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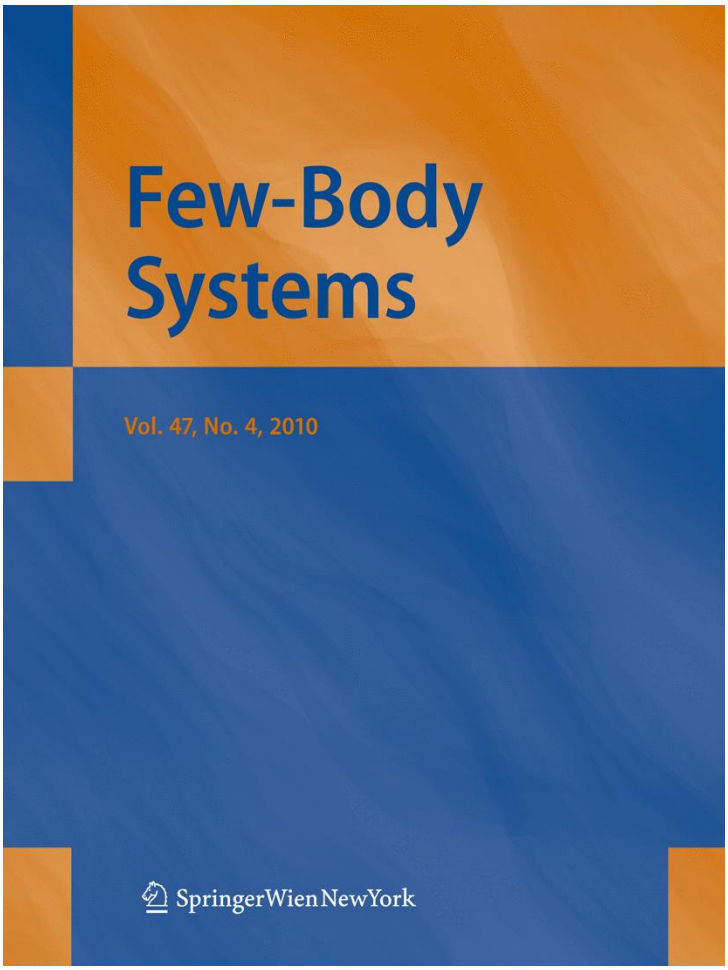

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Three-Body Spectrum of ^{18}C and its Relevance to *r*-Process Nucleosynthesis

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Abstract The ^{18}C spectrum has been studied in a three body $n + n +^{16}$ C model that includes deformation and the 2^+ excitation of the ^{16}C core as well as Pauli projection of forbidden states. The $^{16}\mathrm{C} - n$ interaction employed in this study has been fitted to reproduce the experimental spectrum of ^{17}C. The calculations show that two neutron separation energy in ^{18}C in consistent with three-body structure of this nucleus and predict more states bound with respect to three-body decay. The comparison of their position to known excited states in ^{18}C is discussed. These calculations suggest also that a few states may exist in astrophysically relevant region between the $^{17}\mathrm{C}+n$ and $^{16}\mathrm{C}+2n$ decay thresholds. The most important of them is 1^- as it can give a large $E1$ resonant contribution to $^{17}\mathrm{C}(n,\gamma)^{18}\mathrm{C}$ neutron capture. The calculations also suggest that a virtual s-wave state may exist above the $^{17}\mathrm{C}+n$ threshold that can give rise to non-negligible $M1$ contributions to the $^{17}\mathrm{C}(n,\gamma)^{18}\mathrm{C}$ reaction rate. The presence of these states in the ^{18}C spectrum can lead to an increased $^{17}\mathrm{C}(n,\gamma)^{18}\mathrm{C}$ reaction rate, which can significantly influence the abundances of uranium and thorium synthesized in the r-process in the supernovae explosions.

1 Introduction

It has been recently realised that ^{18}C is a semi-waiting point in r-process nucleosynthesis, which occurs in core collapse supernovae explosions [1]. The calculations of Sasaqui et al. [1], performed using models with neutrino-driven winds, have shown that the ^{232}Th, ^{235} U and ^{238}U abundances are influenced by the neutron capture reactions $^{15}\mathrm{C}(n,\gamma)^{16}\mathrm{C}$ and $^{17}\mathrm{C}(n,\gamma)^{18}\mathrm{C}$, which are a part of the chain $^{14}\mathrm{C}(n,\gamma)^{15}\mathrm{C}(n,\gamma)^{16}\mathrm{C}(n,\gamma)^{17}\mathrm{C}(n,\gamma)$ $^{18}\mathrm{C}(\alpha,n)^{21}\mathrm{O}(n,\gamma)^{22}\mathrm{O}(n,\gamma)^{23}\mathrm{O}$. In particular, for some choice of parameters of the fast flow model, the actinide abundances change by a factor of two or even more when the $^{17}\mathrm{C}(n,\gamma)^{18}C$ reaction rate is varied by the 2σ uncertainty.

When calculating the r-process nucleosynthesis, Sasaqui et al. used the $^{17}\mathrm{C}(n,\gamma)^{18}\mathrm{C}$ reaction rate predicted earlier by Herndl et al. in [2] on the basis of the shell model calculations with the WBP interaction [3]. According to these calculations, no resonances occur in the low-energy region above the $n +^{17}$ C threshold relevant to supernovae temperatures, so that the rate of this reaction is determined by non-resonant (n,γ) capture.

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At the time Herndl et al. published their results, little was known about the spectrum of ^{18}C. Only the first excited 2^+ state at $E_x = 1.6$ MeV had been seen. Recently, new levels have been identified at 2.5 and 4.0 MeV from the study of in-beam γ-ray spectroscopy [5], inelastically scattered ^{18}C nuclei [6] and one neutron removal from ^{18}C [7]. According to the latter work, the 4.0 MeV state can have a spin-parity of $(2^+, 3^+)$. This assignment, as well as the position of the first excited state at 1.6 MeV, is reproduced by the shell model calculations of Herndl et al. The modification of the shell model matrix elements made in Ref. [5] improves the position of the first 2^+ state in ^{18}C but gives seven levels above 2 MeV where only two states are observed. Also, the modified interaction does not describe the spectrum of ^{17}C. Thus, predictions of the shell model for neutron rich carbon isotopes are not accurate and the conclusion about the absence of low-lying astrophysically relevant $^{17}\mathrm{C} + n$ resonances may be premature.

The ^{18}C isotope belongs to a class of nuclei where the one-neutron and two-neutron decays thresholds are close to each other. In ^{18}C, the gap between $^{16}\mathrm{C} + 2n$ and $^{17}\mathrm{C} + n$ thresholds is only 0.728 MeV. The part of the ^{18}C spectrum relevant to supernovae temperatures lies just between these two thresholds and thus can be strongly influenced by the proximity of the $^{16}\mathrm{C} + 2n$ threshold. The standard shell model is not designed to include threshold effects into its scheme. On the other hand, it could be very difficult to include such effects properly for nuclei away from the closed shells. Therefore, the use of other models that can treat two-neutrons correlations explicitly should be welcome. It seems natural to use a three-body model for such a purpose.

A three-body model has already been used for another carbon isotope, ^{16}C [8,9], where a similarly small gap between one- and two-nucleon thresholds is observed. This model reproduces the two-neutron separation energy and explains abnormally small $B(E2)$ value in ^{16}C, which is due to the contribution from the two valence neutrons only. Recent life time measurements for the $^{18}\mathrm{C}(2_1^+)$ give similarly small $B(E2, 2^+ \rightarrow 0+)$ value [10], which also points out to the dominance of the two valence nucleon contribution to this transition and thus to a possible three-body nature of ^{18}C.

The aim of the present work is to study ^{18}C in a three-body $^{16}\mathrm{C} + n + n$ model. However, unlike what was done for ^{16}C, we have to take the excitation of the ^{16}C core into account since, according to knockout experiments [11], the internal structure of ^{17}C is dominated by the $^{16}\mathrm{C}(2^+) + n$ configuration. We first choose the $^{16}\mathrm{C} + n$ potential that fits the spectrum of ^{17}C and then we calculate the spectrum of ^{18}C. We predict the existence of a few states in the region of interest to supernova nucleosynthesis and we estimate the resonant capture rate due to these states.

In Sect. 2 , the three-body model with deformed core that is allowed to excite is presented. In Sect. 3 , the two-body system $^{16}\mathrm{C} + n$ interaction is studied. Sections 4 and 5 present results of three-body calculations obtained for bound and unbound ^{18}C states, respectively, and their relevance to r-process nucleosynthesis is discussed. Section 6 gives concluding remarks.

2 3-Body Model

We perform three-body calculations for the $^{16}\mathrm{C} + n + n$ system assuming that the ^{16}C core is deformed and allowed to excite to its first excited state, 2^+ at $E_x = 1.77$ MeV. These calculations are similar to those performed in Ref. [12] for the $^{12}\mathrm{Be}(^{10}\mathrm{Be} + n + n)$ system. The three-body Schrödinger equation is,

$$\left(T_{nn} + T_{core-nn} + h_{core}(\xi) + V_{core-n}(|\boldsymbol{r}_c - \boldsymbol{r}_{n_1}|) + V_{core-n}(|\boldsymbol{r}_c - \boldsymbol{r}_{n_2}|) + V_{nn}(|\boldsymbol{r}_{n_1} - \boldsymbol{r}_{n_2}|) - E\right) \Psi^T_{JM} = 0, \tag{1}$$

where $h_{\mathrm{core}}(\xi)$ is the intrinsic Hamiltonian of the core with eigenstates ϕ_I and energy eigenvalues ϵ_I. To solve this we expand the wave function Ψ^T_{JM} onto the hyperspherical harmonics (HH) defined in the T basis,

$$\Psi^T_{JM} = \rho^{-5/2} \sum_{K I l_x l_y L S j} \chi^{LSIJ_3J}_{Kl_xl_y}(\rho) \mathcal{Y}^{LSTIJ_3JM}_{Kl_xl_y}(\Omega_5), \tag{2}$$

where $\rho = \sqrt{x^2 + y^2}$ is the hyperradius, Ω_5 is a set of hyperangles $\{\hat{x}, \hat{y}, \alpha\}$, $\boldsymbol{x}$ and $\boldsymbol{y}$ are normalized Jacobi coordinates for the $n - n$ and $^{16}\mathrm{C} - 2n$ two-body systems shown in Fig. 1, $\alpha = \arctan x/y$, and the HHs are defined as

$$\mathcal{Y}^{LSTIJ_3JM}_{Kl_xl_y}(\Omega_5, \xi) = \varphi^{l_xl_y}_K(\alpha) \left[\left[\left[Y_{l_x}(\hat{x}) \otimes Y_{l_y}(\hat{y}) \right]_L \otimes X_S(1,2) \right]_{J_3} \otimes \phi_I(\xi) \right]_{JM} X_T(1,2). \tag{3}$$

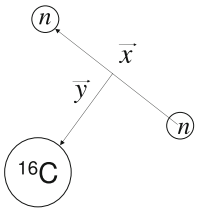

Fig. 1 Normalized Jacobi coordinates for $^{16}\mathrm{C} + n + n$ system

Here X_S and X_T are the spin and isospin functions for the two neutrons, Y_{l_x} and Y_{l_y} are spherical harmonics, K the is hypermoment, l_x and l_y are the orbital angular momentum of the $n-n$ and $^{16}\mathrm{C}-2n$ systems, respectively, I is the spin of the ^{16}C, L, J and M are the total orbital momentum, total angular momentum and the projection of the total angular momentum, respectively.

Usually, the hyperradial part $\chi_\gamma(\rho)$, where $\gamma \equiv \{l_x, l_y, L, S, J, I, J_3, K\}$, is found by solving the coupled set of differential equations

$$\left(-\frac{\hbar^2}{2m}\left[\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} + \frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2}\right] + V_{\gamma\gamma} - E + \epsilon_I\right)\chi_\gamma^J(\rho) = -\sum_{\gamma'\neq\gamma} V_{\gamma\gamma'}(\rho)\chi_{\gamma'}^J(\rho), \tag{4}$$

where $\mathcal{L} = K + 3/2$ and the hyperradial potential is defined as

$$V_{\gamma\gamma'}(\rho) = \left\langle \mathcal{Y}_{\gamma'}^{JM}(\Omega_5)\Big|\sum_{j>i}^{3} V_{ij}(\rho, \Omega_5)\Big|\mathcal{Y}_{\gamma}^{JM}(\Omega_5)\right\rangle. \tag{5}$$

However, when bound eigenstates of the $^{16}C - n$ potential are present as forbidden partial waves, the Schrödinger equation should be solved in a subspace of allowed states. We eliminate forbidden $0s_{1/2}$, $0p_{3/2}$ and $0p_{1/2}$ states in both the $^{16}\mathrm{C}(0^+) - n$ and $^{16}\mathrm{C}(2^+ - n)$ channels using the Pauli Projection technique. Details of this technique are given in Ref. [13]. We should note here that in ^{18}C the $1s_{1/2}$, $0d_{5/2}$ and $0d_{3/2}$ orbitals might be partially filled. At present, our three-body formalism cannot deal with such a situation. However, the effects of partial blocking and/or hindrance of the $0d_{5/2}$ orbital have been studied within the multichannel algebraic scattering theory in Ref. [14]. This theory does not reproduce the ^{17}C spectrum and its binding at the same time, the requirement crucial for application to the ^{18}C nucleus. On the other hand, it is well-known that there is no need to remove the Pauli forbidden states explicitly if supersymmetric potentials are used. They can be found either from a theoretical model or phenomenologically. Thus, fixing the $^{16}\mathrm{C} - n$ potential from the ^{17}C spectrum can already contain implicit information on how much the sd shell is filled. Another argument in favour of allowing all the sd states is given in Ref. [15] where it was pointed out that taking core degrees of freedom into account is a proper alternative to a more fundamental treatment.

Ideally, the summation over K in Eq. (2) should go up to infinity. In reality, this sum has to be truncated up to some $K_{\max}$. In the present paper, the model space has been truncated to $K_{\max} = 28$ for 0^+ states, but for excited states with non zero spin it has to be reduced to smaller $K_{\max}$ ranging from 14 to 18.

For the nn interaction we use the GPT potential [16], that includes central, spin–orbit and tensor interactions in all partial waves. As for the Core$-n$ interaction we use a deformed Wood–Saxon nuclear potential in the core's rest frame,

$$V_{\mathrm{core}-n}(r,\theta,\phi) = \frac{V}{1+\exp\left(\frac{r-R(\theta,\phi)}{a}\right)}, \quad R(\theta,\phi) = R(1+\beta Y_{20}(\theta,\phi)), \tag{6}$$

where $R = r_0 A^{1/3}$, a is diffuseness and β is fractional deformation. We also include a standard undeformed spin orbit term

$$V_{ls}(r) = -\left(\frac{\hbar}{m_\pi c}\right)^2 (2\vec{l}\cdot\vec{s})\frac{V_{\mathrm{so}}}{r}\frac{d}{dr}\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1} \tag{7}$$

as it has been shown that the level shifts due to deformation of the spin–orbit potential are negligible for nuclei lighter than the rare earths [17]. The undeformed spin–orbit potential has been successfully used in three-body calculations of ^{12}Be [12]. The parameters V, r_0, a and β are chosen to reproduce the experimental spectrum of ^{17}C and their choice is discussed in the next session.

It is known that to reproduce exactly the binding energy in three-body calculations a three-body force may be needed. In particular, without this force the ^{6}He binding energy in the $^4\text{He} + n + n$ model is only 0.4 MeV as compared to the experimental value of 0.97 MeV [18], which is mainly because of the t+t clustering in ^{6}He. The relatively small difference of 0.57 MeV is important in this particular case as it makes almost half the experimental binding energy. The latter determines the asymptotic behaviour of the ^{6}He wave function responsible for many ^{6}He observables and the introduction of the three-body force is crucial for correct understanding of the experimental data involving ^{6}He. In ^{18}C, the thresholds for cluster emissions lie much higher than in ^{6}He and, unlike in ^{6}He, any cluster partition includes weakly-bound fragments so that this particular kind of clustering does not contribute much. Instead, core excitations become more important, however, they can be included into the three-body calculations directly. Therefore, one may expect that the role of the three-body force for ^{18}C is less important. Indeed, for other three-body system, $^{10}\text{Be} + n + n$, a very reasonable binding energy of ^{12}Be is obtained without introduction of any three-body force [12].

3 Bound States Properties of ^{17}C and the $^{16}\text{C} - n$ Potential

The three experimental low-lying levels of ^{17}C are shown in Fig. 2. The ground state is $3/2^+$ with a neutron separation energy of 0.728 MeV. The other two bound excited states are $1/2^+$ and $5/2^+$, located at 0.210 and 0.330 MeV, respectively. The spin-parity assignments for these states have been established by analysing the γ-ray spectrum observed in $\text{p} + ^{17}\text{C}$ inelastic scattering [4], and by studying the one neutron removal reaction from ^{18}C [7].

The shell model calculations using the WBP interaction predict the ^{17}C ground state to be $J^\pi = 3/2^+$ [4], consistent with experiment. However, the shell model predicts the first excited state to be $5/2^+$ at $E_x = 0.032$ MeV while the second excited state should be $1/2^+$ at 0.295 MeV. Experimentally, $1/2^+$ is below $5/2^+$, as can be seen in Fig. 2.

The level ordering in ^{17}C can be understood on the basis of a simple Nilsson model. According to this model, in light nuclei the $0d_{5/2}$ level splits into three levels, $[220\frac{1}{2}]$, $[211\frac{3}{2}]$ and $[202\frac{5}{2}]$ [19]. For $\beta > 0$, typical

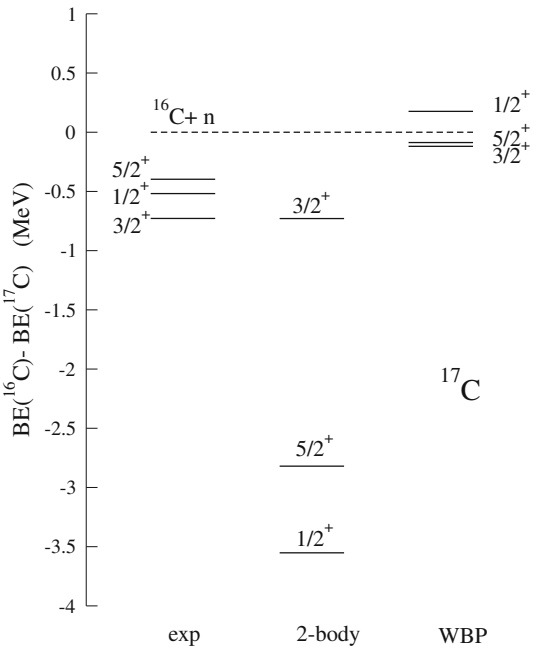

Fig. 2 Low lying experimental spectrum of ^{17}C plotted relatively to the $^{16}\text{C} + n$ threshold and compared to the shell model with WBP interaction from [4] and the two-body model calculations with a $^{16}\text{C} - n$ potential from Ridikas et al. [15]

Table 1 The parameter sets A and B for the $n-^{16}$C potential that fits the ^{17}C spectrum in a deformed two-body model with core excitations, corresponding to two deformations β, and the binding (B.E.) and excitation (E_x) energies of the 0^+_1, $2^+_{1,2}$ and 4^+_1 states in ^{18}C obtained with these sets

	β	$V_{l=0}$	$V^{\text{even}}_{l\neq 0}$	V_{so}	V_{odd}	B.E.(0^+_1)	$E_x(2^+_1)$	$E_x(2^+_2)$	$E_x(4^+_1)$
A									
$A1$	0.55	−30.95	−44.05	−0.58	−44.05	−5.18	3.07	4.23	1.75
$A2$	0.55	−30.95	−44.05	−0.58	0.0	−5.22	1.12	3.89	2.23
B									
$B1$	0.48	−32.15	−44.72	−0.56	−32.00	−4.92	1.77	3.92	1.62
$B2$	0.48	−32.15	−44.72	−0.56	0.0	−5.26	1.11	4.07	2.18

The $V_{l=0}$ and $V^{\text{even}}_{l\neq 0}$ are the depths of the central potentials in $l=0$ and $l\neq 0$ even partial waves, V_{ls} is the depth of l-independent spin–orbit potential. Two values of the central potentials in odd partial waves, V_{odd}, has been used in three-body calculations for each set A and B. The radius and diffuseness for all potentials are fixed as $r_0 = 1.25$ fm and $a = 0.65$ fm. All potential depths and binding energies are in MeV

for the carbon isotopes, the lowest level is $[220\frac{1}{2}]$ followed by $[211\frac{3}{2}]$ and for $\beta > 0.25$ the $[202\frac{5}{2}]$ level rises above $[211\frac{1}{2}]$. The first eight neutrons from ^{17}C fill $0s$ and $0p$ shells. The two out of three remaining neutrons fill the $[220\frac{1}{2}]$ shell. Therefore, the low-lying spectrum of ^{17}C is determined by the single-particle spectrum of the Nilsson model above this shell, which, for typical values of $\beta \sim 0.5$, is $3/2^+$, $1/2^+$ and $5/2^+$.

The successful interpretation of the ^{17}C spectrum on the basis of the Nilsson model, combined with its small separation energy, suggests that ^{17}C could be described by a deformed two-body model. However, the Coulomb breakup [20] and the single-neutron knockout [11] experiments imply that ^{17}C is built mostly on the ^{16}C(2^+) state. Therefore, to understand ^{17}C, the 2^+ excitation of the ^{16}C core should be taken into account.

^{17}C has already been studied in a two-body deformed model with core excitations by Ridikas et al. [15]. At that time, the spin-parity of the ^{17}C ground state was unknown. Therefore, Ridikas et al. have found different sets of the $^{16}\text{C}-n$ interaction, that binds ^{17}C by 0.728 MeV, for each J^π, $1/2^+$, $3/2^+$ and $5/2^+$, separately. In particular, for the $3/2^+$ state to be bound by the experimental value of 0.728 MeV, the set of parameters $V = -54.101$ MeV, $V_{\text{so}} = 6.5$ MeV, $r_0 = 1.083$ fm, $a = 0.65$ fm and $\beta = 0.55$ should be used. However, our calculations of ^{17}C performed with this set of parameters give $1/2^+$ as a ground state at a much lower energy (see Fig. 2). Therefore, we cannot use this potential in three-body $^{16}\text{C}+n+n$ calculations because we would get a strongly overbound ^{18}C.

In this work, we find the $^{16}\text{C}-n$ interaction that exactly reproduces the ^{17}C spectrum. We use the same model as in Ridikas et al., which is a coupled channel problem with a deformed $^{16}\text{C}-n$ potential and non-diagonal interaction described by the rotational model. We use the deformed Wood–Saxon potential and the non-deformed spin–orbit potential given by Eqs. (6) and (7). For r_0 we use the standard value of 1.25 fm, which is widely accepted by various two-body models and which is larger than the value of 1.083 fm used by Ridikas et al., and we use the same diffusseness $a = 0.65$ fm. Both r_0 and a are fixed in these calculations.

In our calculations we have used two values for the deformation parameter β. The first one, $\beta = 0.55$, calculated by Ridikas et al. from the experimental transition probability $B(E2: 0^+_{gs} \rightarrow 2^+_1)$, available at the time, and used in Ref. [21] to study effects of dynamical core deformation on single nucleon knockout reactions. The second one, $\beta = 0.48$ has been calculated by us from the experimental matter deformation length $\delta_m = \beta_m R_m = 1.3$ fm obtained in Ref. [22] from the Coulomb-nuclear interference in the $^{208}\text{Pb}+^{16}$C inelastic scattering. To get this value, we used the ^{16}C matter radius R_m equal to 2.82 fm. This value was used in Ref. [23] to deduce the r.m.s. radii of the carbon isotopes.

To reproduce the ^{17}C spectrum, we vary the depths of the Woods–Saxon and the spin–orbit potentials. There are two important features of the potential we have found. To make the $3/2^+$ state lower than $5/2^+$ we had to use a negative spin–orbit strength, V_{so}, in Eq. (7), which is a non-standard choice, since it gives a repulsive spin–orbit interaction, and to reproduce the narrow splitting between all three states we had to use an l-dependent $^{16}\text{C}-n$ potential, which is stronger in d-wave than in s-wave. We have found one set of parameters for each β: the set A for $\beta = 0.55$ and the set B for $\beta = 0.48$, both presented in Table 1. These potentials are similar to each other, so we use only one of them (namely, the set B) in calculations of the ^{18}C spectrum.

Although our two-body model reproduced very well the ^{17}C spectrum, it cannot explain the probability of the core excitations observed in the Coulomb breakup [20] and one-neutron knockout experiments [11]. In our model, 69% of the ^{17}C wave function is built on the ground state of the ^{16}C core, while only 31% is built on

Table 2 The probabilities P (in per cents) of the $^{16}\mathrm{C}(J^\pi) \otimes nlj$ configuration and their r.m.s. radii $\langle r^2\rangle^{1/2}$ (in fm) for bound states in $^{17}\mathrm{C}$

	$^{16}\mathrm{C}(0^+) \otimes s_{1/2}$		$^{16}\mathrm{C}(0^+) \otimes d_{3/2}$		$^{16}\mathrm{C}(0^+) \otimes d_{5/2}$		$^{16}\mathrm{C}(2^+) \otimes s_{1/2}$		$^{16}\mathrm{C}(2^+) \otimes d_{3/2}$		$^{16}\mathrm{C}(2^+) \otimes d_{5/2}$	
	P	$\langle r^2\rangle^{1/2}$	P	$\langle r^2\rangle^{1/2}$	P	$\langle r^2\rangle^{1/2}$	P	$\langle r^2\rangle^{1/2}$	P	$\langle r^2\rangle^{1/2}$	P	$\langle r^2\rangle^{1/2}$
$^{17}\mathrm{C}(\frac{3}{2}^+)$			69	4.34			8	3.28	16	4.00	5	3.99
$^{17}\mathrm{C}(\frac{1}{2}^+)$	44	6.37							28	4.10	28	4.14
$^{17}\mathrm{C}(\frac{5}{2}^+)$					68	4.53	9	3.57	5	4.05	17	4.07

the 2^+ excitation of $^{16}\mathrm{C}$. Experimentally, the probabilities of $^{16}\mathrm{C}(^+)$ and $^{16}\mathrm{C}(2^+)$ are $19 \pm 9\%$ and $58 \pm 8\%$, respectively. Nevertheless, we have to use potentials A or B in three-body calculations since they give correct binding for the $^{16}\mathrm{C} + n$ two-body subsystem.

The decomposition of the $^{17}\mathrm{C}$ wave function for each bound state into the $^{16}\mathrm{C}(J^\pi) \otimes nlj$ configurations is shown in Table 2 in more details. One can see that the $3/2^+$ and $5/2^+$ states have similar probabilities of the core excitations while the $1/2^+$ state is very different. The r.m.s. radius of the $^{16}\mathrm{C}(0^+) \otimes s_{1/2}$, shown in Table 2 as well, is typical for halo states. However, the probability of this configuration in $^{17}\mathrm{C}$ is only 44%.

4 ^{18}C Bound States

The $^{16}\mathrm{C} - n$ interaction found in previous section is, strictly speaking, defined only for even partial waves as all the bound state of $^{17}\mathrm{C}$ have positive parity. However, in the three-body system $^{16}\mathrm{C} + n + n$, odd partial waves will be present in the $^{16}\mathrm{C} + n$ subsystem too. There are no experimental data which would allow us to fix the V_{odd} interaction in such subsystems. There are three strategies to deal with such a situation: (i) to use $V_{\mathrm{odd}} = 0$. This makes sense as the odd parity states in known neutron-rich carbon isotopes lie relatively high; (ii) To put V_{odd} equal to the V_{even} interaction in one of even partial waves and (iii) to find V_{odd} by fitting the ground state binding energy of $^{18}\mathrm{C}$. In this paper, we try all three options.

For $V_{\mathrm{odd}} \neq 0$, bound states may be present in the $0p$-wave. Such states should be forbidden as the $0p$ neutron shell is occupied. Therefore, we eliminate these states, as well as the $0s$ states, using the Pauli projection technique when $V_{\mathrm{odd}} \neq 0$ is used. We reference the potential sets A and B for $V_{\mathrm{odd}} \neq 0$ as $A1$ and $B1$, while for $V_{\mathrm{odd}} = 0$ we use notations $A2$ and $B2$.

We have investigated convergence of the three-body solutions for the $^{18}\mathrm{C}$ binding energy. For 0^+ states, we were able to go up to $K_{\max} = 28$, for which the ground state energy is practically converged. The other two excites 0^+ states are above the $^{17}\mathrm{C} + n$ threshold. The convergence in their energy is similar to that in ground state, however, one should keep in mind that no convergence could be achieved for states above the two-body threshold since such states are resonances whose energies are defined within their widths. For non-zero spins, computer memory limitations allowed us to go only up to $K_{\max} = 14$ for 4^+ state, $K_{\max} = 16$ for 1^+ and 2^+, and $K_{\max} = 18$ for 1^-. Although for these $K_{\max}$ the eigenenergies have not completely converged, their excitation energies are more stable with respect to increase of $K_{\max}$. Therefore, the all the excitation energies shown below are calculated with respect to the ground state energy at $K_{\max} = 14$.

The $^{18}\mathrm{C}$ ground state binding energy calculated with $V_{\mathrm{odd}} = 0$ is practically the same (see Table 1) for potentials $A2$ and $B2$. Including V_{odd} does not change it to much. Thus, for potential $A1$, where V_{odd} is the same as the V_{even} interaction in $l \neq 0$ it changes only by 33 keV. Such a small change in binding energy despite a huge change in odd potential results from a small probability of the odd partial waves in $^{18}\mathrm{C}$. This probability is approximately equal to 4 and 2% for the $A1$ and $A2$ potentials, respectively. Similar contributions from odd partial waves are obtained for the $B1$ and $B2$ potentials. It should be noticed that all four potentials gave the binding energy of the ground state within 300 keV of the experimental value of -4.908 MeV without any introduction of the three-body force.

The energy of the first excited state 2^+ is more sensitive to the choice of the odd interactions. When V_{odd} is fitted to reproduce the $^{18}\mathrm{C}$ ground state binding energy (potential set $B1$), the excitation energy of the 2^+ is above the experimental one by only 100 keV. This looks like a very good result but for the same potential the first 4^+ level unexpectedly decreases just below 2^+ (see Table 1), which contradicts our knowledge about nuclear structure. It is very difficult to know whether this is a consequence of the two-body potential model chosen, or is a result from unnoticed numerical problems arising in large scale three-body calculations. For

Table 3 The probabilities P (in per cents) of the $^{17}C(J^\pi)\otimes nlj$ configuration for bound states in ^{18}C calculated with $B2$ potential

^{18}C	$^{17}C(3/2^+)\otimes$			$^{17}C(1/2^+)\otimes$			$^{17}C(5/2^+)\otimes$		
	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$
0^+_1		38		11					42
2^+_1	6	12	5		10	16	9	5	14
2^+_2	0	3	22		0	0	0	45	21
4^+_1		32						32	12
1^+	0	0	42	0	0			41	0

$V_{\rm odd} \neq 0$, the model space is much larger than that for $V_{\rm odd} = 0$ because it includes also the model space of $0p$ forbidden states to which the total three-body wave function should be orthogonal. For 4^+, the number of channels becomes so large that it becomes physically impossible to verify the sensitivity of the three-body solutions to all the parameters that control their quality. We use the same sets of such control parameters both for $V_{\rm odd} \neq 0$ and $V_{\rm odd} = 0$, where no problems were encountered. However, this may not guarantee us from numerical problems in larger model spaces. On the other hand, there may be also a physical reason for abnormally low 4^+ state with the interaction chosen. At least, we have found out that the reason for low position of the 4^+ state obtained with the $B2$ potential was the l-dependence of the $^{16}\mathrm{C}-n$ potential. When this l-dependence is removed, the split between the 0^+, 2^+ and 4^+ states looks more traditional but the ^{18}C three-body ground state becomes overbound. Therefore, below we mainly discuss the results obtained with $B2$ potential for which the ordering of 2^+ and 4^+ states is correct.

For $B2$, the calculated ^{18}C binding energy of the ground state, $E = -5.26$ MeV is only about 350 keV larger than the experimental value of -4.92 MeV. The r.m.s. radius has converged to 2.932 fm. The distance between the two valence neutrons is $R_{nn} = 4.64$ fm and $^{16}\mathrm{C}-n$ distance is $R_{n-^{16}C} = 3.85$ fm. The ^{18}C wave function is built on the ground state of the ^{16}C core rather than on its first excited state, with $P(^{16}\mathrm{C}(0^+)) = 62\%$ and $P(^{16}\mathrm{C}(2^+)) = 28\%$. The composition of the ground state in terms of the $^{17}\mathrm{C}(J^\pi)\otimes nlj$ configurations in shown is Table 3.

The three-body 2^+_1 state is lower that the experimental one by 472 keV. The life time of this state has been recently measured and the $B(E2; 2^+ \to 0^+)$ transition probability has been determined in a lifetime measurement experiment [6] to be $4.3 \pm 0.2 \pm 1.0$ e^{2}fm^4. The calculated three-body value of $B(E2 : 2^+ \to 0^+)$ for ^{18}C is 9.47 e^{2}fm^4. This value has been calculated at $K_{\rm max} = 12$ and it differs from that obtained at $K_{\rm max} = 10$ by $\sim 1\%$. The three-body $B(E2)$ probability is about twice as large as the experimental value. Such an overestimation can result from neglecting interactions in odd partial waves. Indeed, using the potential set $B1$ instead of $B2$ we get 7.79 e^{2}fm^4, which is closer to experiment.

Apart from the 2^+_1 and 4^+ states discussed above, the $B2$ potential predicts another three states below the $^{17}\mathrm{C}+n$ threshold, which are 1^+, 2^+_2 and 2^+_3. The 1^+ state, predicted at 3.68 MeV, is reasonably close the state at 4 MeV observed in Refs. [7,5]. The 1^+ state could not be seen in the recent neutron knockout experiment with the ^{19}C beam because the cross section for population of this state should be very small [7]. On the other hand, the shell model calculations from [2] do not predict any 1^+ states below the $^{16}\mathrm{C}+2n$ threshold, so that the low position of the three-body 1^+ state could have the same origin as the abnormal decrease of the 4^+ state, namely, the l-dependence of the $^{16}\mathrm{C}+n$ interaction. As well as the 4^+ state, the 1^+ state is built on the d^2 configuration (see Table 2), where the interaction is stronger. Including non-zero $V_{\rm odd}$ (using potential $B1$) pushes 1^+ into the region between the $^{17}\mathrm{C}+n$ and $^{16}\mathrm{C}+2n$ thresholds. Interestingly, the modification of the shell model interaction made in Ref. [5] brings the shell model 1^+ state into the vicinity of the $^{17}\mathrm{C}+n$ threshold too.

The last two bound states, predicted with $B2$, are the two 2^+ states at 3.85 and 4.07 MeV. Both of them are close to the observed 4 MeV state with the tentative $(2^+, 3^+)$ parity assignment. For $B1$, the 2^+_2 state almost does not change its position, slightly shifting to 3.92 MeV, while the 2^+_3 state becomes unbound. Although the energy of the 2^+_3 is not converged, its decrease with $K_{\rm max}$ suggests that 2^+_3 becomes a two-body $^{17}\mathrm{C}+n$ resonance. Both calculations support the 2^+ interpretation of the observed 4 MeV state. Such an interpretation is also consistent with the modified shell model calculations from Ref. [5]. However, this interpretation is not consistent with large cross sections for the neutron removal reaction of ^{19}C [7]. This contradiction could be resolved if two states, 2^+ and 3^+, existed in ^{18}C very close to each other near $E_x = 4$ MeV.

Finally, the three-body model cannot offer an explanation for the observed state at 2.5 MeV. Although it predicts a 4^+ state at 2.18 MeV, our current knowledge about the split between the first 0^+, 2^+ and 4^+ levels suggest that the lowering of the 4^+ state is more likely a by-product of the chosen two-body interaction. On the other hand, if a 4^+ excited state existed very close to 2.5 MeV next to either a hypothetical 0^+ or 2^+ state, then its deexcitation by γ-ray emission would be almost indistinguishable from that from the 0^+ or 2^+ states. A hypothetical 4^+ state near 2.5 MeV would not have been seen in neutron knockout from ^{19}C either as it would require an $l = 4$ momentum transfer, which should be suppressed for dynamical reasons.

5 Unbound States Above the $^{17}C + n$ Threshold and their Contribution to the $^{17}C(n, \gamma)^{18}C$ Reaction Rate

The three-body model predicts five states above the $^{17}C + n$ but below the $^{16}C + 2n$ threshold: two 0^+ states along with a 2^+, 3^+, and 1^-. They should be seen as resonances in the $^{17}C+n$ two-body continuum. However, we predicted these states by solving a bound state problem and therefore do not know their widths. We estimate these widths in a single channel two-body $^{17}C + n$ model with a standard spherical Woods–Saxon potential ($r_0 = 1.25$ fm, $a = 0.65$ fm) whose depths is fitted to reproduce the calculated positions of theoretical resonances. The resonance energy of a state was assumed to be the difference between its three-body energy and the ^{17}C two-body energy. The single-particle widths obtained from potential phase shifts were multiplied then by the probabilities of the corresponding $^{17}C(3/2^+) \otimes nlj$ configurations. The 0^+_2, 0^+_3 and 3^+ states are pure d-wave resonances, and their estimated widths are 0.04, 0.4 and 2 keV, respectively. The p-wave resonance 1^- has an estimated width of 150 keV. The 2^+_4 state has s-wave and d-wave components built on $^{17}C(3/2^+)$, the probability of the d-wave channel being about three times larger than that in the s-wave channel. The partial width in the d-wave channel is easily estimated in a similar way to be ∼4keV. However, since there is no centrifugal barrier in the s-wave channel, we cannot provide any estimate for its width. It is questionable whether such a state could exist at all as the absence of any barriers could spread it over all the continuum. It is possible that it can exist as a virtual state, similar to ^{10}Li or ^{9}He, however, at present we cannot make any predictions for its scattering length.

The resonance energies of 0^+_3, 3^+ and 2^+_4 states, 0.211, 0.277 and 0.290 MeV, respectively, are very close to those that corresponding to supernovae temperatures. We have calculated the contribution from these states to the resonant $^{17}C(n, \gamma)^{18}C$ reaction rate $N_A\langle\sigma v\rangle_R$. For an isolated narrow resonance i this rate is [2]

$$N_A\langle\sigma v\rangle_R = 1.54 \times 10^5 \mu^{-3/2} T^{-3/2} \sum_i (\omega\gamma)_i \times \exp(-11.605 E_i/T_9) \quad \mathrm{cm^3 mole^{-1} s^{-1}}, \tag{8}$$

where E_i is the resonance energy in MeV, T_9 is the temperature in $10^9 K$, μ is the reduced mass $m_n m_{^{17}C}/(m_n + m_{^{17}C})$, where m_n is the neutron mass and $m_{^{17}C}$ is the mass of ^{17}C, and $\omega\gamma$ is the resonance strength (in eV) given by:

$$\omega\gamma = \frac{2J+1}{2(2j_t+1)} \frac{\Gamma_n \Gamma_\gamma}{\Gamma_{\mathrm{tot}}}. \tag{9}$$

Here J and j_t are the spins of resonance and the target nucleus ^{17}C, respectively, Γ_n and Γ_γ are the partial widths of the entrance and the exit channels, respectively, and the total width Γ_{tot} is the sum over the partial widths of all channels. The partial widths Γ_γ are calculated from the electromagnetic reduced transition probabilities $B(J_i \to J_f; L)$ which is calculated in the same three-body model. Two types of electromagnetic transitions are possible for the 0^+_3, 3^+ and 2^+_4 resonances, $E2$ and $M1$, for which the Γ_γ partial widths are given by expressions [2]

$$\Gamma_{E2}[\mathrm{eV}] = 8.13 \times 10^{-7} E_\gamma^5[\mathrm{MeV}]\, B(E2)[\mathrm{e^2 fm^4}], \tag{10}$$

$$\Gamma_{M1}[\mathrm{eV}] = 1.16 \times 10^{-2} E_\gamma^3[\mathrm{MeV}]\, B(M1)[\mu_N^2]. \tag{11}$$

To calculate the electromagnetic transitions $E2$ and $M1$ in the three-body model, two terms in the transition operator are required: one that acts on the two valence neutrons and the other treating the collective excitation of the ^{16}C core. No effective charges were applied for neutrons in this work. Effective charges arise normally when the electromagnetic transitions have strong contributions from the model spaces absent in the

Table 4 Reduced transition probabilities $B(J_i \to J_f; \sigma\lambda)$ (in $e^2fm^{2\lambda}$ for $\sigma\lambda = E\lambda$ and in μ_N^2 for $\sigma\lambda = M1$), the corresponding partial widths Γ_γ and resonance strength $\omega\gamma$ (both in eV)

J_i^π	$E_{\rm res}$	J_f^π	E_γ	$\sigma\lambda$	$B(\sigma\lambda)$	Γ_γ	$\omega\gamma$
0_2^+	0.047	2_1^+	3.46	$E2$	0.30	1.24×10^{-4}	1.55×10^{-5}
0_3^+	0.21	2_1^+	3.63	$E2$	0.54	2.82×10^{-4}	3.53×10^{-5}
		1^+	1.07	$M1$	3.51	4.99×10^{-2}	6.23×10^{-3}
3_1^+	0.28	2_1^+	3.71	$E2$	0.18	1.01×10^{-4}	8.84×10^{-5}
				$M1$	0.012	9.80×10^{-3}	8.58×10^{-3}
		2_2^+	0.96	$E2$	2.50	1.69×10^{-6}	1.48×10^{-6}
		2_3^+	0.75	$M1$	0.023	1.10×10^{-4}	9.68×10^{-5}
		1_1^+	1.14	$E2$	6.11	9.44×10^{-6}	8.26×10^{-6}
		4_1^+	2.64	$E2$	0.24	2.56×10^{-5}	2.24×10^{-5}
				$M1$	0.055	1.2×10^{-2}	1.05×10^{-2}
2_4^+	0.29	2_1^+	3.72	$E2$	0.30	1.71×10^{-4}	1.07×10^{-4}
				$M1$	2.2×10^{-4}	1.31×10^{-4}	8.19×10^{-5}
		2_2^+	0.98	$E2$	2.33	1.69×10^{-6}	1.06×10^{-6}
		1_1^+	1.15	$E2$	2.28	3.73×10^{-6}	2.33×10^{-6}
		0_1^+	4.83	$E2$	0.033	7.14×10^{-5}	4.46×10^{-5}
		4_1^+	2.66	$E2$	0.46	4.93×10^{-5}	3.08×10^{-5}
1_1^-	0.61	2_1^+	4.03	$E1$	0.21	14.45	5.42
		0_1^+	5.15	$E1$	0.018	2.53	0.95

The resonance energies $E_{\rm res}$ and the γ-ray energies E_γ are in MeV. Transitions with very small contributions are not included in the table

model Hamiltonians. This is the case in conventional shell model which does not account for clustering and correct asymptotic behaviour outside the range of the nucleon–nucleus interaction. Microscopic cluster models include them explicitly and do not use effective charges. Whether effective charges are needed in few-body models is still unclear. The three-body calculations of ^{16}C have shown that effective charges that reproduce the $B(E2)$ value for ^{15}C lead to overestimated $B(E2; 2^+ \to 0^+)$ value for ^{16}C [8]. In the case of ^{18}C, the model space is much larger than was used for ^{16}C in [8], so the introduction of effective charges appears to be less necessary. The contributions that come from effective charges can be compensated by various other effects like, for example, two-body potentials in odd partial waves or contribution from other excited states of the core.

The $E2$ transition involves two matrix elements $\langle {}^{16}\mathrm{C}(0^+)|E2|{}^{16}\mathrm{C}(2^+)\rangle$, responsible for the core excitation, and $\langle {}^{16}\mathrm{C}(2^+)|E2|{}^{16}\mathrm{C}(2^+)\rangle$, which induces reorientation and is related to the quadrupole moment of ^{16}C. Both of these terms were taken to be consistent with the experimental deformation length of $\delta_m = \beta_m R_m = 1.3$ fm used to calculate the three-body ^{18}C wave functions. As for the $M1$ transition, we require the matrix element $\langle {}^{16}\mathrm{C}(2^+)|M1|{}^{16}\mathrm{C}(2^+)\rangle$. We calculate it in the *spsdpf* shell model with the WBP interaction [3] using the code *NuShell@MSU* [24].

The calculated Γ_{E2} are presented in Table 4. All of them are much smaller than neutron widths, which means that the resonance strength $\omega\gamma$ depends only on Γ_γ:

$$\omega\gamma \approx \frac{2J+1}{2(2j_t+1)}\Gamma_\gamma. \tag{12}$$

The resonant $E2$ reaction rates for 0_3^+, 3^+ and 2_4^+ are presented in Fig. 4, where they are compared to the non-resonant reaction rate,

$$N_A\langle\sigma v\rangle_{NR} = 3417.69\, T_9 - 358.029\, T_9^{1.66}\,\mathrm{cm^3 mole^{-1} s^{-1}}, \tag{13}$$

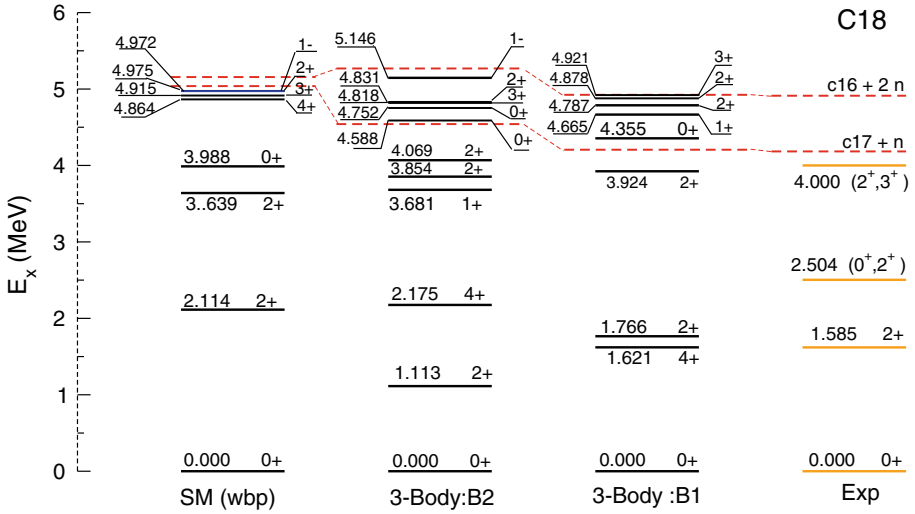

Fig. 3 The three-body ^{18}C spectrum obtained with potentials $B2$ and $B1$ in comparison to shell model spectrum from Ref. [4] and the latest experimental spectrum from Ref. [7]

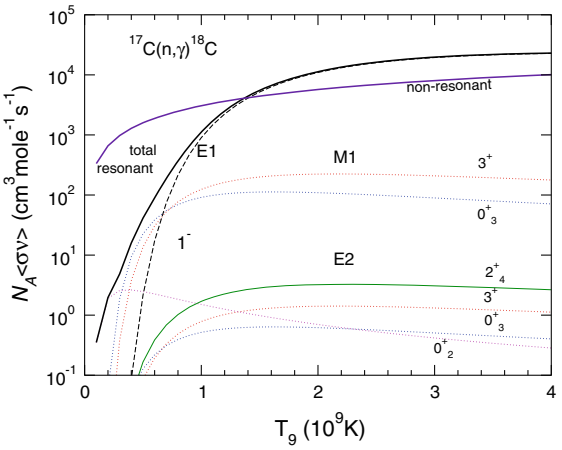

Fig. 4 The resonant $E2$ (*dot curves*), $M1$ (*dot-dashed curves*) and $E1$ (*dashed curves*), contributions to the ^{17}C $(n, \gamma)^{18}$C reaction rate, obtained in this work for individual resonant states J^π, in comparison with non-resonant reaction rate from [2] (*violet solid line*). The total resonant neutron capture is shown by a *black solid line*

obtained by Herndl et al. earlier in Ref. [2]. The resonant rates are about three order of magnitude smaller. This happens because the non-resonant $E1$ capture proceeds via the p-wave entrance channel where the centrifugal barrier is smaller.

In calculating $B(M1)$, we encoutered two types of situations. For some transitions, this value is large for small $K_{\max}$ but decreases with increasing model space, and this decrease becomes faster with increasing $K_{\max}$. The $B(M1)$ in such situations is not converged so we cannot estimate the corresponding $M1$ reaction rate. For only five transitions, $2^+_4 \to 2^+_1$, $0^+_3 \to 1^+$, $3^+ \to 2^+_1$, $3^+ \to 2^+_3$ and $3^+ \to 4^+$, have we obtained converged $B(M1)$ values, which are shown in Table 4. The corresponding $M1$ contributions to the resonant reaction rate, arising from 0^+_3 and 3^+ resonances, are shown in Fig. 3. They are two orders of magnitude larger than the corresponding $E2$ reaction rates, but still smaller than the non-resonant rate from Ref. [2]. The $M1$ contribution from 2^+_4 is about an order of magnitude smaller than the $E2$ one for the same state and it is not shown.

The three-body model predicts existence of the 1^- state around 600 keV, which is close to the energies relevant for supernovae explosions. We have calculated the resonant $E1$ contribution to ^{17}C$(n, \gamma)^{18}$C reaction rate due to this state. The width Γ_γ for this case is [2]

$$\Gamma_{E1}[\text{eV}] = 1.06 E^3_\gamma[\text{MeV}]\, B(E1)[\text{e}^2\text{fm}^2]. \tag{14}$$

To calculate the $E1$ transition, only the part of the $E1$ operator is required that acts on the valence neutrons in ^{18}C because no core states of negative parity are present. The calculated $B(E1)$ transition probabilities and Γ_{E1} are given in Table 4. The resonant $E1$ capture rate, shown in Fig. 4, dominates the non-resonant capture calculated by Herndl et al. at $T_9 \geq 1.2$. We have to mention, however, that the $\omega\gamma \sim 10$ eV obtained in our three-body calculations is much larger than the shell model value of 0.022 eV from Ref. [2]. If the shell model value is used in our calculations, the resonant reaction rate would still be lower than the non-resonant rate.

The resonant rates discussed above were obtained using the $B2$ potential set where interaction in odd partial waves is absent. Including V_{odd} (by using potential $B1$) pushes the 1^-, 3^+, 0^+_3 and 2^+_4 states above the $^{16}\text{C}+2n$ threshold. The 0^+_2 state will take place of the 0^+_3 state and the 1^+ state becomes unbound with respect to neutron emission. The 2^+_3 state will be just under the $^{16}\text{C}+2n$, which is a bit far from the astrophysical relevant energies. In this case, the only important contribution to the resonant reaction rate may come from the $M1$ transitions from 1^+. However, we do not calculate it here.

6 Summary and Conclusions

We have studied the ^{18}C spectrum within a three-body deformed $^{16}\text{C}+n+n$ model with 2^+ excitation of the ^{16}C core. This study reveals that the bound state energy of ^{18}C is compatible with the three-body structure of this nucleus. Our model gives the energy of the first excited 2^+ state within 0.4 MeV of the experimentally observed one and suggest that there should exist another 2^+ state around 4 MeV, which is compatible with recent experimental studies. However, our model gives low positions of the first 4^+ and 1^+ levels, which could be a drawback of the $^{17}\text{C}+n$ interaction used. Although this interaction reproduces exactly the observed spectrum of ^{17}C, its composition in terms of probabilities of the core states is not reproduced. Including more excitation, for example, by adding the $^{16}\text{C}(4^+)$ into the coupled channel problem, could improve this situation, however it would increase the number of channels in the three-body problem to such extent that it would make such calculations impossible.

Our calculations suggest that there may be resonances in the region between the $^{17}\text{C}+n$ and $^{16}\text{C}+2n$ thresholds, the most important of which is 1^-. Interestingly, the recent analysis of the two-neutron knockout from ^{19}C [25] suggests that the important part of the removal cross section comes from population of the intermediate 1^- state in ^{18}C. If this resonance has a three-body structure and a large $E1$ strength associated with it, then it will give a large contribution to the $^{17}\text{C}(n,\gamma)^{18}\text{C}$ capture rate. The increased reaction rate may significantly influence the abundances of actinides synthesized in the r-process and, therefore, a search for this resonance is an important and timely task.

The three-body calculations suggest that a 2^+ state may be present in astrophysically relevant region between the $^{16}\text{C}+2n$ and $^{17}\text{C}+n$. This state should have a significant s-wave component in which the valence neutron does not see any centrifugal barrier. Such a state may manifest itself as a virtual state in the $^{17}\text{C}+n$ continuum. To calculate the contribution from this state correctly, a scattering three-body problem including a binary channel should be solved. This problem should also include core excitations and Pauli projection of forbidden states, which is too complicated at the moment. However, it is very important to perform such calculations properly because the presence of a virtual s-wave states could lead to an enhanced neutron $M1$ capture rate. Predictions for strong $M1$ enhancement have been recently made for a similar reaction, $^{17}\text{F(p},\gamma)^{18}\text{Ne}$, in the Gamow shell model [26].

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