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Review

Theoretical and experimental study of the light deflection by a frequency modulated ultrasonic wave

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ABSTRACT

A formula that describes angular excursion variation of an acousto-optical deflector is theoretically demonstrated and experimentally confirmed. This deflector is obtained using a laser beam interaction with a frequency modulated ultrasonic sinusoidal wave in a liquid medium. The obtained results show that each diffracted order position varies sinusoidally around its central position, in the same rhythm as the modulating signal. Moreover, the scanning frequency of the diffraction order increases linearly according to the modulating signal frequency. Furthermore, the increase in the frequency excursion leads to the increase of the angular excursion. All the theoretical results are confirmed experimentally. Finally, the frequency modulation index has been easily obtained with good precision using experimental measurements of the diffracted order angular excursion.

Contents

1. Introduction

Acousto-optic devices are based on photoelastic or elastooptic effect according to which an ultrasonic wave applied to an elastic medium, produces a strain which changes its refractive index. The light interaction with this medium provokes the diffraction phenomenon [1]. The obtained diffraction orders depend on the shape, the amplitude and the frequency of this ultrasonic wave. When this last is sinusoidal, the intensity and the position of the diffracted orders are constant, this diffraction has been explained by Raman and Nath and many other authors [1,2]. In case where sinusoidal ultrasonic wave is amplitude modulated (AM), the diffraction orders position remains constant, it was also observed that besides these diffracted orders, the spectrum showed satellite diffracted orders. This diffraction was performed for the first time by Pancholy and Parthasarathy and explained mathematically by Mertens and Hereman [3,4].

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In the case where ultrasonic wave is frequency modulated (FM) by a periodic signal, the position of the diffracted orders varies according to this modulating signal as presented by some authors who used these techniques for making deflectors and scanners for different scientific fields for making deflectors and scanners for different scientific fields [5–8]. To our knowledge, a rigorous theoretical development of light diffraction by a frequency modulated signal has not been done before. In this work we propose, analyze and experimentally demonstrate this phenomenon, starting from the diffraction relation [9,10], to finally reach a very important relationship between the diffraction orders positions and the modulating signal. Experiments on the scanning frequency and the angular excursion variation of the diffraction orders, as a function of the modulation frequency, were performed to confirm the obtained relationship.

In addition, we demonstrated that it is possible to obtain experimentally the frequency modulation index by measuring the angular excursion of the diffracted order. This operation was performed before using only a spectrum analyzer [11].

2. Theoretical development

Let $A(t)$ be an electrical frequency modulated signal, which its instantaneous frequency is modified according to a linear law by a modulating signal $S(t)$ [11]:

$$A(t) = A_a \cdot \cos\left[\omega_a t + C\int S(t)dt\right] \tag{1}$$

where A_a and ω_a stand for the amplitude and the pulsation of the carrier wave respectively and C represents the sensitivity of the modulator.

If $S(t)$ is cosinusoidal:

$$S(t) = A_m \cdot \cos(\omega_m t) \tag{2}$$

where A_m and $\omega_m = 2\pi f_m$ are the amplitude and the pulsation of the modulating signal.

Eq. (1) becomes:

$$A(t) = A_a \cdot \cos\left[\omega_a t + \frac{\Delta f}{f_m}\sin(\omega_m t)\right] \tag{3}$$

where $\Delta f = A_m \cdot C$, represents the frequency excursion. The instantaneous frequency $f(t)$ of the signal $A(t)$ can be written as follows:

$$f(t) = f_a + \Delta f\cos(\omega_m t) \tag{4}$$

This electrical signal $A(t)$ is converted to a longitudinal acoustic wave via piezoelectric effect [12]. The output acoustic power delivered by the transducer depends on the mismatch between the outer acoustic impedance of the transducer and the liquid acoustic impedance [13]. When this acoustic wave propagates in a liquid medium it gives rise to density variations $\Delta\rho/\rho_o$ (called condensation) brought about by hydrostatic pressure [14]. In water, the elasto-optical tensor is reduced to a constant value equals 0.31 [15]. The traveling acoustic wave sets up a spatio-temporel modulation of the refractive index [16]. In addition, the highest acoustic signal frequency that can propagate into a liquid medium cannot exceed 50 MHz due to acoustic losses [17].

The medium refractive index n_o, which was initially constant, becomes $n(x,\ t)$. It varies according to the electrical signal $A(t)$. The obtained variation of the refractive index can be written as follows:

$$n(x,t) = n_0 + \Delta n\left\{\sin\left[\omega_a t - k_a x + \frac{\Delta f}{f_m}\sin(\omega_m t - k_m x)\right]\right\} \tag{5}$$

where n_0 is the average refractive index of the medium, Δn stands for the index variation amplitude due to the acoustic wave, k_a *and* k_m are the wave numbers of the carrier and the modulating wave respectively and x is the propagation direction of the acoustic wave.

It is clear from Eq. (5) that the refractive index varies in time and space. We note that for $\Delta f = 0$ the refractive index will be given without frequency modulation, as it was given in previous references [1,19–21].

The acoustic wave propagation in the transparent medium provides a moving phase grating which may diffract portions of an incident laser beam into one or more directions. Under Raman-Nath regime, the output laser beam field $E_o\ (x,\ z,\ t)$ can be written as [3,21,22]:

$$E_o(x,z,t) = E_0 \exp j(\omega_0 t - k_0 z)\exp -jk_0 L\cdot n(x,t)$$

where E_0, k_0 and ω_0 are the amplitude, the wave number and the pulsation of the incident laser beam respectively, z represents the laser beam propagation direction and L is the interaction length.

At a certain distance from the AO cell, the diffracted light field can be described in the plane (X, Y) by the following relationship [9,10]:

$$\begin{aligned} E(X,t) = {} & E_0\cdot\frac{\exp(jk_0 z)}{j\lambda_0 z}\cdot\exp j\left(k_0\frac{X^2}{2z}\right) \\ & \cdot\exp j\left(\omega_0 t - k_0 z - \frac{2\pi n_0 L}{\lambda_0}\right)\cdot\sum_{p=-\infty}^{+\infty}\exp -jp(\omega_a t) \\ & \cdot J_p\left(\frac{2\pi L\Delta n}{\lambda_0}\right)\cdot\exp -jp\frac{\Delta f}{f_m}[\sin(\omega_m t)] \\ & \cdot\delta\left[\frac{X_p(t)}{\lambda_0 z} - \frac{p}{\lambda_a} - p\frac{\Delta f}{V}\cos(\omega_m t)\right] \end{aligned} \tag{6}$$

where $J_p(\Psi = 2\pi L\Delta n/\lambda_0)$ is the p^{th} order Bessel function of the first kind and of parameter Ψ [3], V is the ultrasonic wave velocity in the medium and δ represents the Dirac function.

Starting from Eq. (6) and using the Dirac function definition (Appendix A), we deduce a very interesting relationship of the diffraction orders:

$$\frac{X_p(t)}{z} = p\frac{\lambda_0\cdot f_a}{V} + p\frac{\lambda_0\cdot\Delta f}{V}\cos(\omega_m t)$$

$$\Rightarrow \theta_p(t) \approx \tan[\theta_p(t)] = \frac{X_p(t)}{z} = \theta_{pmed} + \Delta\theta_p\cos(\omega_m t) \tag{7}$$

With $\Delta\theta_p$ is the angular excursion and $\theta_p(t)$ represents the diffraction angles for $p = 0, \pm1, \pm 2, \pm3\ldots$.

- This equation contains two parts; a constant part $\theta_{p\ med}$ which represents the angle of p^{th} diffracted order without modulation as shown in Fig. 1a and the second one, which depends on time, describes theoretically the diffracted orders deflection around a central position $\theta_{p\ med}$ of the scanned area as presented in Fig. 1b.
- It is obvious, from Fig. 1b, that the diffracted orders positions vary in sinusoidal manner with time, where ($T_m = 1/f_m$) represents the period of the modulating signal as presented in Eq. (2). In addition, the angular excursion of each diffracted order $\Delta\theta_p$ depends on two parameters; the diffracted order number p and the frequency excursion Δf as indicated in the following relationship:

$$\Delta\theta_p = p\cdot\frac{\Delta f\cdot\lambda_0}{V} \tag{8}$$

3. Experimental setup and procedure

Details of the experimental setup are given in Fig. 2.

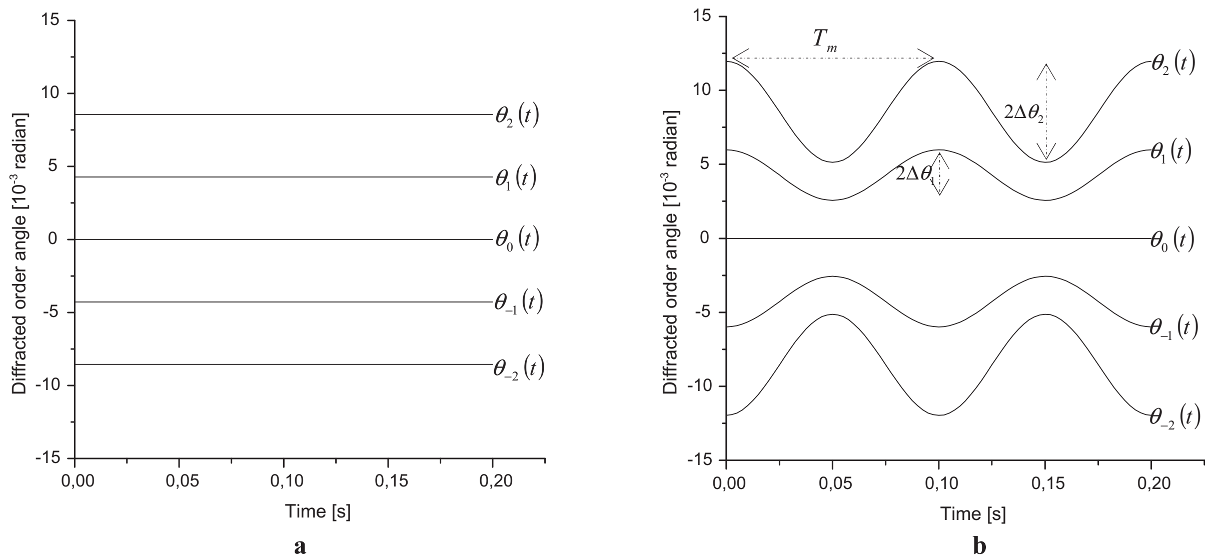


Fig. 1. Diffracted orders angles for $p = 0, \pm1, \pm\,2$ as a function of time for two cases: (a) Ultrasonic wave without modulation ($\Delta f = 0$) and (b) Ultrasonic wave is frequency modulated ($\Delta f \neq 0$).

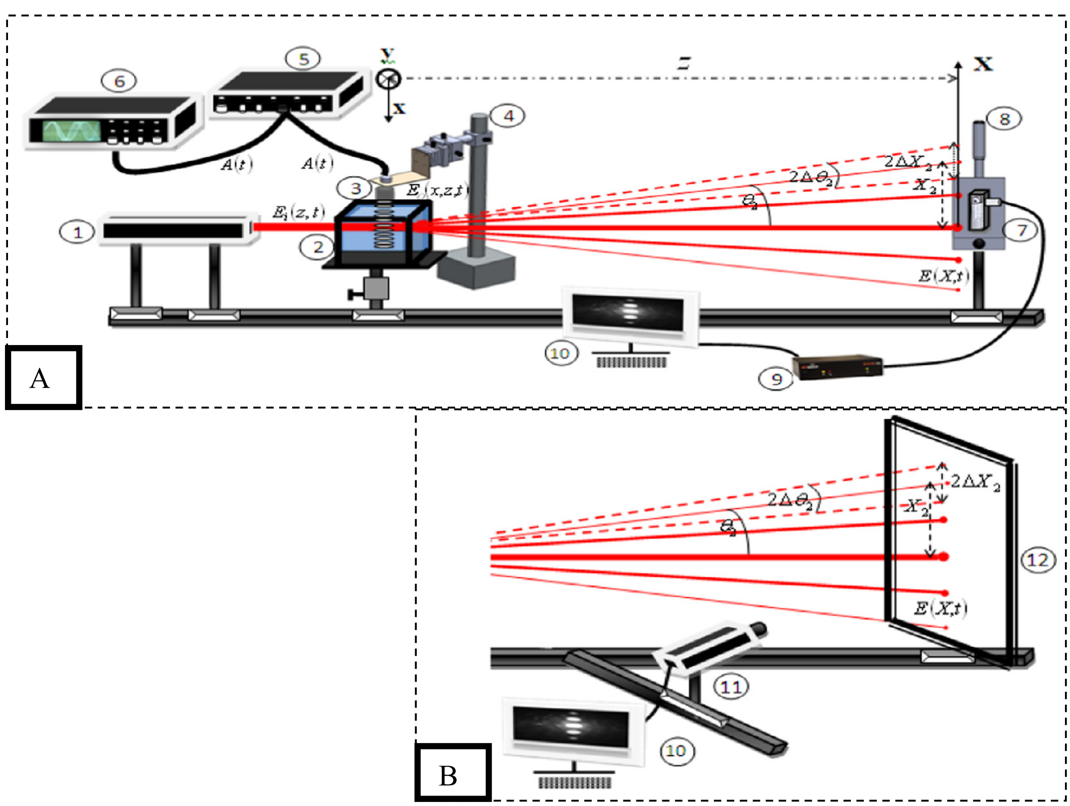
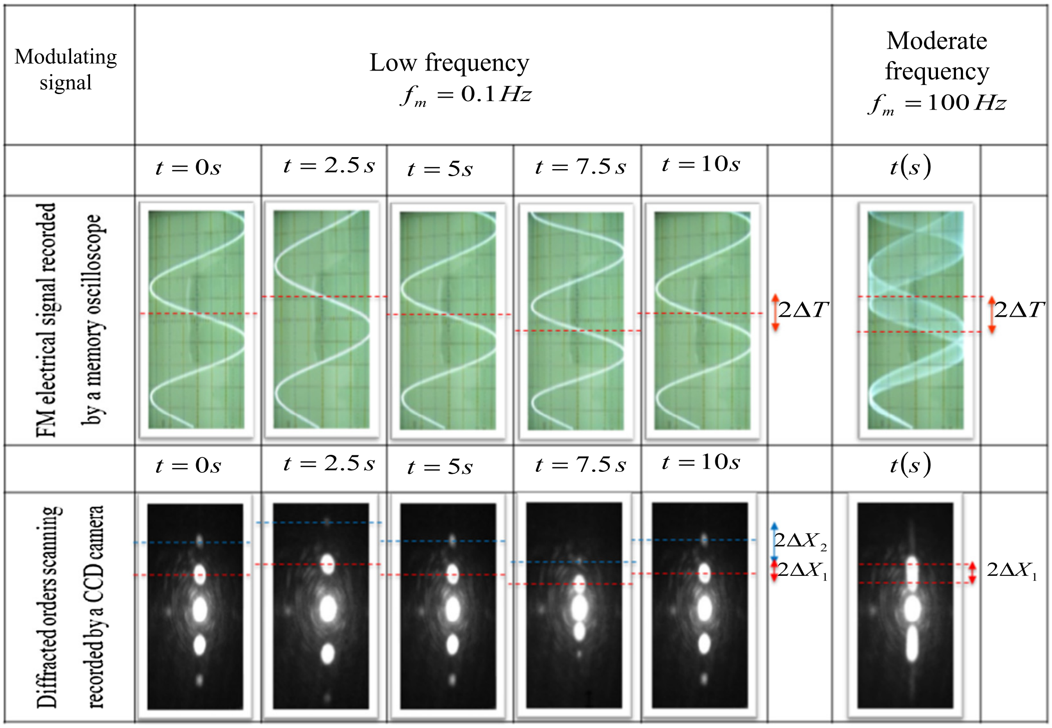


Fig. 2. Experimental set-up.

The experimental setup consists of: 1- He-Ne laser source (output power 30 mW at $\lambda_0 = 632.8$ nm) 2- Parallelepiped AO cell, 3- A circular piezoelectric transducer (Panametrics INC, with 19 mm in diameter and with $f_r = 10$ MHz as a resonance frequency), 4- A transducer holder. 5- A frequency generator (FI 5500GA) with maximum frequency $f_{a\ \max} = 25$ MHz, $U_{max} = 20$ V and a modulation frequency $f_{m\ \max} = 20$ kHz, 6- An oscilloscope (Philips) with a maximum detectable frequency $f_{max} = 80$ MHz, 7- An ultrafast photodiode (UPh) with a detection specter ranging from $\lambda = 170$ to 1100 nm, 8- A UPh holder, 9- An acquisition card, 10- A Computer, 11- A CCD Camera with resolution 1034×779 pixel and a pixel size equals 4.65 μm × 4.65 μm), 12- Screen.

The presented optical arrangement allows us to study the acousto-optical interaction using a frequency modulated acoustical signal. An He-Ne laser beam illuminates in the z direction a progressive ultrasonic wave inside a parallelepiped AO cell made of transparent glass and filled with distilled water. A frequency modulated (FM) ultrasonic wave is generated by a piezoelectric circular

Table 1
Presentation of FM electrical signals and their corresponding diffracted orders scanning for low and moderate frequencies f_m.



transducer made of $LiNbO_3$ and driven by a variable frequency generator, where the carrier signal is sinusoidal and presents a high frequency f_a equals 10 MHz and a maximum amplitude U equals 20Vpp. The modulating signal in turn, presents a variable frequency f_m .We assume, like Raman and Nath that ultrasound can act as a pure phase grating. To ensure progressive wave propagation, an absorbing material is used in the cell bottom. Vibrations of the piezoelectric transducer caused variations of the refractive index of the medium inside the AO cell. Hence, so the distribution of the refractive index is expressed by Eq. (5).

Having left the AO cell, the intensity of the diffracted light in the far field can be observed at a position z equals 4 m, as shown in Fig. 2. The acquisition card which is connected to the computer and to the ultrafast photodetector (UPh), as shown in Fig. 2A allows us to obtain the scanning frequency of each diffracted order. Whereas its scanning excursion is recorded by a CCD camera as illustrated in Fig. 2B. The obtained figures of the electrical signals and their respective deflected orders are presented in Table 1 for different modulating frequencies.

From the obtained images, one can observe that the increase of the modulating signal frequency leads to a faster variation of the instantaneous period, as well as the diffracted orders scanning frequency. This can be displayed on the spectral plane by a luminous band.

4. Results and discussion

In order to check the proposed theoretical development, a series of experiments have been conducted: the first one consists of observing the influence of a modulating signal frequency f_m on the scanning frequency f_S for each diffracted order, the second consists of observing the influence of the modulating signal frequency f_m and the frequency excursion Δf on the diffracted order angular excursion $\Delta\theta_p$.

The perpendicularity between the ultrasound field and the light beam must be kept at an exact value during the experiment. For this, two goniometers, with an angular resolution of 12′, are placed in two perpendicular directions (y and z) to monitor the angular position of the piezoelectric transducer. The perpendicularity is checked at maximum diffraction efficiency. In addition, the UPh is mounted on a holder with two dimensionally moving benches with a step of 10 μm, along x and y directions.

4.1. Influence of the modulating signal frequency f_m on the scanning frequency f_S for each diffracted order

The experimental setup shown in Fig. 2 is carried out in order to observe the effect of the modulating signal frequency f_m on the scanning frequency f_S for each diffracted order ($p = \pm 1, \pm 2$).

First, we start the experiment by feeding the transducer with an electrical signal without modulation. The UPh is located at a distance z equals 4 m from the parallelepiped AO cell and exactly on the diffracted order position. Then, the previous electrical signal is frequency modulated with a frequency excursion of $\Delta f = 2$ MHz and a modulating signal of $f_m = 100$ Hz. The refractive index inside the AO cell takes consequently the form given by Eq. (5). Each diffracted order will deflect around its central position $\theta_{p\ med}$. The obtained signal of the first diffracted order intensity, recorded by the UPh, is presented in Fig. 3.

What happens practically is that the diffraction order oscillates around a central position with a given period Ts = 1/fs. When the UPh is placed in the central position of the scanned area, it would detect three peaks for two sweeps of the excursion range. The measurement time between the first and the third peak corresponds to the scanning period ($T_S = 10$ ms).

In order to check the exact scanning frequency value of first diffracted order, many measurements have been conducted for different frequencies f_m and for each frequency three UPh positions on X

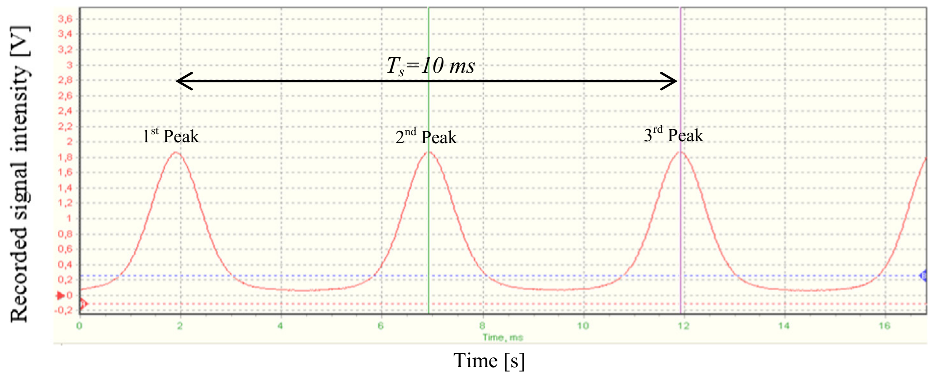


Fig. 3. Recorded signal intensity of the first diffracted order vs time, with an UPh position in the central position of the scanned area.

Table 2
The scanning frequency values f_s of the first diffracted order for different positions of the UPh.

The modulating signal frequency of the generator f_m [Hz]	Measured scanning frequency f_S [Hz]		
	θ_{1max}	θ_{1min}	θ_{1med}
0.10	0.098	0.097	0.099
1.00	0.970	1.050	1.000
10	9.888	10.010	9.780
100	105.000	103.500	100.000
1000	1000.000	1040.000	1040.000

axis have been chosen: the highest reached position of the deflected order (θ_{1max}), the lowest (θ_{1min}), and the medium (θ_{1med}) one. The Table 2 presents the obtained values.

It is clear from the obtained results that the scanning frequency ($f_S = 1/T_s$) is very close to the modulating signal one f_m and the relative difference between the two frequencies varies from 0 to 5%. We note that, the same results were recorded for the remaining diffracted orders.

To explain theoretically the obtained results, it is sufficient to consider Eq. (7). A double excursion $2(\theta_{pmax} - \theta_{pmin})$, takes place for time t = $1/f_m$. This means that, theoretically, the scanning frequency f_s is exactly equal the modulation frequency f_m, which is observed experimentally in Table 2.

4.2. Influence of the modulation frequency f_m on the angular excursion of the diffracted order

This experiment has been conducted in order to observe the influence of the modulating frequency f_m on the angular excursion $\Delta\theta_p$. For this purpose the same electrical signal without modulation, previously used, has been employed to measure the medium angle θ_{1med} using a micro displacement of the UPh holder. Then, we choose Δf = 2 MHz and measure the maximum and the minimum reached angles of the first diffracted order θ_{1max} and θ_{1min} respectively, for different values of f_m. Fig. 4a illustrates the obtained average values of the deflected angles with their standard deviations. The same experiment was done for the remaining diffracted orders as indicated in Fig. 4b. It will be noted that the theoretical curves are plotted using Eq. (7) for an acoustic velocity V equals 1488 m/s.

From the obtained curves presented in Fig. 4, one can observe that the experimental results are very close to theoretical ones. In addition, the maximum and the minimum deflection angles

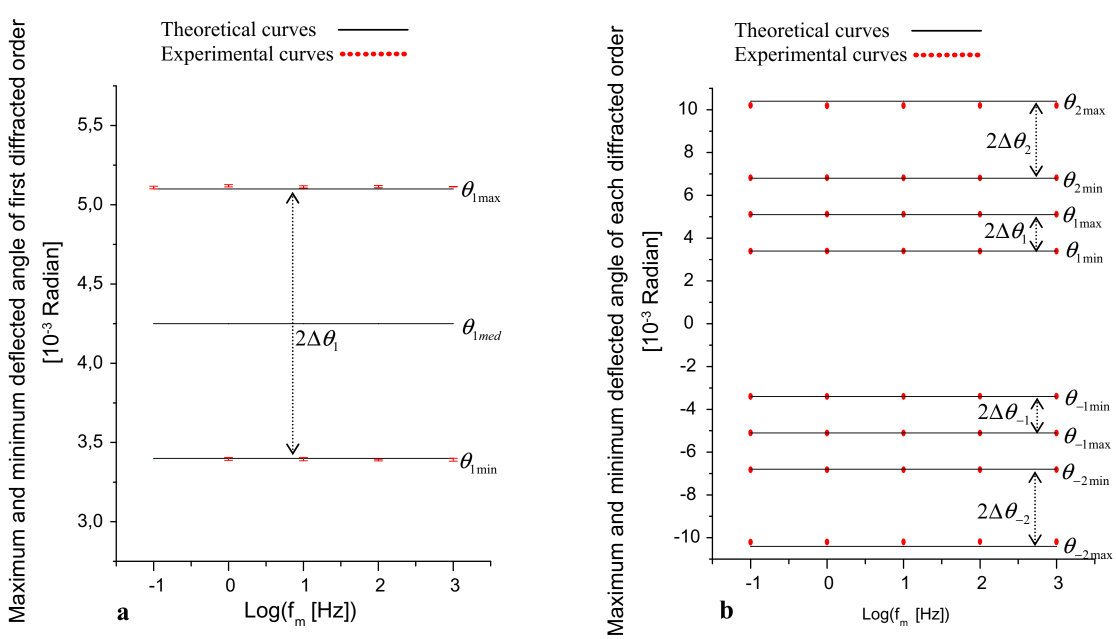


Fig. 4. The angular excursion $\Delta\theta_p$, the maximum and minimum deflected angles according to the modulating signal frequency f_m. (a) For the first diffracted order, (b) for each diffracted order.

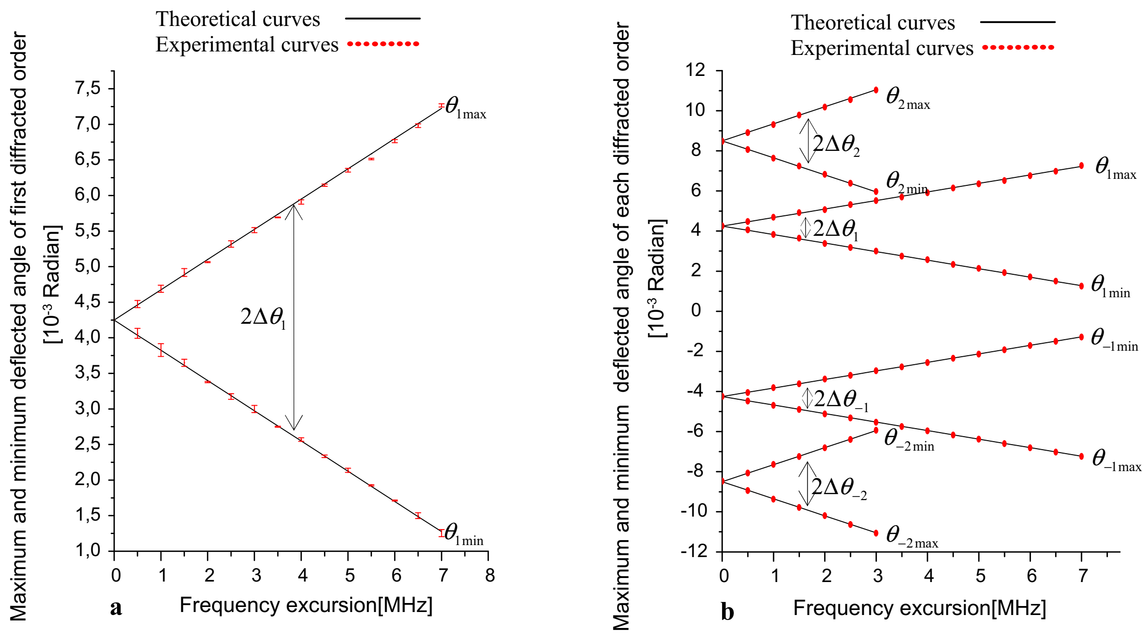


Fig. 5. Angular excursion $\Delta\theta_p$, the maximum and minimum deflected angles as a function of the frequency excursion Δf. (a) For the first diffracted order, (b) for each diffracted order.

are symmetrical to a central position θ_{1med} of the diffraction order. Moreover, the angular excursion $\Delta\theta_p$ doesn't depend on the variation of the modulating signal frequency f_m. Furthermore, the angular excursion of the p[th] diffracted order is p times the first diffracted order one, as presented by the following relationship:

$$\Delta\theta_p = p(\lambda_0 \Delta f / V) = p \cdot \Delta\theta_1$$

4.3. Influence of the frequency excursion Δf on the angular excursion of the diffracted order

To conduct this experiment, the same previous optical arrangement was undertaken, except in this case the modulating frequency f_m is constant (f_m = 100 Hz) and the frequency excursion Δf varies by a step of 0.5 MHz and for each value, the angular excursion $\Delta\theta_1$ of the first diffracted order is measured. The obtained results are summarized in Fig. 5a. To generalize these results for the rest of the diffracted orders, the same experiment was undertaken. The obtained results are illustrated in Fig. 5b.

The obtained curves show evidently a large concordance between theoretical and experimental results. In addition, a linear relationship is observed between the frequency excursion Δf and the angular excursion $\Delta\theta_p$ of the diffracted orders, each pair of the curves for the same diffracted orders starts from a common point corresponding to an $\Delta f = \Delta\theta_p = 0$ (diffraction orders without deflection). The obtained linearity is clearly justified mathematically using Eq. (8) where $\Delta\theta_p$ and Δf are linearly related.

4.4. Determination of the frequency modulation index using acousto-optical method

In this experiment, a new method has been performed to determine with good accuracy a frequency modulation index of an FM signal. This parameter is generally obtained using an electronic spectrum analyzer [11].

The mathematical expression of the frequency modulation index is given by the following formula [11,18]:

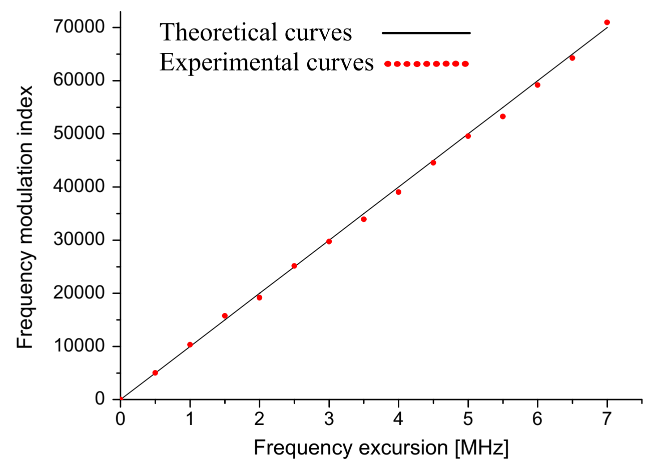


Fig. 6. Frequency modulation index variation according to the frequency excursion Δf for the first diffracted order.

$$\beta = \frac{\Delta f}{f_m}$$

Combining this theoretical formula and the relationship given by Eq. (8), the frequency modulation index can be rewritten as follows:

$$\beta = \frac{\Delta f}{f_m} = \frac{\Delta\theta_p \cdot f_a}{\theta_{p\,med} \cdot f_m}$$

This last relationship indicates that is possible to obtain the frequency modulation index β experimentally by a simple measurement of the angular excursion $\Delta\theta_p$ and the diffraction angle $\theta_{p\,med}$ (without modulation). The following figure will present the values of β obtained theoretically using the generator parameters (Δf and f_m), and experimentally using the values of $\Delta\theta_p$ and θ_p taken from previous experiment. The results are summarized in Fig. 6.

It is clear from the obtained values, that the experimental results of the frequency modulating index are very close to the values given by the rf generator, the maximum relative error doesn't exceed 5%. It should be noted that the frequency modulation index can also be obtained using the 2nd diffraction order but the bandwidth of the piezoelectric transducer will limit this measurement.

5. Conclusion

Acousto-optical deflectors (AOD) provide precise spatial controls of an optical beam. Whether performing 1D or 2D scanning or executing beam deflection through a fixed angle. The light beam deflection is obtained by modulating the acoustical signal frequency. Therefore, the angular excursion, the light beam scanning frequency and the frequency modulation index are important parameters in the study of the frequency modulation.

In our work a theoretical development of the acousto-optical interaction using a frequency modulated acoustical signal has been performed, followed by an experimental work to confirm the theoretical obtained results. During this work, we have confirmed that the scanning frequency f_s is very close to the modulating frequency f_m given by the rf generator. Furthermore, we have demonstrated that the angular excursion $\Delta\theta_p$ don't depend on the frequency modulation f_m, in the same time, it presents a linear relation with the frequency excursion Δf. In addition, we have noted that for the p^{th} diffracted order, the angular excursion $\Delta\theta_p$ is p times the first diffracted order one. Finally, we have explained theoretically and demonstrated experimentally the possibility of using the (AOD) to obtain the frequency modulation index β.

Acknowledgments

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Appendix A

The diffraction formula of Fraunhover is written as follows:

$$E(X,Y,t) = \frac{\exp(jk_oz)}{j\lambda_0 z}\cdot \exp j\left(k_0\frac{X^2+Y^2}{2z}\right)\cdot \iint E_0(x,y,t) \times \exp\left[-jk_0\left(\frac{X\cdot x+Y\cdot y}{z}\right)\right]dx\,dy$$

In our case, the laser beam diffraction is observed only in X direction so the previous integral can be simplified to one-dimensional integral as follows:

$$\begin{aligned}E(X,t) = {}& E_0\cdot\frac{\exp(jk_0z)}{j\lambda_0 z}\cdot \exp j\left(k_0\frac{X^2}{2z}\right)\cdot \exp j(\omega_0 t-k_0 z)\\ &\cdot \exp j\left(-\frac{2\pi n_0 L}{\lambda_0}\right)\cdot\int_{-\infty}^{+\infty}\\ &\times \exp\left(-j\frac{2\pi}{\lambda_0}L\Delta n\left[\sin\left(\omega_a t-k_a x+\frac{\Delta f}{f_m}\sin(\omega_m t-k_m x)\right)\right]\right)\\ &\cdot \exp\left[-jk_0\left(\frac{X\cdot x}{z}\right)\right]dx\end{aligned}$$

Employing the well-known Jacobi relation $\left(\exp[-j\Psi\cdot\sin(\alpha)] = \sum_{p=-\infty}^{+\infty}J_p(\Psi)\cdot\exp[-jp(\alpha)]\right)$ the previous equation becomes:

$$\begin{aligned}E(X,t) = {}& E_0\cdot\frac{\exp(jk_0z)}{j\lambda_0 z}\cdot \exp j\left(k_0\frac{X^2}{2z}\right)\cdot \exp j(\omega_0 t-k_0 z)\\ &\cdot \exp j\left(-\frac{2\pi n_0 L}{\lambda_0}\right)\cdot\int_{-\infty}^{+\infty}\sum_{p=-\infty}^{+\infty}J_p\left(\frac{2\pi L\Delta n}{\lambda_0}\right)\\ &\cdot \exp\left(-jp\left(\omega_a t-k_a x+\frac{\Delta f}{f_m}\sin(\omega_m t-k_m x)\right)\right)\\ &\cdot \exp\left[-jk_0\left(\frac{X\cdot x}{z}\right)\right]dx\end{aligned}$$

$$\begin{aligned}E(X,t) = {}& E_0\cdot\frac{\exp(jk_0z)}{j\lambda_0 z}\cdot \exp j\left(k_0\frac{X^2}{2z}\right)\cdot \exp j(\omega_0 t-k_0 z)\\ &\cdot \exp j\left(-\frac{2\pi n_0 L}{\lambda_0}\right)\cdot\sum_{p=-\infty}^{+\infty}J_p\left(\frac{2\pi L\Delta n}{\lambda_0}\right)\cdot\exp(-jp\omega_a t)\int_{-\infty}^{+\infty}\\ &\times \exp\left(-jp\left(-k_a x+\frac{\Delta f}{f_m}\sin(\omega_m t-k_m x)\right)-jk_0\left(\frac{X\cdot x}{z}\right)\right)dx\end{aligned}$$

By developing this integral and using the Dirac function definition we obtain:

$$\begin{aligned}E(X,t) = {}& E_0\cdot\frac{\exp(jk_0z)}{j\lambda_0 z}\cdot \exp j\left(k_0\frac{X^2}{2z}\right)\cdot \exp j\left(\omega_0 t-k_0 z-\frac{2\pi n_0 L}{\lambda_0}\right)\\ &\cdot\sum_{p=-\infty}^{+\infty}\exp -jp(\omega_a t)\cdot J_p\left(\frac{2\pi L\Delta n}{\lambda_0}\right)\cdot\exp -jp\frac{\Delta f}{f_m}[\sin(\omega_m t)]\\ &\cdot\delta\left[\frac{X_p(t)}{\lambda_0 z}-\frac{p}{\lambda_a}-p\frac{\Delta f}{V}\cos(\omega_m t)\right]\end{aligned}$$

As it is well known, the Dirac function is defined as follows: $\delta(\varepsilon)=\int_{-\infty}^{+\infty}\exp(-j2\pi\cdot\varepsilon)\cdot xdx$ where it is always equal to zero except for $(\varepsilon = 0)$, hence:

For the case where : $\varepsilon\neq 0 \Rightarrow \frac{X_p(t)}{\lambda_0 z}-\frac{p}{\lambda_a}-p\frac{\Delta f}{V}\cos(\omega_m t)\neq 0$

$$\Rightarrow \delta(\varepsilon)=0\Rightarrow E(X,t)=0$$

And if : $\varepsilon = 0 \Rightarrow \frac{X_p(t)}{\lambda_0 z}-\frac{p}{\lambda_a}-p\frac{\Delta f}{V}\cos(\omega_m t)=0\Rightarrow \delta(\varepsilon)\neq 0$

$$\Rightarrow E(X,t)\neq 0$$

In this last case, when the electrical field E(X, t) exists, a very interesting relation of the diffraction orders angles has been deduced as follows:

$$\frac{X_p(t)}{z}=p\frac{\lambda_0\cdot f_a}{V}+p\frac{\lambda_0\cdot\Delta f}{V}\cos(\omega_m t)$$

$$\Rightarrow \theta_p(t)\approx\tan[\theta_p(t)]=\frac{X_p(t)}{z}=\theta_{pmed}+\Delta\theta_p\cos(\omega_m t)$$

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