho_j}\left|e_{j,\rho_j}\right| + \omega_{j,\rho_j}} \tag{34}$$

Consider the Lyapunov function candidate

$$V_{j,\rho_j} = V_{j,\rho_j-1} + \frac{1}{2}e_{j,\rho_j}^2 + \frac{1}{2\gamma_{lj,\rho_j}}\tilde{a}_{lj,\rho_j}^2 + \frac{1}{2\gamma_{rj,\rho_j}}\tilde{a}_{rj,\rho_j}^2 + \frac{1}{2\lambda_{j,\rho_j}}\tilde{q}_{j,\rho_j}^2 \tag{35}$$

According to (17) and (25), (35) can be rewritten as

$$V_{j,\rho_j} = V_{j,\rho_j-1} + \sum_{p=1}^{\rho_j}\frac{1}{2}e_{j,p}^2 + \sum_{p=1}^{\rho_j}\frac{1}{2\gamma_{lj,p}}\tilde{a}_{lj,p}^2 + \sum_{p=1}^{\rho_j}\frac{1}{2\gamma_{rj,p}}\tilde{a}_{rj,p}^2 + \sum_{p=1}^{\rho_j}\frac{1}{2\lambda_{j,p}}\tilde{q}_{j,p}^2 \tag{36}$$

The time derivative of V_{j,ρ_j} is

$$\dot{V}_{j,\rho_j} = \dot{V}_{j,\rho_j-1} + e_{j,\rho_j}\dot{e}_{j,\rho_j} + \frac{1}{\gamma_{lj,\rho_j}} a_{lj,\rho_j}\dot{\hat{a}}_{lj,\rho_j} + \frac{1}{\gamma_{rj,\rho_j}} a_{rj,\rho_j}\dot{\hat{a}}_{rj,\rho_j} + \frac{1}{\lambda_{j,\rho_j}} q_{j,\rho_j}\dot{\hat{q}}_{j,\rho_j} \tag{37}$$

By substituting (33) and (34) into (37), we have

$$\begin{aligned}\dot{V}_{j,\rho_j} &= \dot{V}_{j,\rho_j-1} - e_{j,\rho_j-1}e_{j,\rho_j} - k_{j,\rho_j}e^2_{j,\rho_j} \\ &+ \frac{1}{2}\left[\theta^{*T}_{lj,\rho_j}\xi_{lj,\rho_j}\left(E_{j,\rho_j}\right)e_{j,\rho_j} - \frac{\hat{a}^2_{lj,\rho_j}e_{j,\rho_j}\left\|\xi_{lj,\rho_j}\left(E_{j,\rho_j}\right)\right\|^2}{\hat{a}_{lj,\rho_j}\left|e_{j,\rho_j}\right|\left\|\xi_{lj,\rho_j}\left(E_{j,\rho_j}\right)\right\| + \varepsilon_{lj,\rho_j}} + \frac{1}{\gamma_{lj,\rho_j}}a_{lj,\rho_j}\dot{\hat{a}}_{lj,\rho_j}\right] \\ &+ \frac{1}{2}\left[\theta^{*T}_{rj,\rho_j}\xi_{rj,\rho_j}\left(E_{j,\rho_j}\right)e_{j,\rho_j} - \frac{\hat{a}^2_{rj,\rho_j}e_{j,\rho_j}\left\|\xi_{rj,\rho_j}\left(E_{j,\rho_j}\right)\right\|^2}{\hat{a}_{lj,\rho_j}\left|e_{j,\rho_j}\right|\left\|\xi_{rj,\rho_j}\left(E_{j,\rho_j}\right)\right\| + \varepsilon_{rj,\rho_j}} + \frac{1}{\gamma_{rj,\rho_j}}a_{rj,\rho_j}\dot{\hat{a}}_{rj,\rho_j}\right] \\ &+ \left[\left(\Delta_{j,\rho_j} + \delta_{j,\rho_j}\left(E_{j,\rho_j}\right)\right)e_{j,\rho_j} - \frac{q^2_{j,\rho_j}e^2_{j,\rho_j}}{q_{j,\rho_j}\left|e_{j,\rho_j}\right| + \omega_{j,\rho_j}} + \frac{1}{\lambda_{j,\rho_j}}q_{j,\rho_j}\dot{\hat{q}}_{j,\rho_j}\right]\end{aligned} \tag{38}$$

Select the following adaptation laws

$$\dot{\hat{a}}_{lj,\rho_j} = -0.5\sigma_{lj,\rho_j}\hat{a}_{lj,\rho_j} + 0.5\gamma_{lj,\rho_j}\left|e_{j,\rho_j}\right|\left\|\xi_{lj,\rho_j}\left(E_{j,\rho_j}\right)\right\| \tag{39}$$

$$\dot{\hat{a}}_{rj,\rho_j} = -0.5\sigma_{rj,\rho_j}\hat{a}_{rj,\rho_j} + 0.5\gamma_{rj,\rho_j}\left|e_{j,\rho_j}\right|\left\|\xi_{rj,\rho_j}\left(E_{j,\rho_j}\right)\right\| \tag{40}$$

$$\dot{\hat{q}}_{j,\rho_j} = -\eta_{j,\rho_j}\hat{q}_{j,\rho_j} + \lambda_{j,\rho_j}\left|e_{j,\rho_j}\right| \tag{41}$$

where $\sigma_{lj,\rho_j} > 0$, $\sigma_{rj,\rho_j} > 0$ and $\eta_{j,\rho_j} > 0$ are the design constants.

Then we have

$$\begin{aligned}&\theta^{*T}_{j,\rho_j}\xi_{j,\rho_j}\left(E_{j,\rho_j}\right)e_{j,\rho_j} - \frac{\hat{a}^2_{lj,\rho_j}e_{j,\rho_j}\left\|\xi_{lj,\rho_j}\left(E_{j,\rho_j}\right)\right\|^2}{\hat{a}_{lj,\rho_j}\left|e_{j,\rho_j}\right|\left\|\xi_{lj,\rho_j}\left(E_{j,\rho_j}\right)\right\| + \varepsilon_{lj,\rho_j}} \\ &+ \frac{1}{\gamma_{lj,\rho_j}}a_{lj,\rho_j}\dot{\hat{a}}_{lj,\rho_j} \\ &+ \theta^{*T}_{rj,\rho_j}\xi_{rj,\rho_j}\left(E_{j,\rho_j}\right)e_{j,\rho_j} - \frac{\hat{a}^2_{rj,\rho_j}e_{j,\rho_j}\left\|\xi_{rj,\rho_j}\left(E_{j,\rho_j}\right)\right\|^2}{\hat{a}_{rj,\rho_j}\left|e_{j,\rho_j}\right|\left\|\xi_{rj,\rho_j}\left(E_{j,\rho_j}\right)\right\| + \varepsilon_{rj,\rho_j}} \\ &+ \frac{1}{\gamma_{rj,\rho_j}}a_{rj,\rho_j}\dot{\hat{a}}_{rj,\rho_j} \le 2\varepsilon_{lj,\rho_j} - \frac{\sigma_{lj,\rho_j}}{\gamma_{lj,\rho_j}}\tilde{a}^2_{lj,\rho_j} \\ &+ \frac{\sigma_{lj,\rho_j}}{\gamma_{lj,\rho_j}}a^2_{lj,\rho_j} + 2\varepsilon_{rj,\rho_j} - \frac{\sigma_{rj,\rho_j}}{\gamma_{rj,\rho_j}}\tilde{a}^2_{rj,\rho_j} + \frac{\sigma_{rj,\rho_j}}{\gamma_{rj,\rho_j}}a^2_{rj,\rho_j}\end{aligned} \tag{42}$$

$$\begin{aligned}&\left(d_{j,\rho_j} + \delta_{j,\rho_j}\left(E_{j,\rho_j}\right)\right)e_{j,\rho_j} - \frac{q^2_{j,\rho_j}e^2_{j,\rho_j}}{q_{j,\rho_j}\left|e_{j,\rho_j}\right| + \omega_{j,\rho_j}} \\ &+ \frac{1}{\lambda_{j,\rho_j}}q_{j,\rho_j}\dot{\hat{q}}_{j,\rho_j} \le \omega_{j,\rho_j} - \frac{\eta_{j,\rho_j}}{2\lambda_{j,\rho_j}}\tilde{q}^2_{j,\rho_j} + \frac{\eta_{j,\rho_j}}{2\lambda_{j,\rho_j}}q^2_{j,\rho_j}\end{aligned} \tag{43}$$

In step $\rho_j - 1$ it has been obtained that

$$\begin{aligned}\dot{V}_{j,\rho_j} &= e_{j,\rho_j-1}e_{j,\rho_j} - \sum_{p=1}^{\rho_j-1}k_{j,\rho_j}e^2_{j,p} \\ &- \sum_{p=1}^{\rho_j-1}\left(\frac{1}{2}\left(\frac{\sigma_{lj,p}}{2\gamma_{lj,p}}\tilde{a}^2_{lj,\rho_j} + \frac{\sigma_{rj,p}}{2\gamma_{rj,p}}\tilde{a}^2_{rj,\rho_j}\right) + \frac{\eta_{j,p}}{2\lambda_{j,p}}\tilde{q}^2_{j,p}\right) \\ &- \sum_{p=1}^{\rho_j-1}\left(\frac{1}{2}\left(\frac{\sigma_{lj,p}}{2\gamma_{lj,p}}a^2_{lj,\rho_j} + \frac{\sigma_{rj,p}}{2\gamma_{rj,p}}a^2_{rj,\rho_j} + \varepsilon_{lj,p} + \varepsilon_{rj,p}\right)\right. \\ &\left. + \frac{\eta_{j,p}}{2\lambda_{j,p}}q^2_{j,p} + \omega_{j,p}\right)\end{aligned} \tag{44}$$

By substituting (42), (43) and (44) into (38), we obtain

$$\begin{aligned}\dot{V}_{j,\rho_j} &= -\sum_{p=1}^{\rho_j}k^*_{j,\rho_j}e^2_{j,p} - \sum_{p=1}^{\rho_j} \\ &\left(\frac{1}{2}\left(\frac{\sigma_{lj,p}}{2\gamma_{lj,p}}\tilde{a}^2_{lj,\rho_j} + \frac{\sigma_{rj,p}}{2\gamma_{rj,p}}\tilde{a}^2_{rj,\rho_j}\right) + \frac{\eta_{j,p}}{2\lambda_{j,p}}\tilde{q}^2_{j,p}\right) \\ &- \sum_{p=1}^{\rho_j}\left(\frac{1}{2}\left(\frac{\sigma_{lj,p}}{2\gamma_{lj,p}}a^2_{lj,\rho_j} + \frac{\sigma_{rj,p}}{2\gamma_{rj,p}}a^2_{rj,\rho_j} + \varepsilon_{lj,p} + \varepsilon_{rj,p}\right)\right. \\ &\left. + \frac{\eta_{j,p}}{2\lambda_{j,p}}q^2_{j,p} + \omega_{j,p}\right)\end{aligned} \tag{45}$$

Let

$$\begin{aligned}\beta_j &= \min_{1\le p\le \rho_j}\left\{4k_{j,p}, \sigma_{lj,p}, \sigma_{rj,p}, 2\eta_{j,p}\right\} \\ \phi_j &= \sum_{p=1}^{\rho_j}\left(\frac{\sigma_{lj,p}}{4\gamma_{lj,p}}a^2_{lj,\rho_j} + \frac{\sigma_{rj,p}}{4\gamma_{rj,p}}a^2_{rj,\rho_j} + \frac{\eta_{j,p}}{2\lambda_{j,p}}q^2_{j,p} + \frac{1}{2}\varepsilon_{lj,p} + \frac{1}{2}\varepsilon_{rj,p} + \omega_{j,p}\right)\end{aligned} \tag{46}$$

Following (44) yields

$$\begin{aligned}\dot{V}_{j,\rho_j} &\le -\sum_{p=1}^{\rho_j}k_{j,\rho_j}e^2_{j,p} - \sum_{p=1}^{\rho_j}\left(\frac{1}{2}\left(\frac{\sigma_{lj,p}}{2\gamma_{lj,p}}\tilde{a}^2_{lj,\rho_j} + \frac{\sigma_{rj,p}}{2\gamma_{rj,p}}\tilde{a}^2_{rj,\rho_j}\right) + \frac{\eta_{j,p}}{2\lambda_{j,p}}\tilde{q}^2_{j,p}\right) + \phi_j \\ &\le -\sum_{p=1}^{\rho_j}k_{j,\rho_j}e^2_{j,p} - \sum_{p=1}^{\rho_j}\left(\frac{1}{2}\left(\frac{\sigma_{lj,p}}{2\gamma_{lj,p}}\tilde{a}^2_{lj,\rho_j} + \frac{\sigma_{rj,p}}{2\gamma_{rj,p}}\tilde{a}^2_{rj,\rho_j}\right) + \frac{\eta_{j,p}}{2\lambda_{j,p}}\tilde{q}^2_{j,p}\right) \\ &+ \phi_j \le -\beta_j V_{j,\rho_j} + \phi_j\end{aligned} \tag{47}$$

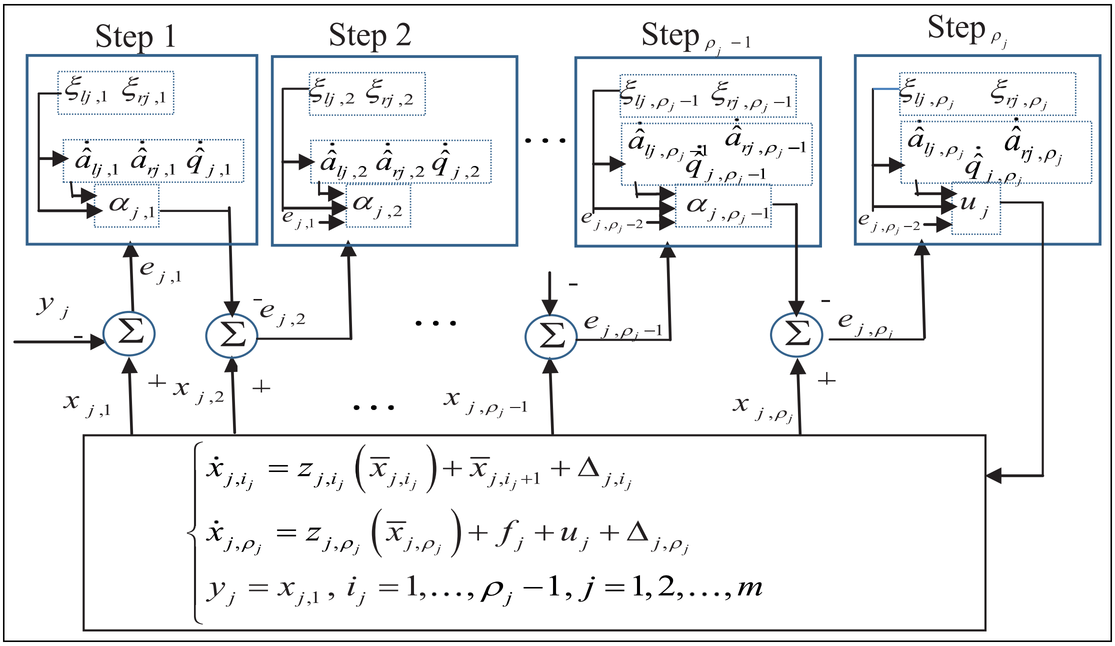

Figure 2. Schematic diagram of fuzzy type 2 backstepping controller.

Let

$$V = \sum_{j=1}^{m} V_{j,\rho_j} = -\frac{1}{2}\sum_{j=1}^{m}\sum_{i_j}^{\rho_j} e_{j,i_j}^2 + \frac{1}{2}\sum_{j=1}^{m}\sum_{i_j}^{\rho_j} \frac{1}{2\gamma_{lj,i_j}}\tilde{a}_{lj,i_j}^2 + \frac{1}{2\gamma_{rj,i_j}}\tilde{a}_{rj,i_j}^2 + \frac{1}{\lambda_{j,i_j}}\tilde{q}_{j,i_j}^2 \tag{48}$$

$$\dot{V} = \sum_{j}^{m} \dot{V}_{j,\rho_j} \le -\sum_{j}^{m} \beta_j V_{j,\rho_j} - \phi_j \le -\beta V + \phi \tag{49}$$

where

$$\begin{cases} \beta = \min\{\beta_1, \beta_2, \dots, \beta_m\} \\ \phi = \sum_{j=1}^{m} \phi_j \end{cases} . \tag{50}$$

The design procedure of the controller can be visualized from the block diagram shown in Figure 2.

The following theorem guarantees the stability of the closed loop system.

Theorem 1: Consider the system (1). By choosing the virtual control inputs (15) and (22), the practical control input (33), and the adaptation laws (18), (19), (20), (25), (26), (27), (39), (40) and (41), under assumptions 1–3, the overall fuzzy control system proposed in the aforementioned design procedure can guarantee the following:

(1) There exist sufficiently large compact sets Ω_{j,i_j} such that all the signals in the resulting closed-loop system are uniformly bounded for all $E_{j,i_j} \in \Omega_{j,i_j}$, $i_j = 1, \dots \rho_j$, $j = 1, \dots, m$.

(2) The tracking error variable $E_{j,1}$ converges to compact set defined by

$$\Omega_s = \left\{ e_{j,1} \middle| \lim_{t\to\infty} \left|e_{j,1}(t)\right| = \sqrt{2\frac{\phi}{\beta}}, j = 1, \dots, m \right\} \tag{51}$$

Remark 2: In the earlier analysis, it can be seen from (46) and (50) that the size of $e_{j,1}(t)$ lies on the design parameters k_{j,i_j}, σ_{rj,i_j}, σ_{lj,i_j}, η_{j,i_j}, γ_{rj,i_j}, γ_{lj,i_j}, λ_{j,i_j}, ε_{lj,i_j}, ε_{rj,i_j} and ω_{lj,i_j}, $i_j = 1, \dots \rho_j$, $j = 1, \dots, m$. If we fix $\sigma_{rj,i_j} > 0$, $\sigma_{lj,i_j} > 0$ and $\eta_{lj,i_j} > 0$, the following are true:

(1) Increasing k_{j,i_j} might result in larger β_j and subsequently larger β.

(2) Both decreasing ε_{lj,i_j}, ε_{rj,i_j} and ω_{lj,i_j} and increasing γ_{rj,i_j}, γ_{lj,i_j} and λ_{lj,i_j},will reduce ϕ_j and, subsequently, ϕ. Thus, if we increase k_{j,i_j}, ε_{lj,i_j}, ε_{rj,i_j} and ω_{lj,i_j} and decrease γ_{rj,i_j}, γ_{lj,i_j} and λ_{lj,i_j}, it will help to reduce $\frac{\phi}{\beta}$, which implies that the tracking errors can be made arbitrarily small by appropriately choosing the design parameters.

Proof: Multiplying both sides in the inequality (49) by $e^{\tau t}$, we obtain

$$\frac{d}{dt}\left(V(t)e^{\beta t}\right) \le \phi e^{\tau t} \tag{52}$$

Integrating (51) over [0, t] leads to

$$0 \le V(t) \le \left[V(0) - \frac{\phi}{\beta}\right] e^{-\beta t} + \frac{\phi}{\beta} \tag{53}$$

Since β and ϕ are positive constants, the aforementioned inequality implies that

$$0 \le V(t) \le V(0)e^{-\beta t} + \frac{\phi}{\beta} \tag{54}$$

From (47), we can see that

$$\sum_{i_j}^{\rho_j} e_{j,i_j}^2 \le V(t), i_j = 1, ..., \rho_j, j = 1, ..., m \tag{55}$$

$$\frac{1}{4\gamma_{lj,i_j}} \tilde{a}_{lj,i_j}^2 \le V(t), i_j = 1, ..., \rho_j, j = 1, ..., m \tag{56}$$

$$\frac{1}{4\gamma_{rj,i_j}} \tilde{a}_{rj,i_j}^2 \le V(t), i_j = 1, ..., \rho_j, j = 1, ..., m \tag{57}$$

$$\frac{1}{2\lambda_{j,i_j}} \tilde{q}_{j,i_j}^2 \le V(t), i_j = 1, ..., \rho_j, j = 1, ..., m \tag{58}$$

By making use of (53) and assumption 2, we obtain

$$\left|e_{j,i_j}\right| \le \sqrt{2\left(V(0)e^{-\tau t} + \frac{\mu}{\tau}\right)}, i_j = 1, ..., \rho_j, j = 1, ..., m \tag{59}$$

$$\left|\tilde{a}_{lj,i_j}\right| \le \sqrt{4\gamma_{lj,i_j}\left(V(0)e^{-\beta t} + \frac{\phi}{\beta}\right)}, i_j = 1, ..., \rho_j, j = 1, ..., m \tag{60}$$

$$\left|\tilde{a}_{rj,i_j}\right| \le \sqrt{4\gamma_{rj,i_j}\left(V(0)e^{-\beta t} + \frac{\phi}{\beta}\right)}, i_j = 1, ..., \rho_j, j = 1, ..., m \tag{61}$$

$$\left|\tilde{q}_{j,i_j}\right| \le \sqrt{2\lambda_{j,i_j}\left(V(0)e^{-\beta t} + \frac{\phi}{\beta}\right)}, i_j = 1, ..., \rho_j, j = 1, ..., m \tag{62}$$

Since $\tilde{a}_{lj,i_j} = \hat{a}_{lj,i_j} - a_{lj,i_j}$, $\tilde{a}_{j,i_j} = \hat{a}_{rj,i_j} - a_{rj,i_j}$ and $\tilde{q}_{j,i_j} = \hat{q}_{j,i_j} - q_{j,i_j}$, with (59), (60) and (61), we obtain

$$\left|\hat{a}_{lj,i_j}\right| \le \sqrt{4\gamma_{lj,i_j}\left(V(0)e^{-\beta t} + \frac{\phi}{\beta}\right)} + \left|a_{lj,i_j}\right|, i_j = 1, ..., \rho_j, j = 1, ..., m \tag{63}$$

$$\left|\hat{a}_{rj,i_j}\right| \le \sqrt{4\gamma_{rj,i_j}\left(V(0)e^{-\beta t} + \frac{\phi}{\beta}\right)} + \left|a_{rj,i_j}\right|, i_j = 1, ..., \rho_j, j = 1, ..., m \tag{64}$$

$$\left|q_{j,i_j}\right| \le \sqrt{2\lambda_{rj,i_j}\left(V(0)e^{-\beta t} + \frac{\phi}{\beta}\right)} + \left|q_{rj,i_j}\right|, i_j = 1, ..., \rho_j, j = 1, ..., m \tag{65}$$

Therefore, from (59), (63), (64) and (65), we can conclude that all signals in the resulting closed loop system are uniformly bounded. From (53) and (55), we have

$$\left|e_{j,1}\right| \le \sqrt{2\left(V(0) - \frac{\phi}{\beta}\right)e^{-\beta t} + 2\frac{\phi}{\beta}}, j = 1, ..., m \tag{66}$$

If $V(0) = \frac{\phi}{\beta}$, then the following inequality is true

$$\left|e_{j,1}\right| \le \sqrt{2\frac{\phi}{\beta}}, j = 1, ..., m \tag{67}$$

If $V(0) \ne \dfrac{\phi}{\beta}$ from (66), it can be concluded that for any given $\tau_e > \sqrt{2\phi/\beta}$, for all $j = 1, ..., m$, there exists T_e such that for any $t > T_e$, we have $\left|e_{j,1}\right| \le \tau_e$. Specifically, given any

$$\tau_s \le \sqrt{2\left(V(0) - \frac{\phi}{\beta}\right)e^{-\beta T_e} + 2\frac{\phi}{\beta}}, j = 1, ..., m, \quad V(0) \ne \frac{\phi}{\beta} \tag{68}$$

and by taking $T_e = -(1/\beta)\ln\left[(1/\tau_s^2 - (2\phi/\beta))/2 \quad (V(0) - (\phi/\beta))\right]$ we have

$$\left|\lim_{t\to\infty}\left|e_{j,1}(t)\right|\right| = \sqrt{2\frac{\phi}{\beta}}, j = 1, ..., m \tag{69}$$

It can be then concluded that all the signals in the closed-loop system remain bounded and the tracking errors can be made arbitrarily small if the design parameters are chosen appropriately. The proof is thus completed.

Simulation example

Consider a second order chaotic system such as the well-known duffing's equation describing a special nonlinear circuit or a pendulum moving in a viscous medium under control (Loria et al., 1998)

$$\left\{\ddot{x} = -p\dot{x} - p_1 x - p_2 x^2 + q\cos(wt) + u\right. \tag{70}$$

where p, p_1, p_2 and q are real constants; t is the time variable w is the frequency.

It can be shown that without control, that is, $u(t) = 0$, the system is chaotic. The chaotic motion of the duffing system is shown in Figure 3.

We apply the proposed controller to control the three interconnected chaotic systems, where each subsystem is duffing chaotic system.

$$\begin{aligned}
&q_1\begin{cases}\dot{x}_{11} = x_{12} + x_{31} + \Delta_{11}\\ \dot{x}_{12} = -0.1x_{12} - 0.5x_{11} - x_{11}^3 + 12\cos(t) + x_{21} + x_{31} + x_{22} + u_1 + \Delta_{12}\\ y_1 = x_{11}\end{cases}\\
&q_2\begin{cases}\dot{x}_{21} = x_{22} + x_{11} + \Delta_{21}\\ \dot{x}_{22} = -0.1x_{22} - 0.5x_{21} - x_{21}^3 + 12\cos(t) + x_{11} + x_{12} + x_{32} + u_2 + \Delta_{22}\\ y_2 = x_{21}\end{cases}\\
&q_3\begin{cases}\dot{x}_{31} = x_{32} + x_{21} + \Delta_{31}\\ \dot{x}_{32} = -0.1x_{22} - 0.5x_{21} - x_{21}^3 + 12\cos(t) + x_{11} + x_{12} + x_{21} + u_3 + \Delta_{32}\\ y_3 = x_{31}\end{cases}
\end{aligned} \tag{71}$$

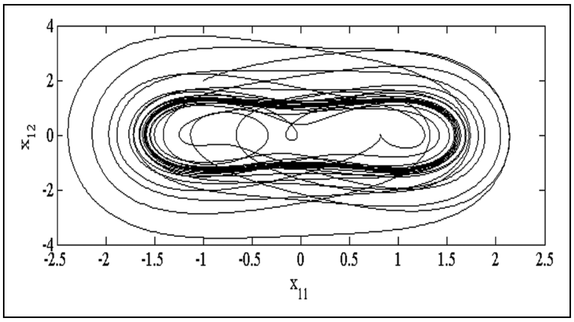

Figure 3. Chaotic attractor of duffing system.

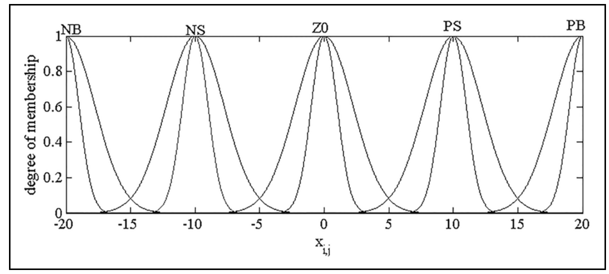

Figure 4. Interval type 2 membership functions.

where $\Delta_{11} = 0.5\cos(x_{11}^2 x_{31} x_{22})\sin(t)$, $\Delta_{12} = 0.2\cos(x_{21}^2 + x_{31}^2)\cos(t)$, $\Delta_{21} = 0.6\sin(x_{21}x_{11}x_{32})\sin(t)$, $\Delta_{22} = 0.2\sin(x_{21}^2 + x_{32}^2)\sin^2(t)$.

$$\Delta_{31} = 0.6\sin(x_{31}x_{11}x_{12})\sin(t),$$
$$\Delta_{32} = 0.2\sin(x_{21}^2 + x_{22}^2)\sin^2(t).$$

The control objective is to design an adaptive backstepping type 2 fuzzy controller such that the outputs y_j follows reference signals $y_{d,j}$, $(j = 1, 2, 3)$. In simulation, five fuzzy sets are defined for each variable x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, and x_{32}(Figure 4).

According to the proposed method, the controller law, virtual controller and the adaptive laws are defined as follows

The control inputs

$$u_1 = -e_{1,1} - k_{1,2}e_{1,2} - f_1 - \frac{\hat{a}_{l1,2}^2 e_{1,2}\|\xi_{l1,2}(E_{1,2})\|^2}{2(\hat{a}_{l1,2}|e_{l1,2}|\|\xi_{1,2}(E_{1,2})\| + \varepsilon_{l1,2})} - \frac{\hat{a}_{r1,2}^2 e_{1,2}\|\xi_{r1,2}(E_{1,2})\|^2}{2(\hat{a}_{r1,2}|e_{1,2}|\|\xi_{r1,2}(E_{1,2})\| + \varepsilon_{r1,2})} - \frac{\hat{q}_{1,2}^2 e_{1,2}}{\hat{q}_{1,2}|e_{1,2}| + \omega_{1,2}} \tag{72}$$

$$u_2 = -e_{2,1} - k_{2,2}e_{2,2} - f_2 - \frac{\hat{a}_{r2,2}^2 e_{2,2}\|\xi_{r2,2}(E_{2,2})\|^2}{2(\hat{a}_{r2,2}|e_{2,2}|\|\xi_{r2,2}(E_{2,2})\| + \varepsilon_{r2,2})} - \frac{\hat{a}_{r2,2}^2 e_{2,2}\|\xi_{r2,2}(E_{2,2})\|^2}{2(\hat{a}_{r2,2}|e_{2,2}|\|\xi_{r2,2}(E_{2,2})\| + \varepsilon_{r2,2})} - \frac{\hat{q}_{2,2}^2 e_{2,2}}{\hat{q}_{2,2}|e_{2,2}| + \omega_{2,2}} \tag{73}$$

$$u_3 = -e_{3,1} - k_{3,2}e_{3,2} - f_3 - \frac{\hat{a}_{r3,2}^2 e_{3,2}\|\xi_{r3,2}(E_{3,2})\|^2}{2(\hat{a}_{r3,2}|e_{3,2}|\|\xi_{r3,2}(E_{3,2})\| + \varepsilon_{r3,2})} - \frac{\hat{a}_{r3,2}^2 e_{3,2}\|\xi_{r3,2}(E_{3,2})\|^2}{2(\hat{a}_{r3,2}|e_{3,2}|\|\xi_{r3,2}(E_{3,2})\| + \varepsilon_{r3,2})} - \frac{\hat{q}_{3,2}^2 e_{3,2}}{\hat{q}_{3,2}|e_{3,2}| + \omega_{3,2}} \tag{74}$$

The virtual control inputs

$$\alpha_{1,1} = -k_{1,1}e_{1,1} - \frac{\hat{a}_{l1,1}^2 e_{1,1}\|\xi_{l1,1}(E_{1,1})\|^2}{2(\hat{a}_{l1,1}|e_{l1,1}|\|\xi_{1,1}(E_{1,1})\| + \varepsilon_{l1,1})} - \frac{\hat{a}_{r1,1}^2 e_{1,1}\|\xi_{r1,1}(E_{1,1})\|^2}{2(\hat{a}_{r1,1}|e_{1,1}|\|\xi_{r1,1}(E_{1,1})\| + \varepsilon_{r1,1})} - \frac{\hat{q}_{1,1}^2 e_{1,1}}{\hat{q}_{1,1}|e_{1,1}| + \omega_{1,1}} \tag{75}$$

$$\alpha_{2,1} = -k_{2,1}e_{2,1} - \frac{\hat{a}_{l2,1}^2 e_{2,1}\|\xi_{l2,1}(E_{2,1})\|^2}{2(\hat{a}_{l2,1}|e_{l2,1}|\|\xi_{2,1}(E_{2,1})\| + \varepsilon_{l2,1})} - \frac{\hat{a}_{r2,1}^2 e_{2,1}\|\xi_{r2,1}(E_{2,1})\|^2}{2(\hat{a}_{r2,1}|e_{2,1}|\|\xi_{r2,1}(E_{2,1})\| + \varepsilon_{r2,1})} - \frac{\hat{q}_{2,1}^2 e_{2,1}}{\hat{q}_{2,1}|e_{2,1}| + \omega_{2,1}} \tag{76}$$

$$\alpha_{3,1} = -k_{3,1}e_{3,1} - \frac{\hat{a}_{l3,1}^2 e_{3,1}\|\xi_{l3,1}(E_{3,1})\|^2}{2(\hat{a}_{l3,1}|e_{l3,1}|\|\xi_{3,1}(E_{3,1})\| + \varepsilon_{l3,1})} - \frac{\hat{a}_{r3,1}^2 e_{3,1}\|\xi_{r3,1}(E_{3,1})\|^2}{2(\hat{a}_{r3,1}|e_{3,1}|\|\xi_{r3,1}(E_{3,1})\| + \varepsilon_{r3,1})} - \frac{\hat{q}_{3,1}^2 e_{3,1}}{\hat{q}_{3,1}|e_{3,1}| + \omega_{3,1}} \tag{77}$$

Also, the adaptive laws

$$\dot{\hat{a}}_{lj,i} = -0.5\sigma_{lj,i}\hat{a}_{lj,i} + 0.5\gamma_{lj,i}|e_{j,i}|\|\xi_{lj,i}(E_{j,i})\| \tag{78}$$

$$\dot{\hat{a}}_{rj,i} = -0.5\sigma_{rj,i}\hat{a}_{rj,i} + 0.5\gamma_{rj,}|e_{rj,i}|\|\xi_{rj,i}(E_{j,i})\| \tag{79}$$

$$\dot{\hat{q}}_{j,i} = -\eta_{j,i}\hat{q}_{j,i} + \lambda_{j,i}|e_{j,i}| j = 1,2,3, \quad i = 1,2 \tag{80}$$

In this simulation, the initial conditions are chosen as $[x_{1,1}(0), x_{2,1}(0), x_{1,2}(0), x_{2,2}(0), x_{3,1}(0), x_{3,2}(0)] = [2, -1, 0.9, -1, 1, -0.5]$, the initial values of the adjusted parameters are given to be $\hat{a}_{l1,1}(0) = \hat{a}_{r1,1}(0) =$ $q_{1,1}(0) = \hat{a}_{r1,2}(0) = \hat{a}_{r1,2}(0) = \hat{a}_{r3,2}(0) = \hat{a}_{l3,2}(0) = 1$, $q_{1,2}(0) = \hat{a}_{l2,1}(0) = \hat{a}_{r2,1}(0) = \hat{a}_{r3,1}(0) = q_{2,1}(0) = 1$ and $\hat{a}_{l2,2}(0) = \hat{a}_{r2,2}(0) = q_{2,2}(0) = q_{3,2}(0) = 2$. The $\hat{a}_{l3,1}(0) = q_{3,1}(0) = 1$ controller parameters are chosen as $k_{1,1} = k_{1,2} = k_{2,1} = k_{2,2} = 15$ $k_{3,2} = k_{3,1} = 15$, $\varepsilon_{l1,1} = \varepsilon_{l1,2} = \varepsilon_{l2,1} = \varepsilon_{l2,2} = \varepsilon_{l3,2} = \varepsilon_{l3,1} = 10$, $\varepsilon_{r1,1} = \varepsilon_{r1,2} = \varepsilon_{r2,1} = \varepsilon_{r3,1} = 10$ $\varepsilon_{r3,2} = \varepsilon_{r2,2} = 10$, $\omega_{1,1} = \omega_{1,2} = \omega_{2,1} = \omega_{2,2} = \omega_{3,1} = \omega_{3,2} = 10$, $\eta_{1,1} = \eta_{1,2} = \eta_{2,1} = \eta_{2,2} = 0.1$,

Table 1. Performance comparison of the proposed approach and the traditional adaptive backstepping controller.

	$\frac{1}{T}\int_0^T e_{1,1}(t)\,dt$	$\frac{1}{T}\int_0^T e_{2,1}(t)\,dt$	$\frac{1}{T}\int_0^T e_3(t)\,dt$	$\frac{1}{T}\int_0^T u_1(t)\,dt$	$\frac{1}{T}\int_0^T u_2(t)\,dt$	$\frac{1}{T}\int_0^T u_3(t)\,dt$
T. B. C.	12.3132	9.9002	16.1445	988.1773	627.2641	805.4903
Proposed approach	4.5512	5.5368	6.1541	380.5819	221.5331	230.0215

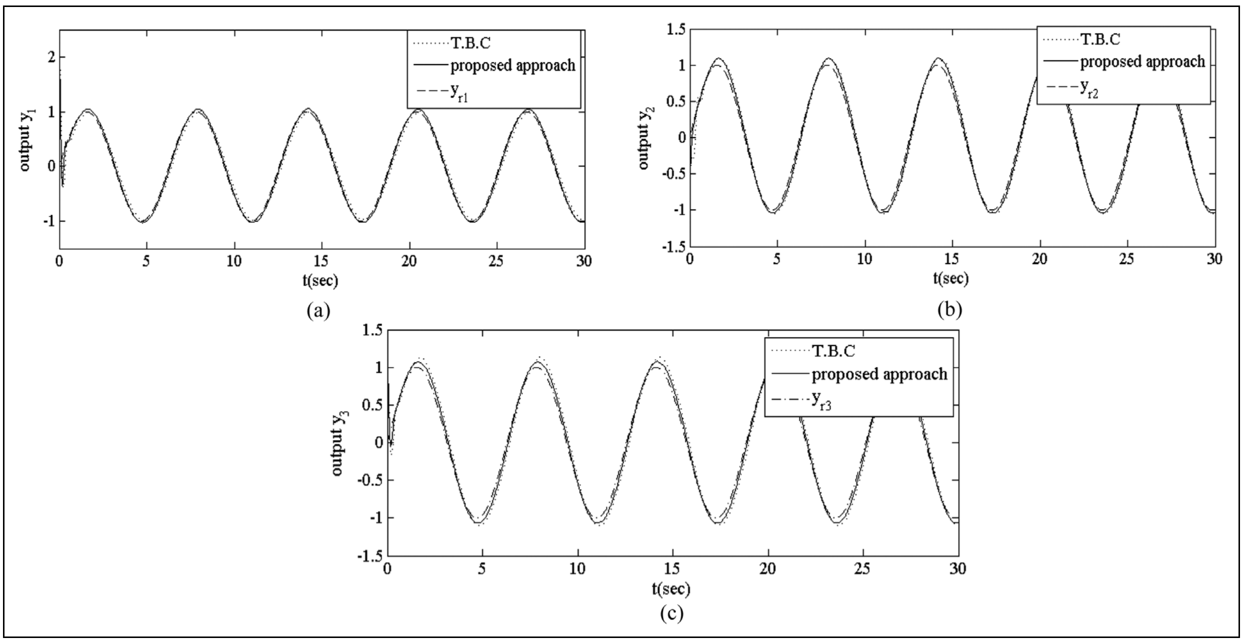

Figure 5. Output trajectories (a) y_1, (b) y_2, (c) y_3.

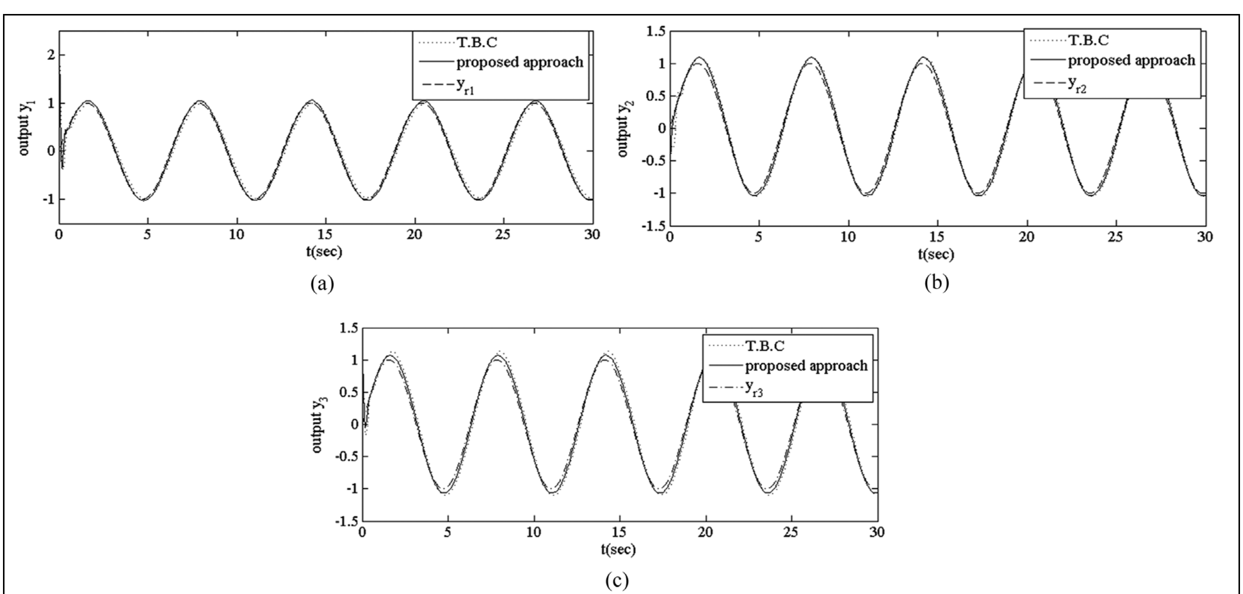

Figure 6. Output trajectories of the (a) y_1, (b) y_2 (c) $y_3$with noise.

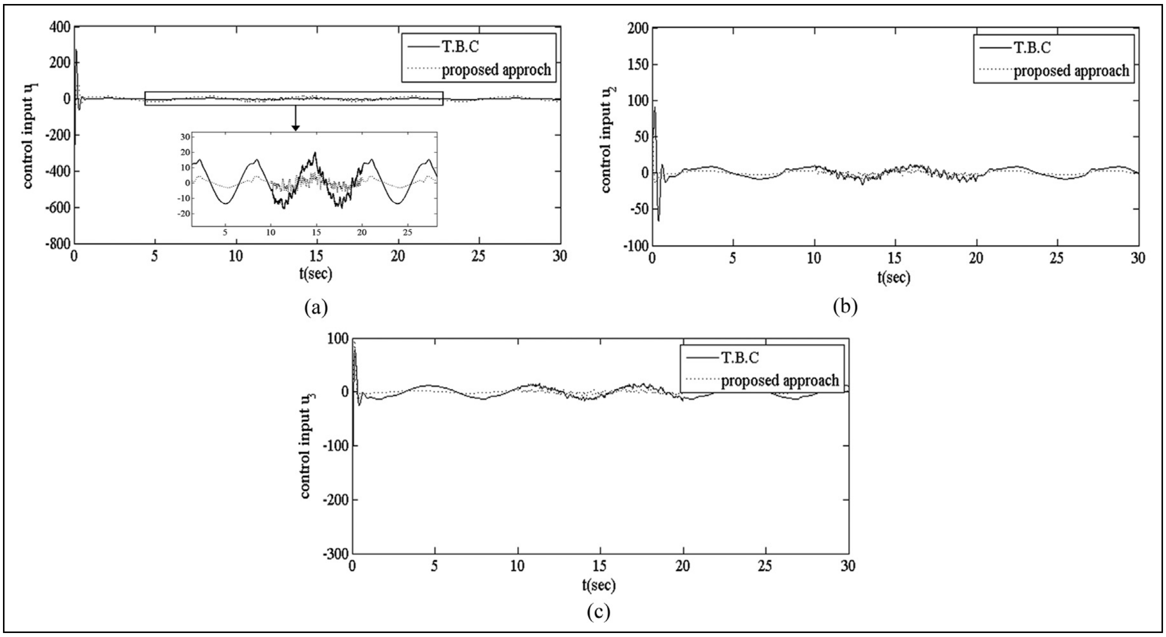

Figure 7. Control inputs (a) u_1, (b) u_2, (b) $u_3$with noise.

Table 2. Performance comparison of the proposed method using type 1 and type 2 fuzzy systems.

	$\frac{1}{T}\int_0^T e_{1,1}(t)\,dt$	$\frac{1}{T}\int_0^T e_{2,1}(t)\,dt$	$\frac{1}{T}\int_0^T e_{3,1}(t)\,dt$	$\frac{1}{T}\int_0^T u_1(t)\,dt$	$\frac{1}{T}\int_0^T u_2(t)\,dt$	$\frac{1}{T}\int_0^T u_3(t)\,dt$
T1FS	5.8292	6.5402	9.3775	1.0042e + 003	628.0019	844.3434
T2FS	4.5512	5.5368	6.1541	380.5819	221.5331	230.0215

$\eta_{3,3} = \eta_{3,2} = 0.1$ $\gamma_{l1,1} = \gamma_{l1,2} = \gamma_{l2,1} = \gamma_{l2,2} = \gamma_{l3,1} = \gamma_{l3,2} = 0.1$, $\gamma_{r1,1} = \gamma_{r1,2} = \gamma_{r2,1} = \gamma_{r2,2} = 0.1$ and $\lambda_{1,1} = \lambda_{1,2} = \lambda_{2,1} = \lambda_{2,2} = \lambda_{3,1} = \lambda_{3,2} = 0.1$.

To investigate the effectiveness of the proposed backstepping controller, a comparison between a traditional adaptive backstepping controller (T. B. C) and the proposed approach is made. Table 1 shows the performance obtained.

It can be seen that the proposed method has better tracking error performance with less control effort.

Figure 5 shows the evolution of outputs y_1, y_2 and y_3, which are obtained by applying the control inputs (71), (72) and (73) to the chaotic system (70) for tracking the reference signals $y_{r,j} = \sin(t)\ j = 1,2,3$. We can observe a good tracking performance.

Figure 6 shows that even in the presence of external disturbances (Gaussian noise) in the time interval $t \in [10, 20]$, the system remains performing in terms of rapidity of convergence and robustness. Figure 7 shows the control signals that are bounded and smooth. The effect of model uncertainty and external disturbances is well minimized.

The simulation results show that the favorable tracking performance can be achieved by applying the proposed approach.

Figure 8 shows that the type 2 fuzzy controller eliminates the chaotic motion and takes the system response to a stable orbit.

Remark 3: Table 2 shows the performance obtained by the proposed method using T1FSs and T2FSs.

It can be seen that the proposed method with interval type 2 fuzzy systems has better tracking error performance with less control effort.

Conclusion

In this paper, an adaptive fuzzy robust control approach based on backstepping design is proposed for uncertain MIMO chaotic system. The type 2 fuzzy systems are used to approximate unknown nonlinear functions (interconnections between the chaotic subsystems and derivative of the virtual control). By only adjusting estimations of unknown bounds, the proposed control approach reduced computation burden. This method has been applied to control the interconnected duffing chaotic system to track a reference trajectory. The proposed scheme can guarantee that all the close loop signals are bounded and that the outputs of the system converge to a

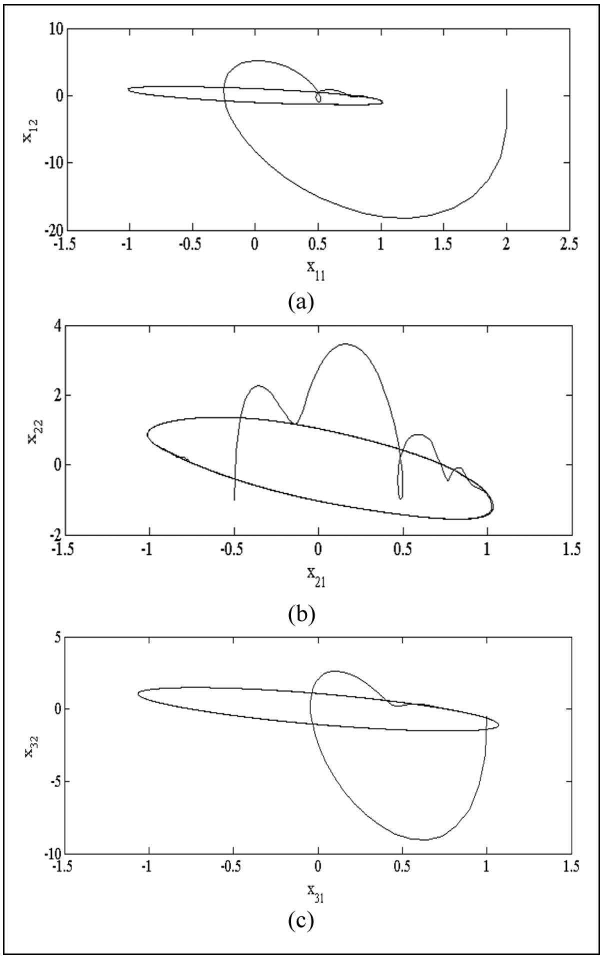

Figure 8. Chaotic attractor of duffing system under control.

small neighborhood of the desired trajectory. Simulations results show that the proposed method is very effective and robust against system uncertainty.

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